Comment

David S. Moore

My first reaction on rereading Hotelling's classic essays is distress that the situation he described has improved so little in nearly half a century. My second reaction is that we largely deserve our fate.

Despite great progress in statistical science itself, in the application of statistical methods to many areas of study and in the organization of statistics as a profession, statistics remains inadequately recognized as an independent discipline. The word calls to the mind of most scholars in other fields a few more or less routine methods learned by their graduate students—regression to the economist, control charts to an industrial engineer, repeated measures designs to the psychologist and so on—rather than "a coherent, unified science" ... "embodying the modern version of the most important part of inductive logic."

In so describing statistics, Hotelling was preaching to the converted of the 1940s. In agreeing here that statistics is a separate and fundamental discipline, we are preaching to the converted of the 1980s. The number of the converted remains small. Statistical methods are certainly much more widely applied than in Hotelling's day. In the past, even routine use of the more complex methods required a specialist, so that data analysis was a collaborative effort. The resulting demand for working statisticians has been an important justification for university programs in statistics. Now analysis is automated, and software is becoming increasingly capable of directing the user's judgment in design and diagnostics as well. What will the working statistician of the future have to offer the engineer or medical researcher or psychologist? This is the practical version of the question whether statistics is in fact a separate and fundamental discipline. In the absence of a convincing answer, the future of both working statisticians and university programs is in doubt.

Failing to obtain wide recognition as a science in its own right, statistics has also failed to remedy the educational problems that were Hotelling's primary concern. His description of the fragmentation of statistics teaching among more reputable disciplines could have been written yesterday. Many academic statisticians would also accept his corollary that this arrangement leads to inferior quality and productivity in teaching and places a burden of divided intellectual loyalty on teachers. I am not fully convinced that this corollary is true, given the nature of statistics as taught in many departments of mathematics and some departments of statistics. Our teaching is too dominated by mathematical modes of thinking that do not reflect the separate identity of statistics. The spirit and content of what we teach to students in other disciplines represents our *de facto* case for recognition as a separate science. If that practical case is weak, arguments from principle will gain us few allies.

In the rhetorical spirit that is appropriate in the discussion of such large issues, I want to argue two strongly put propositions. First, that statistics is not only an independent discipline but a fundamental discipline, in fact, one of the "liberal arts" in their modern guise. Second, that the major threat to the independence of statistics in many academic institutions is its self-inflicted subservience to mathematics.

STATISTICS AMONG THE LIBERAL ARTS

Hotelling, no doubt recognizing that his audience accepted statistics as a fundamental discipline, did not offer much in the way of explicit argument to support this opinion. I believe that such an argument, in outline, is as follows. A pervasive aspect of modernization is differentiation, the division into distinct institutions of functions that were once integrated. This sociological process has occurred as clearly in the intellectual area as in any other. It is illustrated by the gradual emergence of statistics as well as many other newer disciplines, including sociology itself. As a result of differentiation and other social changes, there is no longer any core of learning common to all educated persons and to all programs of "liberal education."

Some scholars lament this irreversible change. Attempts to specify a core of liberal knowledge are the focus of debates over the curriculum at many universities, and have reached the best seller list in E. D. Hirsch's book *Cultural Literacy: What Every American Needs to Know*. Such attempts invariably favor the older academic disciplines, which retain a certain prestige. More seriously, the "liberal arts" as reconstituted by those who regret the differentiation of knowledge too often focus on content rather than method, on learning certain facts rather than learning to learn. There is little hope that statistical science will be seen as fundamental from this perspective, although a few statistical facts may appear in a core curriculum as a result of the voting power of social science faculty.

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It is more consistent with the differentiation of knowledge, the diversity of society, and even trends within many of the older disciplines to understand a liberal education as first of all presenting intellectual methods. History is not "1066 and all that," but a wider study of past societies that includes their everyday life and their way of looking at the world. Literature is not a canon of classics (now often seen as too narrow in their social origins), but a study of the art of critically reading texts in the light of their social and historical context as well as for their message and structure. Physics is not Newton's laws, but a way of thinking about natural phenomena that rests on both experiment and idealized principles abstracted from many observations and presented in mathematical form. Which specific societies, or texts, or natural phenomena are studied is secondary to a mastery of the method.

From this point of view, statistical science belongs among the liberal arts. Students should understand historical and literary methods; they should grasp the probing of nature by experimental science and the power of abstraction and deduction in mathematics. Surely reasoning from uncertain empirical data has shown itself to be a similarly powerful and pervasive method of gaining knowledge. There is more than rhetoric to this claim. Nisbett, Fong, Lehman and Cheng (1987) document the possibility of teaching statistical reasoning, and the empirical effect of such teaching on the quality of students' reasoning both in science and in everyday life. What is more, the research surveyed suggests that "higher education does not train the mind as physical exercise trains the muscles." That is, the different intellectual methods are not substitutes for each other in education. In particular, study of statistics improves powers of reason in ways that study of law or chemistry does not.

