The Place of Statistics in the University

Harold Hotelling

Reprinted from *Proceedings of the Berkeley Symposium on Mathematical Statistics and Probability* (J. Neyman, ed.), University of California Press, Berkeley, 1949, pages 21–40.

The teaching of statistics in American colleges and universities, which has for the most part been a development since the first world war and has now reached large proportions, presents a number of unsatisfactory features. Courses in statistical methods are taught in various departments without coordination or intercommunication. These courses cover what is to a large extent the same material, but with many variations in the selection of subjects according to the ideas and abilities of individual instructors, and with illustrative examples drawn in each case from material pertaining to the department in which the course is taught. Thus a student desiring to learn more about statistics than he can obtain in one department must, in taking courses in other departments, repeat a great deal of what he has previously covered. There is a plethora of elementary courses, a dearth of advanced ones. Some departments have excellent statistical laboratories which they reserve for the use of their own students, each with an attendant to keep others away, while other departments have none. Some classes in elementary statistics are too large and some too small, with no one in a position to equalize the sections as between different departments.

The library situation is confused. Books on statistical methods are cataloged and shelved under Sociology, Economics, Business, Psychology, Zoology, Botany, Engineering, and Medicine. Books on probability are divided among Philosophy, Mathematics, Physics, and Chemistry. Books on the method of least squares are for the most part divided among Mathematics, Astronomy, and Civil Engineering, though some get into the Economics, Geology, and Physics reading rooms. Works on the analysis of variance and design of experiments are apt to be concentrated under Agriculture, while methods of approximate evaluation of multiple integrals and similar purely mathematical subjects of use in statistics are, at least in one of our largest universities, to be found only in the library of Biology.

These are minor nuisances. The major evil is that those teaching statistical methods are all too often not specialists in the subject. Their original selection was seldom on the basis of scholarship in this field, they are not encouraged to make advanced studies in it, and their environment is such as to draw their attention in every direction except to the central truths and problems of their science. Frequently they lack the knowledge of mathematics necessary to begin to read the more serious literature of the subject they are teaching. Many have been utterly unable to keep up with the rapid progress which has been taking place in statistical methods and theory, progress which affects even the most elementary things to be taught. There results a widespread teaching of wrong theories and inefficient methods. Students are sent to the government service and to industrial and commercial statistical positions equipped with the skill that results from careful drilling in methods that ought never to be used. Some of these same students are encouraged and assisted to become college and university teachers of statistics without ever making thoroughgoing studies of the fundamentals of the subject, or exhibiting any power of making original contributions to it, or studying any graduate mathematics. Through the method of selection of teachers in general use, and through textbooks written by individuals of this type, there is a perpetuation of obsolete ideas and unsound methods.

All this does not mean that any considerable number of those teaching statistics are unworthy or objectionable members of the academic community. Many, indeed, are of very superior intellect, upright character, personal charm, and undoubted teaching ability. Some are making creative contributions to other subjects. The only trouble is that they are teaching a subject in which they are not specialists, and which progresses so fast that only specialists can keep up with it.

The chief reasons for the extensive teaching of statistical method by those who are not specialists in it appear to be the following:

1. The rapid growth of the subject and multiplication of its applications, creating a very large and very urgent demand for teaching it that could not be met immediately by the small existing number of scholars specializing in statistical method. This difficulty is aggravated by the paucity of university facilities for

training advanced scholars in the field, so that even now the available number of such scholars cannot be expanded with sufficient rapidity to meet the current need. Since specialists have not been available in anything like sufficient numbers, statistical method has inevitably been taught largely by nonspecialists.

2. A confusion between statistical method and applied statistics. Statistical method is a coherent, unified science. "Applied statistics" may mean any of thousands of diverse things. Any particular study in applied statistics will ordinarily utilize some few of the results obtained by the science of statistical method, but will be largely concerned with matters peculiar to the particular application in view and others closely related to it. For example, studies of business cycles utilize statistical methods, good or bad, with a view to drawing inferences from existing data on prices, production, incomes, interest rates, bank reserves, and the like. The main job of the applied statistician in this field is to study the sources and nature of the various series of observations, keeping in mind incidental events which may break the continuity of a series, and watching, with a background of economic theory and knowledge of the facts, for explanations. He should also be well acquainted with statistical theory, since otherwise there is grave danger of wasting or misinterpreting the laboriously accumulated observations. Indeed, an organization studying business cycles, or solar cycles, or rat psychology, or cancer, or practically anything else, would almost certainly benefit from participation by a specialist in statistical method. However, the chief attention in any such study must not be on statistical method, but on features peculiar to its own scope. The specialist in statistical method will do well to participate occasionally in such a study, but if he does so too extensively the needs of the application will so engross his attention that he cannot keep up with the progress of statistical method. He will then cease to be a specialist in it, becoming instead an economist, or a practitioner in the field of his applications of statistics. This indeed, has been a frequent phenomenon. The call of applications is enticing, and has led many young scholars to forsake the cultivation of statistical theory. The applications have benefitted greatly by the process. Moreover, problems brought back in this way from applications have provided valuable inspiration in developing the theory. The mistake has been in supposing that participation in applied statistics is equivalent to specialization in statistical method and theory, and the consequent appointment to teach the latter of persons whose sole concern is with the former.

