based, multidimensional scaling method may be selected along with a higher degree, inner-product based, projection pursuit method. If the same qualitative features are present in such "orthogonal" analyses, the user can be more sure that the corresponding effects are real ones and not just an artifact of a particular method employed.

Finally, I wonder to what extent the OMEGA system could fruitfully be developed along the general lines very briefly sketched in my published discussion of Van der Heijden, de Falguerolles and De Leeuw (1989, page 275). The thrust of those remarks was in favor of a general constructive interplay between two broad approaches to data analysis: the exploratory, graphical approach and the confirmatory, modeling approach. Could OMEGA benefit from blending with the second of these? Some particular possibilities that come to mind are: brushing points that are influential for particular aspects of the analysis; examining the robustness of the methods proposed; borrowing ideas from the *model* choice literature in the present *method* choice context; and filtering to remove uninteresting model effects to see more clearly what remains (the thrust of the original paper).

REMARKS ON THE EXAMPLE

The following remarks concern "color strength: unexpected nonpredictability" (Section 5.2):

To what extent is the reduction from 29 to 5 variables in the PCA-COV analysis a reflection of dominant variation of these variables compared to the rest? Recalling the discussion in Section 3.1, it would be helpful to know to what extent the results go through in a PCA-COR analysis.

The (3, 5) and (4, 5) scatterplots in Figures 6 and 7 seem to reveal an outlier with low STRVI and STRREM values for its STRTRA figure.

The authors note two oddly placed batches in Figure 8: numbers 84 and 93. Could it be that these are ill-fitting points in the dominant PCA plane (perhaps with high loadings on a particular minor component)?

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Comment

N. I. Fisher

I am grateful for the opportunity to comment on this interesting piece of work. I regret that the rude interjection of the Australian holiday season has prevented me from giving the paper the attention it deserves, so I shall confine my remarks to a couple of specific aspects relating to graphical testing and estimation.

The authors are confronted by a common problem: the sheer volume of data sets being presented to the in-house statisticians means that the treatment of all but a very small number of sets must necessarily be

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brisk. Despite this, they wish to have some scope for exploratory analysis because of the complex multivariate nature of the data. The result, for each data set, is a conducted coach tour rather than a leisurely excursion.

This being the case, it is important that each stop on the tour provide the best view possible. In the context of graphical estimation, this means, for example, equipping fitted curves with pointwise or simultaneous confidence bands; and in graphical testing, it means designing plots which have characteristic shapes under the null and alternative hypotheses, and including objective means of assessment. These issues have been argued by Fisher (1987), but go back at least to Gnanadesikan (1985).

Specifically, consider the problem of testing for independence of X and Y. The authors go some way to building more statistics into their graphical display of $(X_1, Y_1), \cdots, (X_n, Y_n)$ by suggesting that plots be made of $(X_1, Y_{i_1}), \cdots, (X_n, Y_{i_n})$ for a few random permutations $Y_{i_1}, \cdots, Y_{i_n}$ of $Y_1, \cdots, Y_n$, to see whether the point clouds look similar. Unfortunately, structure in point clouds can be very difficult to assess. For this reason, Fisher and Switzer (1985) developed a general technique, the χ-plot, whereby transforms (λ_i, χ_i) of the data points (X_i, Y_i) are plotted, to yield patterns of points characteristic of independence, monotone dependence or more complex forms of dependence. Control lines based on permutations or large-sample theory can be added to allow more objective assessment of departure from the model of independence. The technical definition of the (λ_i, χ_i) values is appended below.

The technique can be extended to multivariate data. Fisher (1987) presented a chi-plot (or χ-plot) matrix, to be used in conjunction with a scatterplot matrix: the example from that paper is reproduced here. Figure 1 shows the scatterplot matrix for data based on test scores in the categories Visual Reception (VR), Visual Memory (VM), Auditory Association (AA), Auditory Memory (AM) and Grammatic Closure (GC), measured on 54 children using the Illinois Test of Psycholinguistic Ability (Seber, 1984, pages 122–123). The treatment and control groups were individually centered and then pooled, and the signs of the scores for Auditory Association changed to "−". The eye has some difficulty assessing the nature of possible dependences between the variates. On the other hand, the χ-plot matrix in Figure 2 readily expresses the similarities and differences. (The control lines are as defined below.) The χ-plots not involving Auditory Association all show modest positive association, whereas those involving Auditory Association all show comparable amounts of association but opposite in sign.

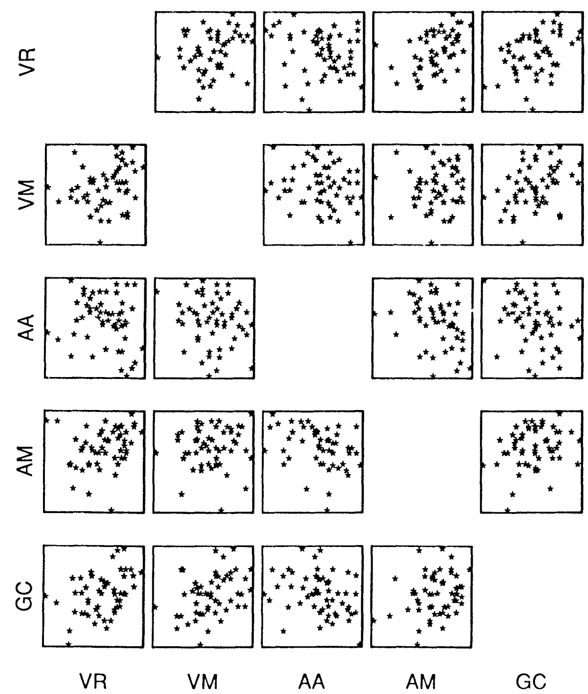

FIG. 1. *Scatterplot matrix for ITPA scores (see text for variable descriptions).*

John Robinson and I have generalized the concept of the χ-plot to testing for independence of two, or even k, vectors, based on the distribution function identity

$$F(x_1, \cdots, x_k) - F_1(x_1) \cdots F_k(x_k) = 0$$

under independence, and will report on this elsewhere.

