Whenever the distribution of the observable vector y is needed in this paper, it is assumed to be normal. However, an analysis of variance makes sense within the context of elliptically contoured distributions. In this case if y has a density, it can be written

$$|\Gamma|^{-1/2} g(y'\Gamma^{-1}y), \tag{12}$$

where $g(x'x)$ is a density in R^n. If

$$\int_0^\infty v^{n/2} g(v)\, dv < \infty, \tag{13}$$

the first two moments of y exist and $Ey = 0$, $Eyy' = \Gamma$. The likelihood function has a maximum at $\xi = (n/v_g)\hat{\xi}$, where v_g is the value of v maximizing $v^{n/2}g(v)$ and $\hat{\xi}$ is the maximum likelihood estimator under normality [Anderson, Fang and Hsu (1986, Theorem 1)]. The uncorrelatedness of $S_\alpha y_t$ and $S_\beta y_u$, $\alpha \neq \beta$, holds, but in general, independence of quadratic forms does not hold. For example, if $y'S_\alpha y$ and $y'(I - S_\alpha)y$ are independent, the distribution of y must be normal [Anderson and Fang (1987, Theorem 1)]. Nevertheless, F-tests are valid [Anderson, Fang and Hsu (1986, Theorem 2)].

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It is a pleasure to read this unified account of the analysis of variance, and the relationship between its many facets, for variance models based on association schemes. The theory of association schemes is an elegant piece of mathematics, as the recent book by Bannai and Itô (1984) shows, with many areas of application. Because this mathematics is so tractable, it is tempting to say, as Speed does in Section 4, that association schemes are the *only* natural framework for analysis of variance. I believe that this temptation should be resisted. The following two examples show that there are both practical and theoretical structures which have an analysis of variance (in the sense of Section 2) and yet which are not based on association schemes. In both cases every matrix A_a, for a in X, is symmetric with constant diagonal entries, and the set $\{A_a\colon a \in X\}$ commutes and spans the algebra it generates, which contains the matrix I. Both examples satisfy the criteria (i)–(iv) given near the end of Section 4.

EXAMPLE 1. Let X be a set of uniform factors on T, and suppose that

(i) for all a, b in X, the supremum $a \vee b$ of a and b is also in X;
(ii) for all a, b in X, the factors a and b are orthogonal modulo $a \vee b$;
(iii) X contains the trivial factor e, which has a different level for every element of T.

For each a in X, define the relationship matrix R_a as in Section 6. As Tjur (1984) and Duquenne (1986) have shown, variance models based on $\{R_a\colon a \in X\}$ have a straightforward analysis of variance, which parallels much of the present paper. Moreover, the sets X and Z in Section 4 can be taken to be identical, and the coefficients in (4.2) can be easily calculated recursively, using the semilattice X, whereas there is no such simple method for calculating those coefficients for a general association scheme. Although this example includes the association schemes of Speed and Bailey (1982), it also includes many structures which neither are association schemes nor can be extended to association schemes by the addition of extra factors. One such structure which does occur in practice is the row-column array in which rows are othogonal to columns but row-column intersections do not all have the same size: here X consists of the row-factor, the column-factor and the two trivial factors. Another such structure, albeit of more theoretical interest, is the Latin cube which is not based on an Abelian group: here X consists of the two trivial factors, the three factors for plane sections—rows, columns and layers—and the letter-factor.

EXAMPLE 2. Let T have the strucutre of the set of treatments in a rectangular lattice design. In the terminology of Bailey and Speed (1986), let S and F be the matrices such that, for t, u in T, the (t, u)-entry of S is equal to the number of spokes which contain both t and u, while the (t, u)-entry of F is equal to the number of fans which contain both t and u. The set $\{I, S, F, J\}$ is not, in general, the basis for an algebra generated by an association scheme, yet it leads to a perfectly tractable analysis of variance theory, just as in the present paper. To be sure, the matrices S and F are not $(0,1)$-matrices: nevertheless, if the covariance matrix Γ is a linear combination of I, S, F and J, then individual covariances depend only on the spoke- and fan-relationships between the relevant elements of T; moreover, a suitable linear combination of equations like (4.3b) does "analyse the variance" of individual elements, just like (2.3).