3. A failure to realize the need for continuing research in the theory of statistics by those who teach it. There is an easy tendency to assume that all the requisite ideas and formulae can be found in some book, and that the duty of the teacher of statistics is simply to transfer this established book knowledge to the minds of the students and impart to them skill in applying it. Similar attitudes applied to other subjects have in the past been a drag on progress, but have long been discarded in respectable universities. They still hang on, however, even in the best institutions, with respect to statistics. The spectacular advances of the last three decades in statistics should make it clear to anyone who has followed them that statistical method is far from static, that the best techniques of present-day statistics may tomorrow be replaced by something better, and that unsolved problems regarding the theory and methods of statistics are sticking out in every direction. A vast amount of research, mostly of a highly mathematical character, is needed and is in prospect. Anyone who does not keep in active touch with this research will after a short time not be a suitable teacher of statistics. Unfortunately, too many people like to do their statistical work just as they say their prayers—merely substitute in a formula found in a highly respected book written a long time ago.

4. The system of making appointments to teach statistics within particular departments devoted primarily to other subjects, on the basis of recommendations by those departments. This means in effect that the teacher of statistical method is selected by economists or sociologists or engineers or psychologists or medical men, according to the department in which he is to teach. Thus the task of selection devolves upon persons unacquainted with the subject, though realizing the need for it in connection with a very specific application. This system results almost inescapably in emphasizing the immediately practical and specific at the expense of the fundamental work of wider applicability and greater long-run importance. Confusion between a science and its applications is most pronounced with those who know little about it, and the distinction between statistical method and applied statistics is likely to be completely lost upon a sociologist or an engineer confronted with the problem of finding someone to teach statistics. If he does make the distinction, he is likely to choose in favor of applied statistics.

But the actual teaching that ensues is bound to consist largely of statistical theory since the students will ordinarily not have had statistical theory elsewhere, and they must have some in order to apply it. What often happens is that a sociologist or an engineer who has made some study of statistics embarks on what he thinks will be a career of teaching the application of statistical method to sociological or engineering problems, only to discover that because of the

ignorance of the students he is compelled to teach the fundamentals of statistics, an entirely different subject, for which he lacks preparation, talent, and interest.

An incident of this sort has been cited previously.[1] A prominent economist was asked to teach a course entitled "Price Forecasting" in a leading university, and accepted. He found, however, that his lectures on this subject were over the heads of the students because he was using statistical concepts unfamiliar to them. He therefore went back over the ground covered so as to explain these particular statistical concepts along with their application. But in explaining them he found himself using other statistical concepts, which in turn called for explanation. At the end of the semester he found that he had not given the course in price forecasting which he had planned and for which the large class had enrolled, but instead had taught a somewhat disordered course in elementary statistics, a subject in which he did not feel particularly competent and for which the students had not come. When he was asked to teach "Price Forecasting" a year later, he proposed that a prerequisite of a course in statistics be imposed, but this proposal was rejected by the chairman of the department, and the course was not repeated.

Appointments by departments of application are not all bad. Some professors in these departments make conscientious excursions into statistics, are well advised by competent specialists in statistics, and bring about the appointment of men of high quality well acquainted with statistical method and theory of the currently best sort. This may work out well if the appointee is an able and energetic scholar deeply devoted to his subject, if he is placed immediately in the highest professorial rank, and if he does not feel under an obligation to devote himself too exclusively to the special interests of the department of which he finds himself a member. He is then free to pursue his specialty, to keep informed on the latest developments in statistical method, and himself to add to the subject, while at the same time transmitting to students a well rounded and up-to-date selection of knowledge. It is in this way that some of the present leaders in statistics have developed.

The outcome is likely to be quite different if the promising young scholar in statistical method is given a junior position in a department of application which wants him to work on its problems and to teach statistical methods with a sole eye to the work of the specific department. He is then under pressure to concentrate on a particular kind of applied statistics. He must study the literature, terminology, techniques, and theories of the application. His usual associates will be in the department in which he is teaching rather than others teaching statistics. If conditions are exceptionally favorable he may still be able to maintain his integrity as a statistical scholar; but if, in addition to the burden of working in two subjects, he is given a heavy load of teaching or other exhausting duties, his statistical ideas will gradually fail to measure up to the best currently available. He will then not have the time or energy to study the continuing output of new ideas and methods in statistics and to contribute his own.

A still less favorable, but unfortunately more common, case is that in which the teacher of statistics is not even selected for scholarship in the theory of statistics. Too often, men are picked to teach statistics without any adequate inquiry into their proficiency in this field or their prospects for research in it. Studies in some other field, with some light dabbling in the application of statistical methods to it, plus a pleasing personality, have all too frequently been thought to comprise sufficient qualifications for teaching statistical methods and theory.

From such methods of selection of teachers of statistics there has resulted a widespread blind leading of the blind. Statistical ideas are not in the course of their teaching subjected to the critical appraisal that would be normal in courses in mathematics or economics or philosophy, for example. The uncritical character of the teaching is reflected in the long line of textbooks written by teachers who have not made any genuinely fundamental study of statistics, but who pass on to students in a magisterial fashion what was passed on to them. Authority takes the place of derivations as regards ultimate sources. It is no wonder that these textbooks, copied from each other, contain increasing accumulations of errors; or that long delays have intervened between the introduction of important new statistical methods and theories in the periodical literature and their appearance in the textbooks and courses put before students.

One of the important weaknesses in much of the current teaching of statistics is a failure to make proper use of the theory of probability. Without probability theory, statistical methods are of minor value, for although they may put data into forms from which intuitive inferences are easy, such inferences are very likely to be incorrect. The objective weighing of the degree of confidence to be placed in inductive conclusions is necessary to avoid fallacies. Indeed, the whole foundation of descriptive statistical methods, of inductive inference, and of the design of experiments, rests upon probability theory. The relevance of probability to much statistical work was indeed questioned

[1] Harold Hotelling, "The teaching of statistics," *Annals of Math. Stat.*, vol. 11 (1940), pp. 457–470.

a quarter century ago by a group of economists impressed by the lack of independence between consecutive observations, and this attitude, in conjunction with an exaggerated and belated remnant of nineteenth-century empiricism, has had a certain influence, particularly on the statistical methods in use by economists. This view is now rapidly giving way to a tendency to use the powerful new statistical methods discovered in the meantime, particularly those of R. A. Fisher, with such adaptations as seem appropriate to particular circumstances. It is now perceived that such efficient objective methods can be used over a much wider range of cases than was formerly supposed, since the independence assumed in their derivations refers not to observations but to residuals from the theoretical model used. Furthermore, research is under way, and has already achieved promising results, as to the extension of accurate methods to still more extensive classes of situations.