Symmetry of a distribution can also be assessed graphically by plotting simply constructed functionals of the data; see, for example, the description and references cited in Fisher (1983, Section 2.2).

In the context of graphical estimation, similar properties of permutation or sign invariance can be exploited in some specific instances. In many other cases, we do not have recourse to such invariance structures and need to use resampling methods such as the bootstrap. Here again, we should be looking to see whether we can do rather better than simply displaying a cloud of grapeshot in the vicinity of a point estimate, or a skein of spaghetti strangling a density or regression estimate; rather, we should be attempting to draw on the increasing body of knowledge about proper use of the bootstrap for accurate estimation.

However, these remarks are at a rather trifling level compared with the main thrust of the authors' work, and I congratulate them on progress to date and look forward to subsequent developments.

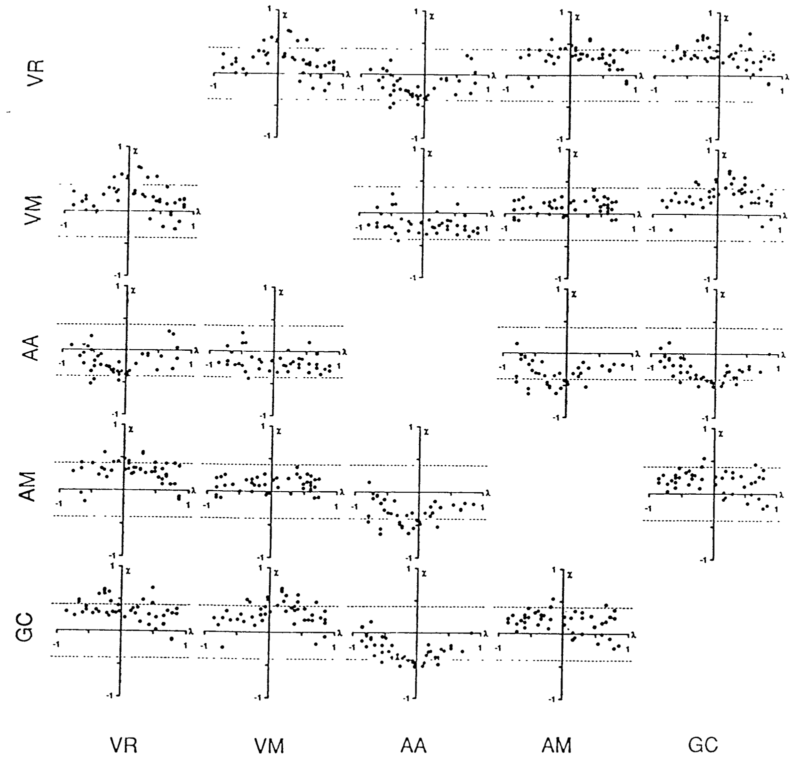

FIG. 2. *Chiplot matrix for ITPA scores plotted in Figure* 1.

APPENDIX: DEFINITION OF (λ_i, χ_i) VALUES IN A χ-PLOT

Let $(X_1, Y_1), \cdots, (X_n, Y_n)$ be a random sample of bivariate data, and define the bivariate and marginal sample distribution functions, evaluated at the data points, by

$$H_{ni} \equiv H_n(X_i, Y_i) = \sum_{j \neq i} I(X_j \leq X_i, Y_j \leq Y_i)/(n-1),$$

$$F_{ni} \equiv F_n(X_i) = \sum_{j \neq i} I(X_j \leq X_i)/(n-1),$$

$$G_{ni} \equiv G_n(Y_i) = \sum_{j \neq i} I(Y_j \leq Y_i)/(n-1),$$

where $I(A) = 1$ or 0 according as the event A is true or false. Then

$$\chi_{ni} \equiv \chi(X_i, Y_i) = \frac{H_{ni} - F_{ni}G_{ni}}{\{F_{ni}(1 - F_{ni})G_{ni}(1 - G_{ni})\}^{1/2}}.$$

Also, set

$$\lambda_{ni} = 4 \ \text{sgn}_{ni} \max\{(F_{ni} - 1/2)^2, (G_{ni} - 1/2)^2\},$$

where

$$\text{sgn}_{ni} = \text{sign}\{(F_{ni} - 1/2)(G_{ni} - 1/2)\}.$$

The value λ_{ni} is a measure of the distance of (X_i, Y_i) from (median (X), median (Y)). A χ-plot is based on the values of $(\lambda_{ni}, \chi_{ni})$. Control lines can be added at $\chi = \pm 1.96\sqrt{n}$, outside which we expect about 5% of points to fall under the hypothesis of independence.

In practice, two modifications are made.

(i) $\sin(\pi\chi/2)$ is used instead of χ; in sampling from the bivariate normal populations, the average value of χ_{ni} near $\lambda = 0$ approximates the correlation between X and Y.

(ii) the approximate normality of each χ_{ni}, when X and Y are independent, breaks down for extreme sample points. Accordingly, the $(\lambda_{ni}, \chi_{ni})$ value is not plotted if $|\lambda_{ni}| \geq 4\{1/(n-1) - 1/2\}^2$.

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