Comment: Harold Hotelling's Views on Statistics

Ralph A. Bradley

I. ARTICLES ON TEACHING

Two, well known articles of Harold Hotelling, "The teaching of statistics" (1940) and "The place of statistics in the university" (1949), have been reprinted above. Several of us have been asked to comment, but, before so doing, it may be helpful to provide some background for these papers and to note some remarks by the original discussants.

Among the leaders in statistics of the forties were Harold Hotelling, Jerzy Neyman, W. Edwards Deming, Burton H. Camp and S. S. Wilks. In 1940, the first four constituted the Committee on the Teaching of Statistics of the Institute of Mathematical Statistics (IMS) with Hotelling as chairman, while S. S. Wilks was both president of the IMS and editor of its journal, *The Annals of Mathematical Statistics*. Hotelling, as a good committee chairman, had drafted the first of the two articles as a position paper for the committee and presented it at an IMS (and mathematical societies) meeting at Dartmouth College in September 1940, 42 IMS members attending.

Olkin, Ghurye, Hoeffding, Madow and Mann (1960) edited a collection of essays in honor of Hotelling on his sixty-fifth birthday. The one Hotelling paper reprinted in the volume was "The teaching of statistics." Neyman (1960) provided a tribute to Hotelling and, in an attempt to explain "the remarkable growth of research in the theory of statistics," he stated

> "However, two single factors seem to dominate all the others. They are the educational and organizational activity of Hotelling (and of a few others such as S. S. Wilks), marked by an outstanding event in 1940, and the appearance in 1946 of an excellent book by Harald Cramér."

Later in his remarks, Neyman noted in reference to the Dartmouth meeting that "Hotelling's paper was received with enthusiasm and, by a unanimous vote, the audience decided to have it published in the *Annals* as an expression of IMS opinion on the matter."

The outstanding event in 1940 was, of course, the appearance of the Hotelling paper reprinted here. The

Ralph A. Bradley is Research Professor, Department of Statistics, University of Georgia, Athens, Georgia 30602.

unanimous vote may have occurred at an IMS business meeting when three resolutions drafted by the Committee on the Teaching of Statistics (1940) were adopted.

The history of the second reprinted article above, "The place of statistics in the university" (Hotelling, 1949), is similar to that of the first article. It was prepared by Hotelling, presented at the 1945/46 Berkeley Symposium, and published with comments of discussants in the proceedings of the symposium in 1949. In the meantime, a further report of the IMS Committee on the Teaching of Statistics (1948) appeared in the *Annals*. (Hotelling remained chairman of the committee; Neyman and Camp had been replaced by Walter Bartky, Milton Friedman and Paul Hoel.) Part II of this report was a condensation attributed to Deming and Friedman of Hotelling (1949), while Part I was described as a summary of conclusions.

There are other Hotelling articles relating to the teaching of statistics and his views of the discipline. The interested reader should consult the bibliography provided by Smith (1978). Some brief comments on several of these articles follow. Hotelling (1930) reported on a visit to Britain and on British statistics and statisticians. In his report he ended the speculation of American students on the identity of "Student," ruling out such guesses as E. S. Pearson and the Prince of Wales, and first showed his interest in distributions of standard statistics under nonstandard assumptions, a topic of his own later research and of the present author's dissertation directed by Hotelling. Hotelling's (1941) address to the Indian Statistical Congress, although short, exemplified his command of language and views of statistics:

> "The chaste beauty and intellectual delights of the theory of statistical inference, regarded as the intellectual offspring of mathematics and inductive logic, are known at present only to a few devotees; but this theory is bound in time to receive a wider appreciation and a higher valuation even apart from its practical usefulness in the form of applications.... An essential part of the development of statistics should be a close attention and a high regard for the mathematical and logical foundations. It is only in this way that

proper teaching of statistics, sound practice, and continued progress can be insured."

The training of social scientists was of interest also to Hotelling (1950) and he provided information (1944, 1948) on the graduate programs that he had developed, first at Columbia.University and then at the University of North Carolina.