The main reasons for the slighting of the theory of probability in so much current teaching of statistics are not to be found in this passing episode related to economic time series. One very substantial reason is that the students of statistics do not know the theory of probability. An even more cogent reason may possibly be found in the state of knowledge of the subject on the part of instructors and authors of textbooks. Probability is a difficult and treacherous subject whose history over the last three centuries is studded with disastrous errors by scholars of great distinction as well as by lesser men. Its proper application to statistics and inductive inference now appears to be quite different in nature from the attempted applications of earlier times. A full and clear view of the situation must enlist the aid of philosophy and mathematics, as well as the newer mathematical statistics. It is small wonder that the sociologist or engineer called on to teach a short course in practical statistics to immature students does not have this full and clear view, and is relieved to find a weight of textbooks and precedent in favor of avoiding probability. What is more surprising is that the cookbook methods and shallow theoretical grounding provided in these courses, which constitute the most common type, are as useful as they are.

The qualifications appropriate for teachers of statistical method and theory are not essentially different from those for teachers of other subjects in the same institutions, except that statistical method and theory are to be substituted for other subjects. This substitution is, however, vital. It must not be imagined that proficiency in some other subject in which statistical methods are used incidentally is equivalent to proficiency in statistical methods sufficient for teaching the latter. The error of such a supposition, if carried over into another field, might lead to the appointment of a man as professor of chemistry on the ground that he could cook.

The first requisite of the college or university professor of any subject is a profound and thorough knowledge of that subject. With this should go an active scholarly concern and research in the field, and the results of this activity should be published. It is customary, in the better institutions at least, to restrict appointments to the rank of assistant professor to persons who have demonstrated scholarly qualifications by work equivalent to that leading to a Ph.D. degree, including an original contribution to the body of knowledge related to the subject the individual is to teach. Promotion to the higher ranks is conditioned upon a number of criteria, among which published research is by far the most important in these institutions.

The professor, or the assistant professor, of statistical method should first of all have a profound and thorough knowledge of statistical method. It is desirable that he have in addition a wide and detailed knowledge of applications of statistical method in divers fields. It is also desirable that he be acquainted with the history of statistics and with the relevant portions of philosophy. It is particularly important that he be able to use effectively some quite advanced mathematics. None of these auxiliaries is, however, sufficient if he does not know the theory of statistics itself.

Research is even more essential in the teacher of statistics than in teachers of most other subjects, since so much remains to be worked out that is of immediate importance. The latest discoveries in the theory of statistics affect what should be taught in elementary courses, and no syllabus in use today can be expected to survive a few more years of research. What is happening is not so much the discovery of errors in what has been believed and taught—this critical process was virtually completed a decade or two ago, though its effect has not yet penetrated the consciousness of some of those concerned—as the development of new statistical methods and ideas of such overwhelming importance as to compete for the limited time available for instruction with material already well established as true and useful. The new material is equally true, but may in some cases be even more useful than matter incorporated in the best of current courses and textbooks. A singularly talented teacher, more than usually in immediate vital contact with research in his field, is needed to understand and evaluate the new ideas. It is hard to imagine an individual sufficiently talented in this way who is not himself engaged in research.

Since research in the theory of statistics requires advanced mathematics, and is indeed largely mathematical in character, a mastery of a substantial amount of higher mathematics must be an essential part of the training of prospective professors of statistics. To specify exactly what or how much mathematics is necessary would be a difficult task. Something of the algebra of matrices and of the theory of functions are minimum necessities, and a good deal of additional knowledge of algebra, geometry, and analysis adds richness and power to the work of the statistical theorist, the inventor of new statistical methods. On the other hand, the time of the graduate student in statistics is much occupied with the theory of statistics itself; and some of it should go into the study of applied statistics. There is a cruel dilemma here, resulting from the delay in learning mathematics imposed by the elementary curricula which have become customary in this country.

The weakness of the mathematical element in the prevailing curricula affects both teachers and students of statistics to an extent justifying some attention from those interested in the improvement of statistics. In the European gymnasium or lycée it has been customary to equip the student with a year of calculus before he enters the university, or, as in England, to give him a much more extensive knowledge of algebra than is obtainable in the usual secondary schools in this country. A student entering the university with such mathematical preparation has a great advantage, whether he specializes in mathematics, statistics, philosophy, economics, engineering, or any of the physical sciences. In American universities it is not often that elementary calculus is taught before the sophomore year, and the more advanced parts of algebra of the English public school type come still later, if at all. The comparison is often challenged on the ground that the European school preparatory to the university is on a more advanced level than ours, and should justly be compared with our first two years of college plus high school rather than to our high school alone. The fact remains, however, that the gymnasium and lycée graduate students to the university at a normal age in the neighborhood of eighteen, just as do our high schools. If they are on a higher level than our schools it is because of superior efficiency and a sounder curriculum, not because of taking more years.

If calculus could be pushed down into the high schools and assumed as a prerequisite for college courses in mathematics, statistics, economics, physics, and several other subjects, the efficiency of instruction in all these departments could be increased. For example, the difficulties experienced by students of economics with ideas of marginal cost, marginal revenue, and the like correspond closely with the difficulties experienced by mathematicians for centuries in trying to define infinitesimals and derivatives, but now successfully overcome. The student who really knows differential calculus need not experience the slightest difficulty with the marginal ideas of economics. The same mathematics is of course useful also in many other subjects.

