Comment

Joe R. Hill

Professor del Pino elegantly documents the many statistical applications of iterative generalized least squares. I thank him for his insightful comments on the relationships between the various algorithms.

My comments concern three issues: (1) estimation of dispersion parameters in mixture models; (2) conditional inference and model checking for normal nonlinear regression; and (3) quasilikelihood versus transformation.

1. DISPERSION PARAMETERS IN MIXTURE MODELS

To be specific, let's assume that:

(1) Given $\theta = (\theta_1, \cdots, \theta_k)$ and known $t_1, \cdots, t_k$, $y_1, \cdots, y_k$ are independent with

$$y_i \mid \theta \sim^{\text{ind}} \text{NEF}[\theta_i, \theta_i/t_i]$$

where NEF$[m, V(m)]$ stands for a natural exponential family with mean m and variance function V (Morris, 1982, 1983), hence

$$y_i \mid \theta_i \sim^{\text{ind}} \text{Poisson}(\theta_i t_i)/t_i.$$

(2) The θ_i are independent with

$$\theta_i \sim^{\text{ind}} \text{CF}[\mu_i = \exp\{x_i' \beta\}, \phi\mu_i]$$

where CF$[m, V(m)]$ is the conjugate family with mean m and variance function V, the x_i' are known $1 \times p$ vectors of covariate values, β is an unknown $r \times 1$ vector of regression coefficients, and ϕ is a dispersion parameter.

Marginally, the y_i are independent with

$$y_i \sim^{\text{ind}} \text{MF}[\mu_i, (1 + \phi t_i)\mu_i/t_i],$$

where MF stands for the marginal family. If ϕ is known, then the quasilikelihood estimator of β is statistically very efficient and computationally simpler compared to the MLE based on the negative binomial likelihood (Hill and Tsai, 1988).

If ϕ is unknown, extended quasilikelihood (Nelder and Pregibon, 1987) provides a method for estimating both β and ϕ. What is the author's opinion of this estimation procedure? Can he suggest a better one?

Also, if β is known, the extended quasilikelihood for ϕ is the likelihood for a curved exponential family (it treats the component deviances as though they had gamma distributions). This suggests that inferences about ϕ should be conditional on the ancillary information in the data. Though this is a bit far afield from the original topic of the article, would the author comment as to how this can be done? This problem also arises in unequal variance normal empirical Bayesian models. In that case, the extended quasilikelihood is the same as the likelihood, and the model is a curved exponential family.

2. NORMAL NONLINEAR REGRESSION

The procedures reviewed by the author pertain to the estimation of the regression parameters. Could he comment on conditional inference based on the resulting estimators? Cox (1980) suggested that the size of intrinsic curvature determines the importance of conditional inference. In unpublished work with Tsai, we found evidence to support this.

Also, what sort of model checking can be done based on the ancillary statistics? Hinkley (1980) gave a persuasive argument for conditional inference to account for model checking involving ancillaries.

3. QUASILIKELIHOOD VERSUS TRANSFORMATION

Finally, a brief comment on the last paragraph in Section 7. Hill and Tsai (1988) and Firth (1988) demonstrated a robustness property of quasilikelihood estimates compared to estimates based on transformations of the data to normality.

Again, I applaud the author for a very stimulating article.

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ADDITIONAL REFERENCES

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