Of course, if the covariance model is to be based on randomization (Bailey, 1981) or symmetry arguments (Speed, 1985), then a group G of permutations of T is involved, as in Section 7. Then the covariance matrix is given by (4.1), where the matrices $\{A_a\colon a \in X\}$ are the adjacency matrices for the orbits of G on $T \times T$. If G is transitive, then conditions (ii)–(iv) at the beginning of Section 4 are satisfied. If, in addition, the symmetrized adjacency matrices commute (cf. Section 8), then they form an association scheme. Hence randomization-based covariance models which have an analysis of variance (in the sense of Section 2) are included in the association scheme theory, although to ignore the group is often to lose useful information. However, Speed himself says (Section 9) that a useful theory of analysis of variance should not be restricted to group-based models; it is not clear to me that the restirction to association-scheme models is any more natural or sensible.

I should like to draw a parallel with two other areas of the design of experiments where association schemes play an important role but do not tell the whole story. In ordinary block designs, association schemes were introduced by Bose and Shimamoto (1952) to clarify partially balanced designs. Many mathematicians have been so beguiled by the theory of association schemes that they have concentrated exclusively on partially balanced designs (including totally balanced designs). While many partially balanced designs do have good statistical properties, several statisticians have recently pointed out that some criteria, such as high efficiency or near-equality of concurrences, may be better satisfied by designs which are not partially balanced. In randomization theory, Bailey and Rowley (1987) have shown that association schemes provide a unifying framework for all the Fisherian designs with valid randomization sets of plans. However, there are other classes of design, not based on association schemes, which also have valid randomization theories.

My second main point is my concern at the attempt to limit the term "analysis of variance" to theory and practice based on (co)variance models. Whatever the individual words "analysis" and "variance" may mean, there can be no doubt that the majority of people who use the term "analysis of variance", or indeed the technique, do so in the context of analysing a designed experiment. Thus the elegant theory presented here is, at best, only half of what is needed. The complete analysis requires, in addition, a similar breakdown of the expectation space and then a specification of how the two decompositions fit together. This last part seems the hardest. Important steps have been taken by Nelder (1965), Houtman and Speed (1983) and Tobias (1986), but much remains to be done.

Incidentally, there appear to be two small mistakes in Section 3. Even if, for all a in X, the matrix A_a has constant row-sums, it does not necessarily follow that $n^{-1}J$ is one of the stratum projection matrices. For example, take $X = \{x\}$ and $A_x = I$. Disconnected distance-transitive graphs provide further counterexamples. Moreover, the facts that the matrices $\{A_a\colon a \in X\}$ commute and span the algebra they generate are not sufficient to ensure that the stratum projection matrices sum to I. For example, take $X = \{x\}$ and let A_x be any nonidentity idempotent. The only simple solution that I can see to these difficulties is to insist that the algebra **A** generated by $\{A_a: a \in X\}$ contain the matrices I and J.

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By consideration of broadly ranging examples (really by an analysis of analysis of variance), Dr. Speed seeks a definition of an analysis of variance. In Section 4 he settles on a formulation that provides lots of insight. My remarks are to the effect that it would seem that his definition might be usefully broadened a bit.

There are practically occurring random process situations where it seems to me an anova exists, yet which escape Dr. Speed's definition, specifically the "equality constraints amongst (co)variances" part. Suppose one has a process $Y(\cdot)$, with stationary increments, for example, a stationary point process. Suppose, and this is usually no real restriction, $Y(0) = 0$. Then, following the work of Kolomogorov [see, e.g., Doob (1953), pages 551–559, Bochner (1947), Itô (1953)