A few secondary schools offer excellent work in mathematics, and their graduates are sometimes looked on with wonder, as if they were freaks, when actually they have a very substantial advantage over others of like age. In most schools the tendency is to weaken the teaching of mathematics in the interest of the peace of mind of the poorer students, or to make room for subjects of greater popular appeal, or because suitable teachers of mathematics cannot be obtained at the salaries and under the conditions prevailing in the schools. The advocates of particular subjects organize violent political campaigns to impose their ideas upon the schools, and when these succeed there is necessarily a diversion of time and attention away from more fundamental but less familiar and popular subjects such as mathematics. An example of this sort of thing occurred recently in New Jersey, where the legislature passed and the governor signed an act to make two years of American history compulsory for every high school student, though extended instruction in the same limited subject is already required in the elementary schools.

The possibilities of teaching quite advanced mathematics to young children have scarcely begun to be explored. Children of kindergarten age are fascinated and thrilled by the wonders of topology, and groups and number theory can be tremendous sensations in the fifth grade, though all these subjects are ordinarily reserved for graduate students specializing in mathematics. What is lacking is teachers who know mathematics and its applications and who possess enough freedom to teach what they know instead of the long, dull, and relatively useless drill on problems of wall-paper-hanging and the like, problems turning on mere conventions which are quickly forgotten—painful, repetitious work which makes children resolve to quit mathematics as soon as possible.

Not a little of the responsibility for the low level of mathematical teaching in American schools must be borne by the teachers' colleges, the superintendents of schools, and the legislatures which follow their recommendations. For several decades these groups have conducted a highly organized and successful drive to make the taking of numerous long courses in pedagogy a leading requirement for licenses to teach. These courses occupy so much of the time of the prospective teachers as to inhibit genuine advanced scholarly work in such a subject as mathematics. Moreover, the licenses to teach are of such general character that mathematics can be and often is taught

by those who have never studied it beyond the course being taught. School salaries are so low and hours of work so long that few mathematicians of ability are drawn into school teaching, and few schoolteachers ever become mathematicians of ability. This is more especially the case because mathematical ability is little prized by principals, licensing authorities, and teachers' colleges, for whom courses in pedagogy and conformity to the ideas and organization of the established hierarchy are more important than deep scientific knowledge or brilliant new ideas.

The weak college curricula in mathematics, resulting in great part from the faulty attitudes toward mathematics prevailing in the elementary and secondary schools, limit the efficiency attainable both by the graduate schools in the training of teachers of statistics and by the colleges themselves in the teaching of statistics, both elementary and advanced. If a graduate school offering advanced work in statistics can get entering students with a knowledge of matrix algebra and theory of functions and additional higher mathematics, such as is obtainable by undergraduates at some institutions, the type of graduate work suitable for these students will be very different from that appropriate to those who have merely gotten past calculus. The latter will need to put in a large part of their graduate study on pure mathematics. The former, besides concentrating chiefly on the theory of statistics, will have time for applied statistics, and should work on applications.

But statistics is an art as well as a science. Work with applications is very important for a theoretical statistician. He is a toolmaker, and needs to know by personal experience something of the lives and collateral problems of those who use his tools. Experience with applications, and the challenge of problems arising out of applications, have played a most important part in the development of statistical theory. Nevertheless, the toolmaker must not put all his time on using the tools he makes; mostly he should work at making the tools. For him the interest is only secondarily in the product of the tools; the main focus on his attention is the tools themselves. So it must be with the academic statistician. His concern must be with statistical methods, and only proximately with the results obtained by applying statistical methods.

For the graduate student in statistics preparing for academic life there is a need for contact with applied statistics which the institution should undertake to provide, or at least facilitate. This need is next in importance after the needs for theoretical statistics and for pure mathematics. The distribution of time among the three—theoretical statistics, mathematics, and applied statistics—is hard to specify exactly, and must in any case depend on the nature of the student's previous work. If his mathematical preparation has been full and rich, more time should be spent on applied statistics in his graduate years than as if he has already had substantial contact with applied statistics in some other way but is deficient in higher mathematics.

Applied statistics entails a somewhat detailed acquaintance with the field of application. Such a field might be life insurance, or mental testing, or industrial quality control, or the work of the Census Bureau, or agricultural economics, or the study of business cycles. Proficiency in any such field calls for rather prolonged study, and it would be too much to expect the embryo statistical theorist to reach the stage of advancement in such a subject which might be reached by one specializing in it and it alone. He should, however, make more than a superficial study of a chosen field of application. This study might or might not be at the university. The requisite familiarity with applied statistics might in some cases be acquired by work in a government bureau, or in a research organization studying business cycles of something else involving applied statistics. What is most desirable is that the work should have brought the student to the point both of applying statistical methods in a reasonably effective way and of perceiving the limitations of existing statistical methods. Perception of these limitations has very frequently been the germ of progress in the subject.

One way in which it is to be hoped that training in applied statistics will be obtained is in the teaching and research of those who are not primarily theoretical statisticians but who know enough theoretical statistics to apply it well. For example, a professor of psychology working with mental tests might enlist the assistance of a young statistical theorist with mutual benefit. The young man might for a short time do some of the drudgery of scoring tests and computing, passing on soon to the problems of test construction and the distributions of various functions of correlation coefficients. This last is on a new and exciting frontier of statistical theory. The advancement of this frontier, which is really the main business of the young man in his capacity as a prospective statistical theorist, would in this way come to him naturally as a problem or series of problems having a tangible meaning additional to its mathematical content. The empirical context is in such cases often of great value in suggesting suitable approaches, for example, suitable approximations in the study of a function not susceptible to simple mathematical representation in terms of elementary functions.