II. VIEWS ON STATISTICS

A summary of the high points in Hotelling's two papers would assist discussion. However, summarization is not easy because Hotelling touches on many issues associated with the campus teaching and organization of statistics. Hotelling showed remarkably keen insights into how the teaching of statistics had evolved, the training of teachers of statistics and the requirements of good teaching. The situations described and the policy issues involved strike home to all who have grappled with such issues and the two papers should be required reading for all who consider academic leadership roles in statistics.

One summary of Hotelling (1940) is available in the form of the first two resolutions of the Committee on the Teaching of Statistics (1940), the third resolution simply relating to publicizing the first two. The two resolutions are:

"1. If the teaching of statistical theory and methods is to be satisfactory, it should be in the hands of persons who have made comprehensive studies of the mathematical theory of statistics, and who have been in active contact with applications in one or more fields."

"2. The judgment of the adequacy of a teacher's knowledge of statistical theory must rest initially on his published contributions to statistical theory, in contrast with mere applications, in a manner analogous to that long accepted in other university subjects."

The two resolutions do not do justice to the article. Doubtless the committee attempted to draft other resolutions without obtaining adequate focus. There is a wealth of interesting detail on the increase in the teaching of statistics, the demand for statisticians, the role of statistics in research in listed areas, the importance of statistics in the economic control of quality of manufactured articles, positions for statisticians in government, the inefficiency of the proliferation of elementary statistics courses, the need for statistics in legal matters, and concerns for training in mathematics. There is discussion also on who should teach statistics, the training of statisticians and the evaluation of teachers of statistics.

In the second article, Hotelling (1949), there is further development of ideas in the first and, as one would expect, new emphasis on the place of statistics in the university. The table of contents of Part II of the Report of the Committee on the Teaching of Statistics (1948) provides a skeleton summary of the article. We choose to selectively elaborate on some topics.

Hotelling argued cogently against the teaching of statistical method in subject matter departments on the grounds that when many variations of the same topics are taught by various departments, examples are taken only from the subject matter of the department, students seeking additional training get repetitions of topics while there is a shortage of advanced courses and a plethora of elementary ones, some classes are too large and others too small, some departments have statistical laboratories with computing facilities but others have none, and library holdings are scattered and in disarray. Even worse, he noted that teachers are not specialists in statistics, lack the knowledge of mathematics that would permit them to learn more, do not keep up with progress and teach wrong theories and inefficient methods with the result that obsolete ideas and unsound methods are perpetuated. Hotelling saw the reasons as (1) an urgent demand for the teaching of statistics, (2) a confusion between statistical method and applied statistics, (3) lack of understanding of the need for research by teachers of statistics:

"Anyone who does not keep in active touch with this research will after a short time not be a suitable teacher of statistics. Unfortunately, too many people like to do their statistical work just as they say their prayers—merely substitute in a formula found in a highly respected book written a long time ago."

and (4) the selection of teachers of statistics by individuals without knowledge of the subject. Another problem noted was that probability was seen as a tricky subject, and one to be avoided, by many teachers, thus limiting teaching to descriptive statistics and avoidance of new statistical methods based on small sample distribution theory.

Note that Hotelling sharply differentiated between statistical method, "a coherent, unified science," and applied statistics, "any of thousands of diverse things." Further, he emphasized the need to participate in research and to publish its results whenever an opportunity arose. Hotelling was fair and admitted the existence of exceptions. He saw that some departments sought advice on statistics appointments and "It is in this way that some of the present leaders in statistics have developed." He admitted also that

"What is more surprising is that the cookbook methods and shallow theoretical grounding provided in these courses, which constitute the most common type, are as useful as they are."

This section concludes with some notes on the training of statisticians and the place of statistics in the university. Other important issues were discussed—expectations of a professor of statistics, the consulting obligations of the statistician and the notion of a statistical consulting organization.

The best paragraphs on the training of research workers and teachers of statistics come from Part I of the 1948 committee report. Hotelling's participation in their construction is evident. We quote:

"The future research workers and teachers of statistical method clearly require far more intensive training in theory than has so far been suggested. A fundamental prerequisite to such training is knowledge of some advanced mathematics. It is difficult to specify exactly what or how much mathematics is necessary, but something of the algebra of matrices and of the theory of functions are minimum necessities, and a good deal of additional knowledge of algebra, geometry, and analysis add richness and power to the work of the statistical theorist."