We statisticians should not base our claim solely on usefulness, but on a clearly presented picture of statistics as an intellectual method related to but distinct from the methods of mathematics and the natural and social sciences. We can legitimately urge an affirmative answer to Hotelling's hesitant question "whether some work in statistics should not be required of all college students as a part of a liberal education."

STATISTICS IS NOT MATHEMATICS

Statistics is concerned with data and with scientific inference in the face of uncertainty. This view is the heart of the argument that statistics is a separate and fundamental discipline. The chief opposing view among academic statisticians, often appearing as an unconscious value judgment rather than as an articulate philosophy, is that statistics is primarily mathematical in nature. Statistics (like economics and physics) certainly makes heavy and essential use of mathematical tools. But I believe that domination of academic statistics, and particularly of the teaching of statistics, by mathematical modes of thought has contributed to the failures that Hotelling noted. I have argued these points at greater length for a general audience in Moore (1988), where readers can also find articulate rebuttals.

In retrospect (and only in retrospect), even Hotelling places too much trust in mathematics. He wrote at a time when it seemed that mathematical statistics might lay a firm and unified foundation for statistical practice. (Wald's *Statistical Decision Functions* was published the year after Hotelling's second essay, for example.) That hope has since faded, except perhaps among the resurgent Bayesians, leaving statistical science more diverse and more eclectic than the vision of the 1940s expected. Hotelling himself had a broad background that included training as a topologist and eminence as an economist. His essays are careful to warn that mathematics alone does not a statistician make. But I see in them an emphasis that dominated academic statistics for the next generation and is only now receding. There is an emphasis on mathematics as the most essential background for both teachers and students, an emphasis on methods rather than on data as the focus of study, and an emphasis on formal, probability-based, methods of inference. Good teaching requires mathematical derivations, and the research that will advance the subject is "mostly of a highly mathematical character."

In the hands of people less broadly experienced than Hotelling, these attitudes lead to the capture of statistics by mathematics. It is worth reminding ourselves and others that statistics is not at all a branch of mathematics.

Statistics did not originate from within mathematics, but in official and private data gathering and from practical problems first in astronomy and surveying and later in the life and social sciences, as Stigler (1986) carefully documents. As argued in the description of statistics among the liberal arts, *the aims and methods of statistics are not those of mathematics.* Statistics is concerned with the production, organization and analysis of data, and with inference from data to the underlying reality. The dominant approach in practice is a complex interplay between the data and a mathematical model. The model may be partly validated by carefully designed data collection, or it may be falsified by the data operating through the diagnostic tools that are a prominent topic of recent research. Mathematical derivation of the consequences of a model is an essential prerequisite for this process, but this is an application of mathematics to statistics parallel to the equally essential applications of mathematics in economics or physics.

Because statistics has different aims than does mathematics, it is not surprising that *the foundational controversies of statistics are unrelated to those of mathematics.* Foundational arguments are in fact much more lively in statistics, and have much greater impact on actual practice. Finally, *statistics does not participate in the interrelationships among subfields that characterize contemporary mathematics.* This fact distinguishes statistics from probability theory, which *is* a branch of mathematics. Martingales and Brownian motion are applied in many fields of mathematics; likelihood functions, sufficient statistics and prior and posterior probabilities are not. This one-way traffic has as a consequence the fact that the importance of research in statistics cannot be judged by mathematical standards or by mathematicians.

Despite the force of these arguments, academic statistics in many institutions has often looked very much like mathematics. Hotelling's evaluation that the best research should be highly mathematical has prevailed, and graduate programs have sometimes contained surprisingly little contact either with real data or with classical statistical methods. At four-year colleges, mathematics departments have felt comfortable in assigning algebraists to teach statistics. These attitudes grew out of the genuine achievements of mathematical approaches in statistics, but also out of the intellectual seductiveness of a clarity and rigor that ignores the messiness of real inference problems, and out of the natural tendency of an academic subculture to support itself, growing away from the roots of the discipline.

More recently, a reaction has set in and is now well advanced. The computing revolution has spurred research on data analysis and diagnostics. Many university departments have placed renewed emphasis on methodological research and on contact with the scientific problems that give statistics its permanent importance. This welcome return to a more balanced view of statistics has not, however, undone the self-inflicted damage of long bondage to mathematics. This is particularly true in the teaching of statistics to students in other fields.

THE TEACHING OF STATISTICS

The teaching of elementary statistics is in effect a presentation to the university at large of our case for recognition as an independent science. If our teaching is dominated by mathematical modes of thought and neglects data and scientific inference, we have abandoned our case. Nisbett and his colleagues offer good evidence that the study of statistics, as taught by psychologists, improves students' ability to deal with uncertainty and variability in quite general settings. If statistics is presented in the deductive mode natural to mathematicians, there is no reason to expect that these specifically statistical intellectual skills will be nourished.