The young man in this hypothetical example might prove so satisfactory from the standpoint of the psychologist as to be invited to continue with the type of mental testing work on which he had embarked and become a psychologist himself. But if he is to become

a professor of statistics his work in psychology should be temporary. There is too much in psychological work that is not theoretical statistics, and there is too much in theoretical statistics that is not psychology. It is hard to do both at once. Besides, discoveries in statistical theory and method should not be confined to a particular kind of application. If the young man succeeds in extending the boundaries of multivariate statistical analysis by discovering the distribution of some new function of correlation coefficients, the chances are that this discovery will also have applications in anthropology, medicine, banking, and other pursuits which in the aggregate will greatly outweigh the application originally in view. The discovery should be regarded primarily as a contribution to the general theory of statistics, and published in a journal devoted to mathematical statistics. It will then become available to a wide circle of teachers of statistics, who may incorporate it into their courses, and its methods and results will be studied by other investigators from the standpoint of possible generalizations and analogs. The importance of the discovery would be much more limited if it were thought of as a development in psychology and published only in a psychological journal. Perhaps dual or multiple publication ought to be permitted in such cases, but the first publication should be in a journal of mathematical statistics. Far too many good statistical ideas have been buried in connection with obscure special applications.

The success of such an experience from the standpoint of preparing the young man for work of a high grade in the theory of statistics would depend partly on his own mathematical-statistical preparation and innate qualities and partly on those of the psychologist under whose auspices he worked. If the psychologist were dogmatically devoted to bad statistical methods inherited from a past epoch the experience might be fatal for the budding statistician. If the young man had not studied psychology and had a contempt for it the results would almost certainly be bad. If on the other hand the psychologist had kept reasonably well in touch with the modern development of statistics, if the young statistician had a sincere interest in and respect for the psychological problems, and if both were genuinely devoted to the advancement of science, there might emerge both a valuable new contribution to psychological technique and a new man fit to take his place in the world of scholarship as a teacher and creator of statistical methods.

Colleges and universities usually expect the members of their faculties to engage in teaching and also in research, with the relative emphasis on these two functions varying greatly from institution to institution and to a lesser extent among departments within the same institution. There is also a considerable variance among individual members of a faculty, which is partly associated with the degree of advancement of the students taught by the various individuals.

Some college teachers do no research. This is usually regarded as deplorable. The evil is, however, of quite different magnitude according to the nature of what is taught by such teachers. If the subject matter of a course has remained static for centuries, with no new points of view or applications and no controversial questions, and if every detail is adequately covered by a textbook which has evolved from generations of such books carefully improved by conscientious scholars until brought virtually to ultimate perfection, then a case can perhaps be made for having such a course taught by an instructor who does no research. There may also be a case for omitting such a subject, if it exists, from the college curriculum.

On the other hand, in a new subject in which sharp differences of opinion exist or have recently existed on fundamental questions, in which current discoveries have an important bearing, and in which there have not yet been the time and consensus necessary for the preparation of an adequate and virtually error-free textbook, teaching without research may have calamitous effects. No skill in pedagogy, no luster of personality, can atone for teaching errors instead of truth. In such a field errors are very likely indeed to be taught by those who do no research, and then the more skillful the pedagogic indoctrination, the greater the harm. Sound educational policy calls for devotion to research of a large fraction of the time and energies of the teaching staff in such a subject. Students also are in particular need of encouragement to original and critical work in relatively new areas of this kind. They must be taught to shun the use of formulae and methods given merely on authority without full and convincing reasons, and to insist on looking closely and critically at assertions.

It may of course be argued that a subject of this kind should not be taught at all. It may be said that where there is no consensus and where textbooks are faulty the specialists in a topic ought to keep it to themselves instead of intruding in cloistered halls dedicated to absolute truth. If doctors disagree, who shall decide? The hollowness of this point of view is easily exposed, for example by reference to the teaching of medicine, which has advanced steadily despite controversies and rapid changes in point of view, to say nothing of demagogic attacks and firmly held popular superstitions. The practical importance of the subject means that it will beyond any question continue to be taught, and on a large and increasing scale.

Statistical theory and method constitute a subject of such great and diversified practical importance as to assure its continued and enlarged teaching. Scarcely

any field in which knowledge is sought can now afford to dispense with statistical methods, and statistical methods can be chosen wisely only in the light of statistical theory. Great advances have been made which provide sound statistical methods for a great variety of cases. However, other urgent practical needs are not yet well provided for, and opinions differ as to the best ways of attacking the outstanding problems thus presented. The searching inevitably turns to deeper questions in the hope of reaching ever more general principles from which ready deductions can be made to fit special cases. Profound puzzles are encountered which call for the penetration of more and more new mathematical frontiers. All this requires research, and a great deal of it.

Even in the teaching of elementary statistical methods for direct practical use by specific occupational groups, where it might be thought that the teaching would most predominate over the research element, the teacher must face difficult questions whose answers call for research in statistical theory. Let us illustrate this by one example out of the many possible. In teaching the analysis of variance for use in agricultural experimentation, questions arising out of the possible non-normality of the underlying distributions must be dealt with in some way. The formulae, even those in the best textbooks, are accurate only if the distribution is normal, and neither this fact nor the non-normality of many distributions should be concealed from the students. Obviously something more needs to be said on the subject at this point. What the teacher can say depends on how deeply he has gone into a whole series of perplexing questions, on some of which the views of scholars are not yet stabilized, and on which a tremendous amount of research is needed before the maximum practical value can be attained for a technique whose usefulness is already amazing.