"In addition to advanced mathematics and advanced work in statistical method, the future statistical theorist needs a good deal of work on applications, in the form either of experience or courses. He will be a toolmaker and needs to know by personal experience something of the problems of those who use his tools."

Some problems were seen in this training. Hotelling compared precollege training in mathematics in Europe and America and found it relatively deficient in the latter. He felt that much of the responsibility could be placed on teachers' colleges with too many courses on pedagogy and too few on subject matter. College curricula in mathematics were consequently weak also and most graduate students of statistics needed much concentration on pure mathematics with the result that only the few with good mathematical training could find time for training in applications of statistics. Hotelling (1949) had a caution also on the training in applications:

"... Nevertheless, the toolmaker must not put all his time on using the tools he makes; mostly he should work at making the tools. For him the interest is only secondary in the product of the tools; the main focus of his attention is the tools themselves. So it must be with the academic statistician."

After considerable comment on possibilities, Hotelling (1949) summarized as follows:

"The teaching of statistics may be organized in any of the following ways:
1. In a two-department Institute of Statistics of the kind suggested above.
2. In a single Department of Statistics.
3. Under an interdepartmental committee.
4. Under the exclusive jurisdiction of the Department of Mathematics.
5. It may as at present be disorganized among a heterogeneous group of departments of application."

It is apparent that the list was in preference order. The notion of an institute with two departments must have stemmed from the plans of Gertrude Cox for the consolidated University of North Carolina, plans that were approaching fruition in 1945–1946 when Hotelling presented his paper at the Berkeley symposium. The Cox plan was for a Department of Mathematical Statistics at the University of North Carolina in Chapel Hill and for a Department of Experimental Statistics at North Carolina State College in Raleigh. The department heads and associate directors of the Institute were to become, respectively, Harold Hotelling and William G. Cochran; perhaps use of the adjective "experimental" rather than "applied" in Raleigh was fortunate. It was clear that only a few large institutions could adopt the institute plan. The interdepartmental committee was regarded as makeshift and Hotelling argued that jurisdiction of a department of mathematics was unsatisfactory. Clearly, he judged the formation of the single department of statistics most likely and desirable in most institutions and this is what has occurred in the better universities through the years.

With the establishment of some central organization of statistics, Hotelling recommended that it be responsible for two different fundamental courses for undergraduates. One of these courses was to require calculus as a prerequisite and the other, first year algebra. He hoped that as calculus became a high school subject, the second course would become extinct. It was also his hope that students would begin statistics training early in order to benefit from this training in other courses. More advanced statistics courses were to be available, some of them courses in applied statistics special to particular departments. Today, courses in statistics are required of undergraduate majors in some disciplines. Why are they allowed to delay the statistics course until the final term of their senior year? Why do few if any of the major discipline courses utilize the statistics training required? The second fundamental course is not extinct!

III. COMMENTS

W. Edwards Deming (1940) had the first opportunity for comment on Hotelling's presentation. The Deming discussion reveals elements of current Deming philosophy for improving quality and productivity (see Mann, 1985). He endorsed Hotelling's recommendations, noting that they were following them closely at the Graduate School of the Department of Agriculture. However, he thought that other phases of statistics needed emphasis also and pointed out the value of descriptive statistics and the importance of statistical control:

> "Most students do not realize that for purposes of prediction the consistency or lack of it between many small samples may be much more valuable than any probability calculations that can be made from them or from the entire lot. Students are not usually admonished against grouping data from heterogeneous sources. Of those that are not guilty of indiscriminate grouping, many are inclined to rely on statistical tests for distinguishing heterogeneity rather than on a careful consideration of the sources of the data. Too little attention is given to the need for statistical control, or . . . too little attention is given to the interpretation of data that arise from conditions not in statistical control."

Deming did not value small sample distribution theory as much as Hotelling and believed that "modern 'theories of estimation' are not theories of estimation at all, but rather theories of distribution . . . ." Nevertheless, he agreed that they were ultimately essential to proficiency in statistics and that the university was the place for such study. On statisticians, Deming said "Above all, a statistician must be a scientist. The skepticism of many first class scientists of today for modern statistical methods should be a challenge to statistical teaching . . . ."

