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CORRECTION

BOOTSTRAP TESTS AND CONFIDENCE REGIONS FOR FUNCTIONS OF A COVARIANCE MATRIX

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Corollary 2 of Section 3.2 is incorrect because the remainder term in equation (3.15) does not behave as asserted. We wish to thank David Tyler for pointing out this error.

Suppose dimension $p = 2$, the cdf F has finite fourth moments, and is such that the eigenvalues of Σ_F both equal $\nu(\Sigma_F) > 0$. Let $Z_F = \{z_{F,ij}\}$ be the random symmetric matrix defined in Section 2. For any constant symmetric 2×2 matrix $A = \{a_{ij}\}$, let

$$K_F(A) = L[(y_F + w_F(A), y_F - w_F(A))'],$$

where

$$y_F = 2^{-1}(z_{F,11} + z_{F,22}),$$

$$w_F(A) = 2^{-1}\Big\{\big[(z_{F,11} - z_{F,22} + a_{11} - a_{22})^2 + 4(z_{F,12} + a_{12})^2\big]^{1/2} - \big[(a_{11} - a_{22})^2 + 4a_{12}^2\big]^{1/2}\Big\}.$$

COROLLARY 2.1. *Under the model described above, $L[J_{n,\lambda}(\hat{F}_n)]$ converges weakly to $L[K_F(Z_F)]$ as $n \to \infty$. However, if $\min(m, n) \to \infty$ while $m/n \to 0$, then $J_{m,\lambda}(\hat{F}_n)$ converges weakly to $L[\lambda(Z_F)]$, in probability.*

Thus, in the equal roots case, the modified bootstrap estimate $J_{m,\lambda}(\hat{F}_n)$, which is based on bootstrap sample size $m = o(n)$, converges to the same limit law as does $J_{n,\lambda}(F)$; but $J_{n,\lambda}(\hat{F}_n)$ does not.

PROOF. Let $\mu_F(r_1, r_2) = E_F(x_{11}^{r_1} x_{12}^{r_2})$. Let D_4 be the set of all nonnegative integer pairs (r_1, r_2) such that $r_1 + r_2 = 4$. Let $\{F_n\}$ be any sequence of cdf's such that $F_n \Rightarrow F$, $\mu_{F_n}(r_1, r_2) \to \mu_F(r_1, r_2)$ for every $(r_1, r_2) \in D_4$, and $n^{1/2}(\Sigma_{F_n} - \Sigma_F) \to A$, a symmetric 2×2 matrix. By a straightforward calculation using Theorem 1 and the formulae for the two eigenvalues, $J_{n,\lambda}(F_n) \Rightarrow K_F(A)$.

Let $W_{n,F} = n^{1/2}(\hat{F}_n - F)$, $Z_{n,F} = n^{1/2}(\Sigma_{\hat{F}_n} - \Sigma_F)$, and $t_{n,F} = \{(\mu_{\hat{F}_n}(r_1, r_2) - \mu_F(r_1, r_2)\colon (r_1, r_2) \in D_4\}$. The empirical processes $\{(W_{n,F}, Z_{n,F}, t_{n,F})\}$ converge in law to a Gaussian process $(W_F, Z_F, 0)$. There exist versions $\{(W_{n,F}^*, Z_{n,F}^*, t_{n,F}^*)\}$

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and $(W_F^*, Z_F^*, 0)$ of these processes such that $\lim_{n\to\infty}(W_{n,F}^*, Z_{n,F}^*, t_{n,F}^*) = (W_F^*, Z_F^*, 0)$ for every realization. The relations $Z_{n,F} = n^{1/2}(\Sigma_{F+n^{-1/2}W_{n,F}} - \Sigma_F)$ and $t_{n,F} = \{\mu_{F+n^{-1/2}W_{n,F}}(r_1, r_2) - \mu_F(r_1, r_2)\}$ are preserved with probability 1 by the versions $\{(W_{n,F}^*, Z_{n,F}^*, t_{n,F}^*)\}$. Define versions $\{\hat{F}_n^*\}$ of $\{\hat{F}_n\}$ by $\hat{F}_n^* = F + n^{-1/2}W_{n,F}^*$. With probability 1, $\hat{F}_n^* \Rightarrow F$, $\mu_{\hat{F}_n^*}(r_1, r_2) \to \mu_F(r_1, r_2)$ for every $(r_1, r_2) \in D_4$, and $n^{1/2}(\Sigma_{\hat{F}_n^*} - \Sigma_F) \to Z_F^*$. In view of the previous paragraph, $J_{n,\lambda}(\hat{F}_n^*) \Rightarrow K_F(Z_F^*)$. This implies the first assertion of Corollary 2.1.

The second assertion is argued similarly: The convergences $\hat{F}_n \Rightarrow F$, $\mu_{\hat{F}_n}(r_1, r_2) \to \mu_F(r_1, r_2)$ for every $(r_1, r_2) \in D_4$, $m^{1/2}(\Sigma_{\hat{F}_n} - \Sigma_F) \to 0$, which occur in probability, imply that $J_{m,\lambda}(\hat{F}_n) \Rightarrow K_F(0) = L[\lambda(Z_F)]$ in probability. □

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