One sign of the domination of statistics teaching by mathematics is a preference for theorems, often accompanied by the lament that our students don't see the relevance of the proofs. The students are right: *an essential distinction between mathematics and statistics is that mathematical theorems are true, but statistical methods are often effective when used with skill.* The proofs, and the theorems themselves in the detailed form that the mathematical mind thinks essential, are at best irrelevant. They may in fact be actively harmful if our goal is to teach statistics as an intellectual method.

Teachers, who approve of the subservience of statistics to mathematics, often argue that the alternative to a mathematical exposition is the "cookbook" style of presenting a list of recipes. But the cookbook is simply another style of teaching mathematics rather than statistics. It is the style often used in teaching calculus, to the discomfort of many mathematicans. Hand-held computers that can do all of introductory calculus present to mathematicians the same dilemma that statistical software presents to statisticians. In both cases, teachers must return to the essentials of their discipline. In mathematics, deduction and the power of abstract structures are central; in statistics, the problems of data, uncertainty and scientific inference.

I accept as fundamental to higher education Hotelling's axiom that the "qualifications of a good teacher of statistics include, first and foremost, a thorough knowledge of the subject." Certainly that knowledge, for a statistician as for a physicist or economist, includes a large dose of mathematics. But, as Hotelling also said, a knowledge of mathematics and of mathematical statistics does not equip one to teach anything other than mathematics and mathematical statistics. *Satisfactory teaching of the science of data requires experience with data.* Most mathematicians could follow the dense mathematics of a graduate microeconomics text and deliver lectures on the subject. Both mathematicians and economists have the good sense to recognize that a purely formal knowledge of economic theory is no basis for teaching economics 101. Why is the case different in statistics 101? A mathematician assigned to teach statistics 101 will teach it as a course in (trivial) mathematics. A statistician who thinks like a mathematician will teach in the same manner. They deserve to lose their audience.

It requires only a bit of delight in playing the devil's advocate to argue that statistics is often better taught by psychologists or economists or engineers. The origins of statistics are closer to these disciplines than to mathematics; statistics has been advanced by both past (factor analysis, multidimensional scaling) and present (pattern recognition) research in these disciplines; faculty in these disciplines are often close to the problems of data and inference. A fully trained and experienced statistician is certainly best prepared to teach statistics, but such persons are still in short supply. In many institutions the mathematics or statistics department will offer instructors lacking any experience with data. A psychologist is preferable. To scorn the psychologist because he cannot read the latest papers in the *Annals of Statistics* is a sign of allegiance to mathematics rather than to the understanding of data.

Although I have been deliberately extreme in arguing for the dominance of data over theorems in the teaching of introductory statistics, the principle is now widely accepted. Reports on the preparation of industrial statisticians (ASA, 1980), on teaching statistics to engineers (Hogg et al., 1985) and on teaching statistics in schools of business (Chicago, 1987) have in common the recommendation of increased experience with data and broader coverage at the expense of mathematical depth. The increasing availability of easy interactive computing is rapidly relieving the drudgery of data analysis, although not the need for thought. Younger faculty are usually well trained in computing, and often have an interest in, and experience with, applied problems. There is every reason to hope that the teaching of statistics will rapidly improve wherever trained statisticians are in charge, and some hope that small institutions will recognize the need to employ a statistician to direct the teaching of statistics.

Our disciplinary infirmity is not necessarily terminal. Statistics is rediscovering itself. The fragmentation of teaching that Hotelling lamented may remain, because campus politics demands that it be so. But we shall at least have a clearer case for control over the introductory teaching of statistics when what we teach is in fact statistics.

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Comment

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The republication of Professor Hotelling's papers is timely. For one thing almost all statistics curriculum planners at academic institutions have taken copies of copies of copies . . . of these ageless classics so that their messages, although immortal, are fading. I was reminded just the other day of the freshness of these articles when I received a memorandum from Department X about the service course that the Department of Statistics had been providing for their students for some years. To preserve confidentiality I would paraphrase the substance of this memo as saying "There is a strong preference for recreating our own course so that it would more directly prepare students for what they will actually do in field X. There is a disjuncture between the current course and the rest of the student's curriculum and they find it difficult to bridge this gap. Members of Department X feel that a different approach is required, which shows how statistics as a tool for conducting research in field X can illuminate the problems of interest in that field." Compare this with Professor Hotelling's 1940 comment that ". . . most students of statistics enter upon the subject not for its intrinsic interest but for the idea of applying statistical methods as a tool . . . " This is followed in 1949 by "The major evil is that those teaching statistical methods are all too often not

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