We have already quoted from Neyman (1960) on Hotelling's role in the growth in quantity and quality of statistics research over 20 years—in Neyman's analogy, the creation of an epidemic. Neyman reported that there were two major obstacles to the development of research in statistics, opposition from mathematicians and resistance from representatives of sciences in which statistical methods were frequently used, the "mathematical front" and the "applied front." Neyman asserted that Hotelling had been victorious on both fronts but then admitted that some clean-up operations were still needed on the applied front.

Hotelling's successes on the mathematical front occurred because of the excellence of his own research and that of the substantial number of people that he recruited to statistics. Both the Department of Mathematical Statistics in Chapel Hill and the graduate curriculum that he developed became models for other departments. New specializations in statistics, for example, sequential analysis, decision theory, nonparametric statistics and Bayesian inference, were integrated into the theory of statistics. The discipline of statistics had achieved new visibility and acceptance.

The applied front may have turned into guerrilla warfare after some initial success. Hotelling had noted that "One of the chief obstacles to efficient organization of teaching is the habit of not prescribing prerequisites outside one's own department." The advent of enrollment-driven university budgets in the early seventies, although not applied strictly at the department level, made concerned departments strive to demonstrate high productivity; one means in subject matter departments was to introduce new applied statistics courses and to vigorously compete for students with old ones.

Other problems have arisen. Mathematical statistics has become more and more theoretical and abstract and young statisticians have had fewer experiences in applications of statistics. Hotelling knew that "statistics is an art as well as a science." Many teachers have not mastered the art even if they have mastered the theory. Recommendation 1 of 1940 is still appropriate but cannot mean that the teaching of statistical methods should be in the hands of research mathematical statisticians.

Hotelling deplored the state of training in mathematics but further deterioration has created a national emergency. (See the 1984 report of the Ad Hoc Committee on Resources for the Mathematical Sciences—the "David Report.") Neyman's "susceptibles" for training in statistics and the "epidemic" of research in statistics have become susceptibles for training in computer science, business administration, engineering and so on. Graduate enrollments in statistics are down and the proportion of foreign graduate students approaches 50%.

Neyman arranged for several discussants from areas of application of statistics to comment on Hotelling (1949). They were not prepared to turn the teaching of statistical methods over to statisticians, noting, for example, that statisticians would not understand the observational difficulties in their fields. Hotelling's suggestion that two basic courses be taught in the department of statistics is useful in this situation. A truce on the applied front often may be reached through the compromise agreement that the basic course (or course sequence) be taught in the statistics department although the subject matter department teaches the applied statistics peculiar to and needed by the subject matter discipline.

Dempster (1949) raised a more fundamental issue in his discussion of Hotelling's paper:

"... it is confusion concerning a logical framework that constitutes the basis of poor teaching, of the use of methods that are not applicable, and of the interpretation of results to mean what cannot be known."

It was also noted that theorists will not enjoy teaching statistical methods and hence will not do it well. It is perhaps not surprising that we have not agreed on a logical framework for statistics. Statistics is a key part of inductive inference and the philosophy of science. Responsible statisticians should search for truth and understanding but not denounce their colleagues with contrary views to the detriment of the discipline.

The discipline of statistics has problems, but they stimulate innovation and research. Statistics plays a big role in the scientific method and must survive, even though statistics and statisticians still strive for recognition and understanding. There is current concern about the public image of statistics and the American Statistical Association is planning an active public information program.

Much has been written on the teaching of statistics. The reader wishing to read additional articles may consider Rustagi and Wolfe (1982), a conference proceedings volume. Part 1, Graduate Programs in Statistics, will be of special interest; Geisser (1982) and Bradley (1982a) both made major references to Hotelling (1940). Bradley (1982b) may be of interest as he discusses the future of statistics as a discipline and Minton and Freund (1977) and Minton (1983) address the organization of statistics in the university and the visibility of statistics as a discipline, respectively. Minton's argument is that public awareness follows academic recognition and that the latter is dependent on viable undergraduate programs.

