one treat (2.2) directly as the underlying generating mechanism of the process and derive forecasts from it? Perhaps one should also consider the purpose of time series analysis in defining influence functionals and gross error sensitivity.

In summary, the paper marks important progress on robustness in time series analysis and I congratulate the authors on their fine work.

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The problem of robust inference in time series is a long-standing one to which Professors Martin and Yohai have made signal contributions. This present paper continues the series of excellent contributions and I am pleased to have the opportunity to comment.

This paper lays out fundamental definitions of influence functionals and gross error sensitivity as a generalization of the corresponding concepts in the traditional i.i.d. case. These are illustrated with computations for several robust estimators of parameters in simple first order autoregressive and moving average models. These are basically toy examples, although the nonboundedness result for the MA(1) model is indeed intriguing. There is, however, a rich mine of further situations to explore, many of which may be formulated as the general replacement model.

Low order autoregressive schemes are useful in a feedback tracking context. That is to say, if $\mathbf{x}_i$ is a position-velocity vector subject to linear control by some guidance system, then a useful model for $\mathbf{x}_i$ may be an autoregressive model. Traditional approaches to such a problem often involve Kalman filtering. Clearly, however, position-velocity sensors may be subject to gross errors, for example, sun glint in an infrared (IR) sensor. Clearly noise in such a system (as opposed to innovations) could be modeled as a mixture distribution. Supposing sun glint did affect an IR sensor, it is likely to persist for some time, highlighting the Martin–Yohai concern with patchiness in the noise structure. The point is that a mildly realistic problem readily suggests many complicated models of general interest—robust estimation of parameters in a general nonlinear process model or robust estimation of the parameters of the Kalman filter, to mention just two.

A related, highly useful time series problem, perhaps the oldest time series problem, is the estimation of the sum of (essentially) deterministic sinusoids in white (often Gaussian) noise. Rotating machinery generates such sinusoids and the application is many-fold, including the obvious naval one. Often the ambient Gaussian noise is contaminated with impulsive noise, which can be either isolated (for example with biological or offshore drilling noises in the ocean acoustic setting, or thunderstorm noises in the electromagnetic setting) or patchy (say in the case of cracking–grinding ice in the arctic acoustic setting). Impulsive noise tends to be heavy tailed compared to the ambient noise process and is frequently modeled with double exponential marginals or even heavier tailed models. The book by Wegman and Smith (1984) contains several papers discussing realistic mixture distribution noise models. In most cases the Martin–Yohai AO model would be highly appropriate. An interesting special case is the reverberation limited case. In such a case, to use the Martin–Yohai notation for the AO model

$$w_i = x_i + \sum_j a_j x_{i-j} + v_i,$$

a_j unknown parameters. In most realistic cases, the fraction of contamination γ would be close to 1 since the reverberation $\sum_j a_j x_{i-j}$ is likely to be present whenever the signal, x_i, is present. Clearly the Martin–Yohai formulation of the IF and GES offers an exceedingly rich context for formulating many interesting time series estimation problems.

Another interesting problem whose robust formulation is unclear to me is the intervention problem. Suppose, for example, that x_i is a process whose fundamental probability structure changes as some unknown time, t_0. How can we estimate t_0?

On the surface, it would appear that a robust technique would tend to attribute deviations of y_i^γ, $i > t_0$, to outliers and hence suppress them. Admittedly I have thought only superficially about this. It is clear, however, that a rather more complex formulation is needed. It may be the case, for example, that x_i, w_i are independent if $i \le t_0$ but dependent for $i > t_0$. Similarly the signal detection problem could benefit from a robust treatment, but its robust formulation is unclear. Indeed, in both cases the statistic involved must distinguish rather subtle differences in underlying models which, in fact, may be masked by the robustified statistic. Still such problems in realistic circumstances often have data which are contaminated by outliers.

The point of this comment is not to critique the authors for what they have not done. In all truly innovative work there is obviously much left to be done. The point is only to mention a few situations which could profit from further exploration. The formulation of IF and GES in the time series context is an important step forward. I cannot resist observing that, in view of the inability to use Gateaux derivatives, it was clearly no piece of cake.

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