In the organization of statistical teaching it is thus of extraordinary importance that colleges and universities emphasize research in the theory of statistics as a leading part of the work of the teaching staff in this field. Hours of teaching and other duties must be kept within such bounds as to make research possible, the initial selection of teachers must be of persons capable of research in statistics, and there must be provision of needed secretarial, computational, and other assistance. The library must be adequate, not only as to publications containing statistical theory, but in the larger field of pure mathematics as well.

In addition to the customary duties of teaching and research, faculty members expert in statistical methods find that they cannot escape a third, namely, advice to their colleagues and others regarding the statistical aspects of their problems. This often takes up a good deal of time. Clearly it is in the interest of the academic enterprise that such services be provided. Scholars in very many departments are finding that their work is improved and facilitated by competent statistical advice not only in the interpretation of their data but also in the design of their experiments and other investigations. Advice needs often to be supplemented by further service. The statistician, like the physician, often finds that one interview at which a prescription is dispensed does not end the matter satisfactorily. The initial diagnosis and treatment may need to be supplemented by further observation, and additional work should often be done.

When hours of teaching are being set, administrators should keep in mind this service to the rest of the institution on the part of those teaching statistics. Taken with the need of research in their own field, it means that teaching hours should be distinctly limited. In some cases, as in that of an enthusiastic young teacher impressed by the amount that needs to be taught and the limited time for teaching it, there may be a need for intervention from above to keep down the number of hours a week a man teaches, and to insist on the taking of sabbatical leaves when due.

One way to handle the problem of statistical service, especially in a large institution, would be through a special organization devoted to this purpose. Such an organization, whether called a Statistical Institute, a Department of Applied Statistics, or something else, might supply not only advice but a more active kind of assistance, including computational and chart-drawing services. It would be one of a possible series of organizations such as those recently suggested by Dr. Paul E. Klopsteg,[2] who proposes a group of "research service laboratories of instrumentology, whose work constitutes a technology consisting of the application of science to science itself." In support of this idea Dr. Klopsteg cites the experience of the National Defense Research Committee, in which it was found "not only desirable but essential to establish a group of mathematicians, known as the Applied Mathematics Panel, to assist the research workers in the various fields." It might be added that the Applied Mathematics Panel found it useful to establish and maintain large Statistical Research Groups at several centers, manned by competent mathematical statisticians and dealing with varied practical problems.

A statistical service organization should be removed from the teaching of statistics only to the extent necessary to gain the advantages of some degree of specialization and to prevent undue interruption of the teachers' other work of teaching and of research

[2] "Increasing the productivity of research," *Science*, vol. 101 (1945), pp. 569–575.

in theory. There are distinct advantages for all parties in a fairly close connection between practical statistical work, research in statistical theory, and statistical teaching. Each of these activities benefits the others, provided only that it does not take away from it too much time. Research in statistical theory, like medical research, needs frequent revitalizing injections of specific practical problems. It also needs the stimulus of contact with students. The teaching of statistical methods is made more vigorous both by research in the subject and by the presence of applications with which students can be confronted. And the needs of applications are better met if through some such an organization as is here envisaged they can be brought to the attention of appropriate specialists, and if, also, students can be enlisted when needed for their treatment.

A university organization dealing with statistics may properly comprise two parts with overlapping personnel, one devoted chiefly to applied statistics, the other to theoretical statistics. The teaching might be done by both, but at least at the more advanced levels would be primarily the concern of the theoretical department. Migration between the two groups ought to be easy and frequent, though some individuals are so definitely adapted to one kind of work or the other as to make it undesirable to have fixed rules calling for periodic transfers.

In smaller institutions it may not be practicable to have two statistical departments, or in the case of still smaller colleges even one. To meet the needs in some of these cases regional centers for advice and service in applied statistics might be established at large universities throughout the country, with access made readily available for sister institutions. These centers might also carry on work in applied statistics in behalf of government agencies and other organizations, much as various agricultural colleges have for years been carrying on cooperative work with the federal Department of Agriculture, or as a great deal of war research is now done in universities and other organizations under contract with the government.

The question how far, if at all, such a university center of applied statistics should go into the market place and engage commercially in service to business concerns is a debatable one. Experiments by universities in commercial service are now under way and are understood to be financially profitable, at least in some cases. There may even be a few slight favorable reactions upon scientific work. On the other hand there are grave dangers to the intellectual integrity of the institution which need serious consideration. The truisms so often brought forward when the question of government grants to universities is raised, such as that the power to give implies the power to withhold, and that he who pays the piper calls the tune, apply equally or with even greater force when the money comes from business concerns. It is not the intention here to enter a final judgment on this question, but only to give a word of well-wishing mixed with caution to those learned institutions which may feel that they have devised means of getting away with the cheese without being caught in the snap of the trap.

Passing from questions of personnel and the research and service functions of academic statisticians to teaching itself, we have to consider problems of departmental organization, of course contents, of systems of prerequisites, and of methods of teaching. All these we consider secondary problems, not in the sense of being unimportant, but because we believe that proper solutions of them will be reached with reasonable promptness when once the kind of personnel described in the second section of this report are at work in some such general setting as has just been described. The ideas recorded below are general in character and are to be regarded as a starting point for developing a program in a particular institution, once suitable faculty members have been obtained. No detailed recommendations on these questions will be attempted at this time.

The teaching of statistics may be organized in any of the following ways:

1. In a two-department Institute of Statistics of the kind suggested above.
2. In a single Department of Statistics.
3. Under an interdepartmental committee.
4. Under the exclusive jurisdiction of the Department of Mathematics.
5. It may as at present be disorganized among a heterogeneous group of departments of application.

