Statistical Science
2003, Vol. 18, No. 2, 133–134

Introduction to the Silver Anniversary of the Bootstrap

A number of times in recent years I have taught a graduate seminar course titled "R. A. Fisher in the 21st Century," a title that I unashamedly stole from Brad Efron's 1996 Fisher Lecture (Efron, 1998), which I had the pleasure of attending. The course starts by reading Brad's article and then three of Fisher's seminal articles (Fisher, 1922, 1925, 1934). After that, we trace our way forward in history, with the students taking the lead. Sometimes they trace ancillarity or sufficiency or likelihood. This latest time they were interested in estimation of the standard error (starting from the asymptotic approximation of Fisher, 1925), which naturally led us to the bootstrap.

The 1977 Reitz Lecture by Brad Efron was "Bootstrap Methods: Another Look at the Jackknife," which was published two years later (Efron, 1979). We were discussing this paper in class last year and suddenly it dawned on me that $2002 - 1977 = 25$ and this was the silver anniversary of the bootstrap! What better reason to have a special issue of *Statistical Science*?

To try and understand the impact that the bootstrap has had on statistics is an interesting exercise. It is obvious that a tool such as the bootstrap—which, at the very least, allows calculation of standard errors that were literally impossible to calculate previously—would have a huge impact on applications. However, the bonus was the impact on theory. The bootstrap has refocused some of our theory, redefining some of our limit theory and making second-order accuracy the benchmark. However, there was more.

The impact of the bootstrap has transcended both theory and applications. The bootstrap has shown us how to use the power of the computer and iterated calculations to go where theoretical calculations cannot, which introduces a different way of thinking about all of statistics. The idea of using computational power and resampling to do our calculations has switched us from thinking in terms of closed-form solutions (deriving formulas and proving theorems) to solving problems by writing algorithms and using iterations, resampling and lots of computer power. This was a huge change in the mindset of statistics (a *paradigm shift* in the words of Kuhn, 1996) that later received another kick forward with the Markov Chain Monte Carlo revolution of the early 1990s. Pushing along this shift in thinking is the most profound impact of the bootstrap.

To make the silver anniversary issue a reality, the first step was to talk with Brad, who gave the go-ahead. The rest was easy. It was wonderful that Brad agreed to provide us with a "Second Look ..." retrospective, and Davison, Hinkley and Young contributed an excellent broad overview of the methodology. There is also an interesting "prehistory" by Peter Hall, that shows that resampling was considered by other statisticians, but was not focused and refined until Efron. What then follows is a marvelous collection of contributions that describe some of the specific impacts that the bootstrap has had, spanning the gamut from the very applied to the very theoretical. We see that not only has the bootstrap allowed us to do calculations that previously were impossible, but there are examples where there may be no other reasonable solution than a bootstrap calculation (such as inference in phylogenetic trees, which needed two articles to do justice to the influence of the bootstrap).

There are many people to thank for helping to make this issue a reality. First of all, having Brad agree and oversee this project was a great help to me. The authors all were marvelous: everyone responded enthusiastically and promptly(!). I must also thank the unnamed referees, whose careful reading only improved manuscripts that were already very nicely done. Last, I really thank the interviewers, who, together with Brad, have provided us with one of the truly great conversations about statistics.

It is interesting to me that I have taught my seminar class three times, and each time that we traced Fisher forward it has led to Efron: once, through ancillarity, to Efron and Hinkley (1978); once, through curvature, to Efron (1975); this latest time to the bootstrap. The influence of Efron's work has been extraordinary, and it is a pleasure to present here a look at the scope of only one of his many contributions. Enjoy.

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