

insist that the algebra \mathbf{A} generated by $\{A_a: a \in X\}$ contain the matrices I and J .

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By consideration of broadly ranging examples (really by an analysis of analysis of variance), Dr. Speed seeks a definition of an analysis of variance. In Section 4 he settles on a formulation that provides lots of insight. My remarks are to the effect that it would seem that his definition might be usefully broadened a bit.

There are practically occurring random process situations where it seems to me an anova exists, yet which escape Dr. Speed's definition, specifically the "equality constraints amongst (co)variances" part. Suppose one has a process $Y(\cdot)$, with stationary increments, for example, a stationary point process. Suppose, and this is usually no real restriction, $Y(0) = 0$. Then, following the work of Kolomogorov [see, e.g., Doob (1953), pages 551–559, Bochner (1947), Itô (1953)

and Masani (1972)], the process has a spectral representation

$$Y(t) = \int_{-\infty}^{\infty} [\exp\{i\lambda t\} - 1]/(i\lambda)Z(d\lambda),$$

with $Z(\cdot)$ a random process satisfying $\text{cov}\{Z(d\lambda), Z(d\mu)\} = \delta(\lambda - \mu)F(d\lambda) d\mu$ with $\delta(\cdot)$ the Dirac delta function and with $F(d\lambda)/(1 + \lambda^2)$ a bounded nonnegative measure. The (co)variance function of the process takes the form

$$\text{cov}\{Y(t), Y(u)\} = \int_{-\infty}^{\infty} [\exp\{i\lambda t\} - 1][\exp\{-i\lambda u\} - 1]/\lambda^2 F(d\lambda).$$

I would submit that these results and in particular the representation

$$\text{var } Y(t) = \int_{-\infty}^{\infty} (\sin \lambda t/2)^2/(\lambda/2)^2 F(d\lambda)$$

constitute an analysis of variance. It should be further mentioned that there are accompanying empirical analyses in the case that $F(\cdot)$ is absolutely continuous see, e.g., Bartlett (1963) and Brillinger (1972).

One way to be led to these results, and indeed corresponding results for stationary random generalized processes, is to apply the ordinary process results to a general linear functional, such as $\int a(t-u) dY(t)$, of the process of interest. This leads me to propose the following extension of Dr. Speed's definition. An anova is said to exist for some group of variates, if they satisfy the conditions of Dr. Speed's definition or if some natural class of functionals of them does. This definition would seem to obviate the need for some of the particular considerations in the manova case. I wonder if it does not lead to a general algebraic result on how a manova structure relates to the corresponding anova structure, quite independently of what the anova design was, for example.

I would like to end by thanking the Editor and Dr. Speed for the opportunity to comment on this important paper.

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The algebraic aspects of the analysis of variance are an intricate, well worked piece of ground. I am grateful to Speed and his coworkers for carefully sifting the