It is likely that the first plan will be adopted only by a few large institutions, and that the second will be found most suitable for the majority. The third should probably be regarded as a makeshift for the transitional period until a proper Department of Statistics can be organized, a step that will not at the moment be reasonably possible for most institutions because the right kind of scholarly personnel does not, and because of limited facilities for graduate study cannot for some years, exist in adequate numbers. It is of course possible that some vestige of an interdepartmental committee, perhaps in the form of an Advisory Board, might be a useful adjunct of a Department of Statistics in order to keep it informed of the needs of applications. It is also possible that something of the sort might function with respect to a Department of Mathematics, or any other department. On the other hand, the desired consultations and adjustments might be accomplished in less formal ways. These possibilities have not been further explored.

To make statistics a subdivision of a Mathematics Department is a solution that will appeal to administrators desirous of keeping down the number of departments. The subject matter of statistics is to a sufficient extent mathematical to give a certain weight to this plan. Statistics uses mathematics of many kinds, and may eventually use every kind. Moreover, some theorems of pure mathematics, for example in geometry, are most easily proved or understood with the help of theorems and concepts familiar in statistics. Thus spherical trigonometry is easy to reconstruct with the help of relations well known to statisticians connecting simple, partial, and multiple correlation coefficients.

On the other hand, statistics has some features uncongenial to traditional mathematics, arising partly from the urgency of practical needs which go beyond what can immediately be provided by rigorous mathematical theory. Again we may cite the problem in the teaching of the analysis of variance of what to do about possible non-normality of the underlying distribution. The user of this technique has the responsibility of verifying that the situation conforms to the assumptions, including that of normality, underlying the tabulated probability criteria. But he is in a very poor position to do this as regards a large proportion of the applications actually made of the analysis of variance. Yet the analysis of variance in some form—possibly through the use of rank order numbers[3] or through a transformation or some other auxiliary device—remains the one powerful means of attacking a very large and important class of practical situations. The practicing statistician needs to do some highly educated guessing on such matters—guessing that will be assisted but not made determinate by knowledge of a considerable range of mathematical truths regarding approaches to the normal distribution, moments of the variance ratio in samples from non-normal populations, asymptotic large-sample theory, and other such matters. This mathematics needs to be supplemented by consideration of the particular subject matter of application. Moreover, it is desirable that students of statistics have some practice with actual empirical data designed to develop the art of guessing in such ways. Another example of nonrigorous mathematics used extensively in statistics is the whole business of asymptotic standard errors found by the differential method. It is desirable that good mathematics replace bad in such connections, but something is to be said for the position into which so many practical statisticians have been driven, that even bad mathematics may be better than none at all. The requisite good mathematics along these lines can come only through those who have made really serious studies of statistics, though a sufficiently interested pure mathematician might eventually be led by such a student of statistics to undertake and complete the necessary research. Practical needs make approximations necessary; the goodness of a particular approximation can often be judged adequately by a statistician familiar with the particular application long before the heavy artillery of advanced mathematical analysis can be brought to bear.

The teacher of statistics must have a genuine sympathy and understanding for applications, and these are not possessed by a great many pure mathematicians, at least in the opinion of some of those concerned with the applications; and it is this opinion rather than the possible fact that is of interest at the moment. For so long as such an opinion is maintained, for example by psychologists and economists, these specialists will be suspicious that courses in statistics given by a department consisting largely of pure mathematicians are unsuitable for their purposes. The result is likely to be a sabotaging of attempts at centralization, the different departments reverting to the old and ultimately objectionable system of teaching their own separate courses in statistical methods.

These difficulties are not necessarily insuperable, and it is to be expected that many medium-sized and small institutions will make their mathematical departments responsible for statistical teaching. But this ought not to be done without a consideration of the possible dangers.

We next consider curricular problems. These may be divided into those of the graduate school and those of the undergraduate college. Those of the graduate school may in turn be divided into those of specialization in statistics and of auxiliary teaching of statistics to students in other departments, such as sociology, who need to use statistical methods, have not studied them sufficiently as undergraduates, and cannot afford to put much time on them. Of these two subdivisions, the number of students at present is greater in the second and the ultimate importance is greater in the first, since the whole future of statistics depends on improvement and enlargement of this graduate teaching.

The incidental teaching of elementary statistical methods to graduate students in such subjects as sociology, without any prerequisite in mathematics or statistics, cannot equip these students with a command of the subject at all comparable to that which could be obtained by a better integration of undergraduate with graduate work. A prospective sociologist ought to study elementary statistical methods and concepts while still an undergraduate, and without

[3] Milton Friedman, "The use of ranks to avoid the assumption of normality implicit in the analysis of variance," *Jour. Amer. Stat. Assoc.*, vol. 32 (1937), pp. 675–701.

special reference to sociology. The features of statistical methods peculiar to their applications in sociology, and going beyond what is taught through illustrations and exercises in an elementary course, may be fit material for a course, graduate or undergraduate, in a Department of Sociology. Such a course should require as a prerequisite an elementary course in a Department of Statistics, or at least one taught by specialists in statistical method and theory rather than in sociology.

Graduate work specializing in statistics will for some years be the province of a few institutions only, for the simple reason that professors are not available to man more than a few. A graduate curriculum must include the theory of statistics as its main content, but needs also to give students an opportunity to strengthen their pure mathematics and to acquire a knowledge of a field of application of statistics and contact with practical statistical work. For the undergraduate college I recommend in place of the sporadic offerings now current in different departments a combination of two general fundamental courses with a number of advanced courses. Of the latter some will be specialized to the work of particular departments or groups of departments.