IV. HAROLD HOTELLING

The Editor has suggested inclusion of some personal reminiscences on Hotelling. We believe that he was a person of charm, with a warm feeling toward others and an advocate of freedom and human dignity. We believe also that he was totally convinced of the importance of intellectual activity and discovery and that he was often so engrossed as to be unaware of needs of others. Hotelling surely had one of the great minds of his age, a rare combination of logical ability and near photographic memory.

Hotelling would astound graduate students by his ability to provide from memory exact references to his published papers and those of others, including page numbers and whether or not the article began on a left-hand or right-hand page. At the famous Hotelling teas, he would discuss at length topics suggested by the general conversation, for example, the history of chess onward from Genghis Khan. At one stage, he became convinced that Laguerre polynomials had a role in the theory of statistics, a solution awaiting a problem. On a number of occasions, he responded to student questions with his own question, "Have you tried Laguerre polynomials?"; a not entirely appropriate question when one student was informing him about his pending marriage. Hotelling often started to walk from his home down the hill at the north end of Franklin Street in Chapel Hill to the University. This was on the author's route also and Hotelling never refused an offered ride. We were never able to determine whether he desired the ride or was too polite to refuse.

In March 1951, the Hotellings were our guests in Blacksburg, Virginia on the occasion of an IMS meeting at the Virginia Polytechnic Institute. The author, chairing his first session ever, was determined that speakers adhere to time allotments and had difficulty with R. C. Bose, speaking on partially balanced incomplete block designs. Bose finally stopped but complained that, if he didn't continue, someone would ask a question necessitating the information. Unswayed, the chairman persisted, only to be thrown into total retreat when Harold Hotelling interrupted and said, "Please consider the question asked." At the time of this visit, we were expecting our first child and Sue Hotelling was convinced that this would be a very special child, an attractive theory to the future parents. On checking that no help was expected from relatives in Canada, she insisted that she be called and that she would visit for a week or so to help out at the appropriate time. Such were the concerns of the Hotellings, an example of their warmth and love.

Considerable biographical information is available on Harold Hotelling. We suggest reading Anderson (1960), Hoeffding (1978), Levene (1974), Madow (1960), Samuelson (1960) and Smith (1983). Smith (1978) quoted from Samuelson's 1970 Nobel Prize acceptance speech: "Economics ... has its heroes, and the letter H that I used in my mathematical equations was not there to honor Sir William Hamilton, but rather Harold Hotelling." Graduate students in Chapel Hill without any disrespect quickly began to refer to Hotelling behind his back as H^2, appropriate for the obvious reason and going along with T^2, the generalized Student statistic.

The reprinting of the two Hotelling papers should serve dual purposes. The first is to bring to the attention of those who did not know him the very significant leadership role that he played in the formative years of modern statistics. The second is to bring new attention to problems that concerned him and require continued and renewed attention today.

REFERENCES

AD HOC COMMITTEE ON RESOURCES FOR THE MATHEMATICAL SCIENCES (1984). *Renewing U. S. Mathematics—Critical Resource for the Future.* National Research Council Report. National Academy Press, Washington.

ANDERSON, T. W. (1960). Harold Hotelling's research in statistics. *Amer. Statist.* **14** 17–21.

BRADLEY, R. A. (1982a). Graduate programs in statistics—a discussion. In *Teaching of Statistics and Statistical Consulting* (J. S. Rustagi and D. A. Wolfe, eds.) 81–91. Academic, New York.

BRADLEY, R. A. (1982b). The future of statistics as a discipline. *J. Amer. Statist. Assoc.* **77** 1–10.

COMMITTEE ON THE TEACHING OF STATISTICS (1940). Resolutions on the teaching of statistics. *Ann. Math. Statist.* **11** 472.

COMMITTEE ON THE TEACHING OF STATISTICS (1948). The teaching of statistics. *Ann. Math. Statist.* **19** 95–115.

CRAMÉR, H. (1946). *Mathematical Methods of Statistics.* Princeton Univ. Press, Princeton, N. J.

DEMING, W. E. (1940). Discussion of Professor Hotelling's paper. *Ann. Math. Statist.* **11** 470–471.

DEMPSTER, E. R. (1949). Untitled discussion. *Proc. Berkeley Symp. Math. Statist. Probab.* 45–47. Univ. California Press.

