I have recently had my say elsewhere [Nelder (1986)] on the relations between statistics, mathematics, science and technology. If I had to make choices as a teacher I would be forced to give first priority to the statistical ideas underlying the analysis of variance (in my general sense) and its generalization, the analysis of deviance. This is not to say that the student should not be aware of the mathematics of balanced random structures; rather that (s)he should also be aware that the scientist and technologist will be interested primarily in the systematic part of the model (signal), and will regard the random part (noise) as an unavoidable nuisance. I believe that the analysis of variance, in its existing wider sense, has a useful part to play in the interpretation of the signal, so I should not like to see its use restricted to the description of the noise only.

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First of all, let me congratulate the author on an interesting paper. Though I find his use of the phrase analysis of variance somewhat unusual, it is certainly important to be aware that the mathematical tools in such seemingly different statistical procedures as that of analyzing a split plot experiment and that of analyzing a time series are, in fact, aspects of the same algebraic theory for spectral decompositions of covariance matrices.

I would like to stress a point related to the discussion of infinite arrays. What is said below can be read between the lines of Section 5, but I think it deserves more attention that the concept of an infinite array constitutes the ideal framework for discussion and clarification of the classical (and, unfortunately, slightly controversial) problem of negative variance components.

Very briefly, there are two kinds of variance component models:

(1) random effect models stated in terms of sums of independent random effects (error terms); and
(2) models stated in terms of covariances (as in Section 4).

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These two types of models are difficult to distinguish, because they are mathematically very similar. In fact, models of type (1) can—under balancedness assumptions—be interpreted as models of type (2) with some inequality restrictions on the eigenvalues of the covariance matrix. Conversely, models of type (2) can be forced into the framework (1) if one allows for negative variance components, i.e., formally negative values of the variances on some of the error terms. Thus, (1) and (2) are just two different ways of stating almost the same model, and this has been the origin of much confusion. However, all difficulties seem to disappear if one thinks of the type (1) models as models for *finite subarrays* of infinite arrays, in the spirit of Section 5. The variances on random effect terms in a type (1) model then become interpretable as proper population variances, which certainly cannot be negative. For example, consider measurements y_{ij} of the same quantity with similar instruments $i = 1, \dots, m$, each repeated n times, $j = 1, \dots, n$. In this case, the variance on the error term ε_i has a straightforward interpretation as the variance on the baseline error of an instrument from the population of instruments of this kind, whereas the variance on ε_{ij} is the variance on measurements corrected for baseline error. Conversely, consider an example of a finite array which cannot be extended arbitrarily in the j-direction, say a field trial with blocks $i = 1, \dots, m$ and plots $j = 1, \dots, n$ within each block. In this case, the random effects interpretation is usually not meaningful, because the plots within a given block cannot be regarded as a sample from some infinite population. The intrablock variance component may very well turn out to be negative (negative correlation between plots in the same block), and even if it is positive, it cannot be taken as a measure of variation to be transferred to some other design with a different block size.

Model (2) is, mathematically, the nice one. Model (1) is a model for incomplete data from an infinite array model of type (2), and as such implies some mathematical problems (estimates on the boundary, etc.). These problems are similar to (though less serious than) those coming up when a stationary time series (also a nice model) is restricted to a finite time interval.

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It is a pleasure to have the opportunity to discuss this paper. Dr. Speed lucidly presents a concise and consistent notion of ANOVA which is yet broad enough to touch on time series and harmonic analysis of groups. His most important contribution, however, may be to have reminded us that the title