Of the two fundamental courses, one will require calculus as a prerequisitie, the other only a knowledge of first-year algebra. This knowledge is of course not the same thing as a record of having passed a course in first-year high school algebra, and must be enforced by examinations, preferably college entrance examinations. I do not believe that the resources of a college should be devoted to any teaching of statistics on a lower level than this. On the other hand, the additional value of statistical teaching in which calculus can be presupposed is so great that strong pressure and inducements should be brought to bear upon students to complete their calculus at an early date and then to study the statistics course based on calculus. Thus it is to be hoped that the less mathematical of these two general courses, instead of being elected by a majority of students, will gradually approach extinction, while the course based on calculus will become the vital point of contact of the student body with the concepts of statistics. One reason for this is that students who, on first acquaintance with statistics, become enthusiastic about it and want to go on should have the opportunity without being subjected to the repetition necessary if they first take the no-calculus course and then, because it is essential to more advanced work, must take the course based on calculus but covering much the same ground. But the chief reason is simply the possibility of covering important materials with the help of calculus which are inaccessible to those who do not have it.

At the same time an effort should be made to get students to study statistics early in their college careers in order to be able to use it later as a tool in their work with other subjects. The two objectives of early study of statistics, and of a first course in statistics based on calculus, can only be reconciled if calculus is taught early. This brings us back to the point made earlier, that there is far too much delay and inefficiency in the teaching of mathematics in schools, due partly to the system of teacher training and licensing, and putting our children at a disadvantage as compared with Europeans. It is desirable that the schools, legislatures, and educational officials take steps to teach calculus more widely in secondary schools, and to obtain teachers capable of more adequate mathematical teaching.

Modern statistical methods are based on the theory of probability, which is therefore essential to a proper understanding of them. The general courses in statistics may therefore begin with elementary probability. The duality between probability and statistical concepts,[4] for example between probability and relative frequency, between mathematical expectation and a sample mean, between parameter and statistic, should be explained. Derivations and the place of the normal distribution should be sketched, and the Student distribution should be derived and applied to a variety of problems in the first course based on calculus. Later courses given by the Department of Statistics, or whoever specializes in statistical theory, will naturally cover other statistical methods and theories. At the same time, useful courses can be offered in Economic Statistics, Mental Testing, and other fields using statistical methods by specialists, regardless of departmental affiliation. Only one limitation should be placed by the institution, as distinct from departments, on these courses in specialized and applied statistics: Students should not be admitted to them until after going through one or the other of the elementary general courses taught by specialists in statistical method. In addition, there might be departmental requirements. For example, the Department of Statistics might offer elementary and advanced courses in Correlation and Multivariate Analysis, and the Department of Psychology might require these also as prerequisites for some of its work in mental testing.

The teaching of statistics should be accompanied by considerable work in applied statistical problems, as well as exercises in mathematical theory, on the part of the students. A large part of this work in applied statistics is best conducted in a laboratory

[4] Cf. article, "Frequency distributions," *Encyclopedia of the Social Sciences* (1931).

equipped with calculating machines, mathematical tables, drafting instruments, and other appurtenances.

Statistical laboratories require supervision, administration, and maintenance. They are needed not only for the purpose of teaching statistics, pure and applied, at all levels, but also by research workers in many fields. There are possible gains of efficiency and economy in a centralized administration of them. One suggestion is that they be under the supervision of the university library. Another is that responsibility for them be lodged in a central Department of Statistics, or in a two-department Statistical Institute. Centralization can be carried too far, and it is likely that some units in a large organization will find it advantageous to have machines which are exclusively their own. The conflicting claims regarding machines and laboratories will require careful weighing.

A question may also be raised as to whether some work in statistics should not be required of all college students as a part of a liberal education. This would be a novel step, but has much to be said for it in view of the widespread use of statistics. The student who can't make up his mind as to his ultimate field of specialization or vocation will do well to study those things that can be used in many fields. Of such things, mathematics and statistics are leading examples. There are more or less sound objections to systems of required studies; but if we are to have them, the claim of statistics should not be rejected merely on grounds of novelty.

Summary.—The teaching of statistics, which has grown rapidly and seems likely to grow much further still, has many unsatisfactory features. The chief of these is the inadequate preparation in statistical theory of a large proportion of those teaching the subject. The evils tend to be perpetuated by the prevailing system of independent courses in elementary statistical method scattered through numerous departments concerned with applications. This system places the selection, supervision, and promotion of teachers of statistical method and theory in the hands of those who are not specialists in this subject. Teachers and prospective teachers of the theory of statistics feel a pressure to divert their efforts away from this theory and into its applications. In consequence, both statistical theory and the underlying mathematics are slighted, with the result that erroneous and inefficient methods continue to be taught and applied.

It is recommended that the preparation of teachers of statistical methods and theory be focused more definitely on this subject itself and the mathematics essential to it. Some study of a field of application, and practice in applications, are also desirable, but should not dominate the graduate curriculum in statistics. Research in the theory of statistics should be a major objective of graduate students and of teachers of statistics.

Organization of the teaching of statistical methods should be centralized, and should provide also for the joint functions of research and of advice and service needed by others in the institution, and possibly outside it, regarding the statistical aspects of their problems of designing experiments and interpreting observations. Beginning courses in statistical methods and theory should be taught only under the supervision of the central statistical organization, but courses in applied statistics, requiring these beginning courses as prerequisites, might be taught in any department. Of these first courses there should be two, one based on calculus and the other requiring no mathematics beyond elementary algebra. The more mathematical of these courses would be the more valuable, and efforts should be made to bring the larger number of students into it. The central statistical group would also teach more advanced courses in the subject.

Schools in this country have lagged behind those of Europe in the teaching of mathematics. If students in general had had calculus at an early stage, statistical teaching could be made both more efficient, in the sense of more material covered in less time, and more useful, in the sense of availability of statistical knowledge to the student at a time when the needs of his other subjects of study call naturally for it. A thoroughgoing reform of school mathematics is greatly needed, including a change in the system of training and licensing teachers so as to insure a better knowledge of mathematics on the part of teachers of the subject.