GEISSER, S. (1982). Observations on graduate programs in statistics and related issues. In *Teaching of Statistics and Statistical Consulting* (J. S. Rustagi and D. A. Wolfe, eds.) 21–33. Academic, New York.

HOEFFDING, W. (1978). Hotelling, Harold. *International Encyclopedia of Statistics* (W. H. Kruskal and J. Tanur, eds.) **1** 439–441. The Free Press, New York.

HOTELLING, H. (1930). British statistics and statisticians today. *J. Amer. Statist. Assoc.* **25** 186–190.

HOTELLING, H. (1941). Presidential address (to Indian Statist. Congress at Madras). *Sankhyā* **5** 127–129.

HOTELLING, H. (1944). Graduate work in statistics at Columbia University. *Biometrics Bull.* **1** 22–23.

HOTELLING, H. (1948). Mathematical statistics and econometrics at the University of North Carolina. *Econometrica* **16** 125–126.

HOTELLING, H. (1950). Needed steps in the mathematical training of social scientists. *Econometrica* **18** 198–199.

LEVENE, H. (1974). Harold Hotelling 1895–1973. *Amer. Statist.* **28** 71–73.

MADOW, W. G. (1960). Harold Hotelling as a teacher. *Amer. Statist.* **14** 15–17.

MANN, N. R. (1985). *The Keys to Excellence—The Story of the Deming Philosophy.* Prestwick Books, Los Angeles.

MINTON, P. D. (1983). The visibility of statistics as a discipline. *Amer. Statist.* **37** 284–289.

MINTON, P. D. and FREUND, R. J. (1977). Organization for the conduct of statistical activities in colleges and universities. *Amer. Statist.* **31** 113–117.

NEYMAN, J. (1960). Harold Hotelling—A leader in mathematical statistics. In *Contributions to Probability and Statistics: Essays in Honor of Harold Hotelling* (I. Olkin, S. G. Ghurye, W. Hoeffding, W. G. Madow and H. B. Mann, eds.) 6–10. Stanford Univ. Press, Stanford, Calif.

OLKIN, I., GHURYE, S. G., HOEFFDING, W., MADOW, W. G. and MANN, H. B. (eds.) (1960). *Contributions to Probability and Statistics: Essays in Honor of Harold Hotelling.* Stanford Univ. Press, Stanford, Calif.

RUSTAGI, J. S. and WOLFE, D. A. (eds.) (1982). *Teaching of Statistics and Statistical Consulting.* Academic, New York.

SAMUELSON, P. A. (1960). Harold Hotelling as a mathematical economist. *Amer. Statist.* **14** 21–25.

SMITH, W. L. (1978). Harold Hotelling 1895–1973. *Ann. Statist.* **6** 1173–1183.

SMITH, W. L. (1983). Hotelling, Harold. *Encyclopedia of Statistical Sciences* (S. Kotz, N. L. Johnson and C. B. Read, eds.) **3** 668–669. Wiley, New York.

Comment: Recollections about Harold Hotelling

W. Edwards Deming

It was in 1936, I believe, when at a meeting of the American Statistical Association in Atlantic City, Harold Hotelling told some of us that a woman in the Mathematics Department at the University of Toronto had written to him for advice. The head of the department had decided that there should be in their curriculum a course on statistics or possibly it was statistical theory. He had delegated the job to her.

W. Edwards Deming is a Consultant in Statistical Studies, Distinguished Lecturer in the School of Business, Columbia University, and Professor of Statistics in the Graduate School of Business Administration, New York University. His mailing address is 4924 Butterworth Place, Washington, DC 20016.

None of the others wished to teach it. She was a woman, and new in the department, so she was elected to teach the course. She wrote to Professor Hotelling with the question, "What should I teach?" His reply to her was merely, in his kind way, "Teach what you know."

This principle pervaded all his work. He tried to stay within his limitations, although I would remark that his limitations were well beyond the horizon. He took the point of view that statistical theory should be taught by someone that knows statistical theory. If one studies bacteriology, he studies it with someone that knows bacteriology. He had profound faith and respect for people that know statistical theory. In my own book, *Out of the Crisis* (1986), the following quotation is at the head of the chapter on page 465.