DISCUSSION

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Professors Martin and Yohai are to be complimented for their topical, thoughtful paper. In the paper they have emphasized population aspects of the material. In my discussion I will emphasize the data side. The two sides are both complementary and intersecting.

There is a circle of interrelated ideas: influence, sensitivity, deletion, resistance, leverage, robustness, and jackknifing. Work appears to progress on all of these fronts more or less simultaneously with algorithmic and computing advances often providing exogenous impetus. I will present a data analysis made possible by some contemporary time series methodology and easy availability of minicomputers.

The concern of Professors Martin and Yohai is to extend the concepts and methods of "influence" to the time series case. They proceed by examining the effects of contaminating the data, by studying for example gross-error sensitivity. In the i.i.d. case an immediate way to study the influence of a possibly incorrect data point is to delete it and to carry through the inference procedure for both the full and depleted data sets. Because of the invariance of the structure under permutations of the data, in the i.i.d. case ways forward are clear; however, as Professors Martin and Yohai emphasize, the permutation invariance is not generally present in the time series case. There is, however, a way to retain the full time series structure and still do deletion/jackknife type studies.

A long time ago (Brillinger, 1966), I suggested that a way to develop jackknife procedures for complex situations was to apply a missing-value technique. Briefly, on deleting the observation one is to act as if the data then consist of what it is but that that observation is missing. Luckily, nowadays we have many conceptual and methodological means for handling data with missing values. A way forward for studying the influence of individual observations in a variety of time series situations is now clear. In that connection it may be remarked that the procedure is a form of sensitivity analysis. Namely one is studying the effect of altering an observation to its "best" estimate based on the remaining data in some sense.

Resulting from the work of Ansley and Kohn (1984), Harvey and McKenzie (1984), Jones (1984), and Shumway (1984) there are a variety of methods to fit finite parameter (ARMA) models to discrete time series data having some missing values. In the calculations to be presented, the method of Jones (1980) was employed. Figure 1 is a graph of the logarithm of the Mackenzie River series of annual Canadian lynx trappings for the years 1821–1934. These data are studied, and much discussed, in Campbell and Walker (1977) and Tong (1977) for

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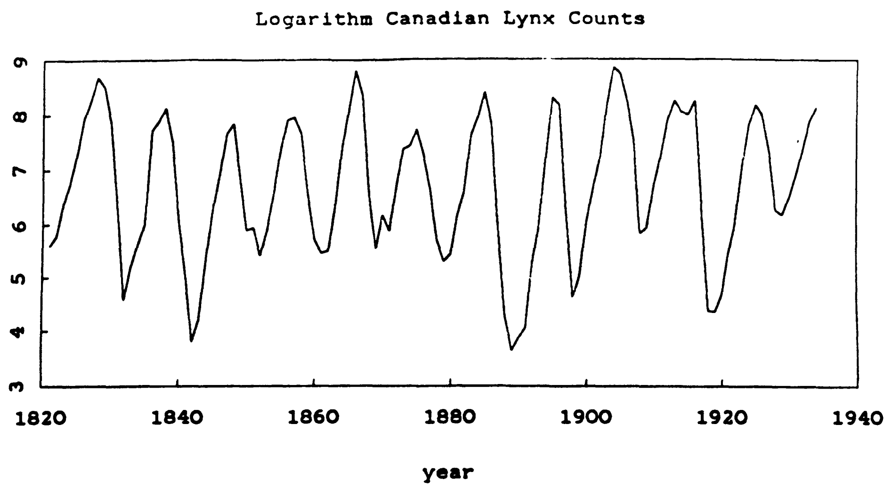

FIG. 1.

example. Taking note of Tong's fit of an ARMA(3, 3) model to this series, that was the principal model that I worked with. The method of fit was maximum likelihood, assuming the process to be Gaussian. Figure 2 gives the residuals (difference between observed and predicted) for the ARMA(3, 3). There are indications of lack of fit (indeed Tong (1983) goes on to fit nonlinear models); however, this model has sopped up a lot of the variation.

The ARMA(3, 3) model was next fitted to the data, by maximizing the likelihood, dropping out each of the 114 observations in turn. Six coefficients and

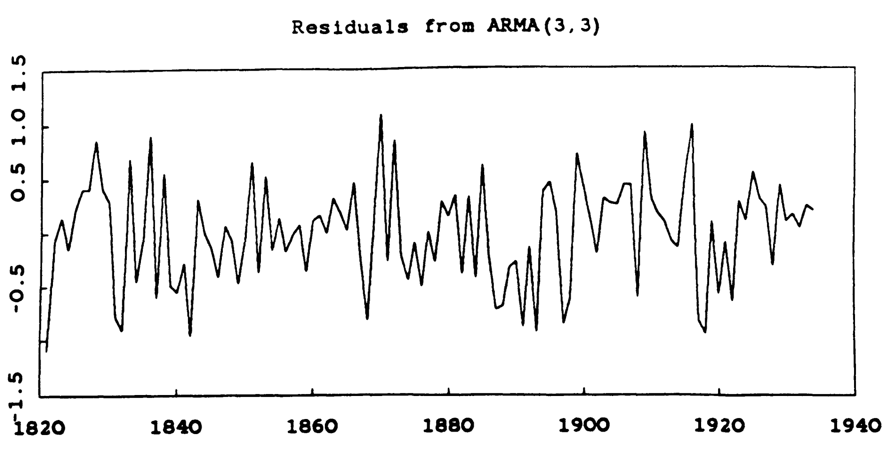

FIG. 2.

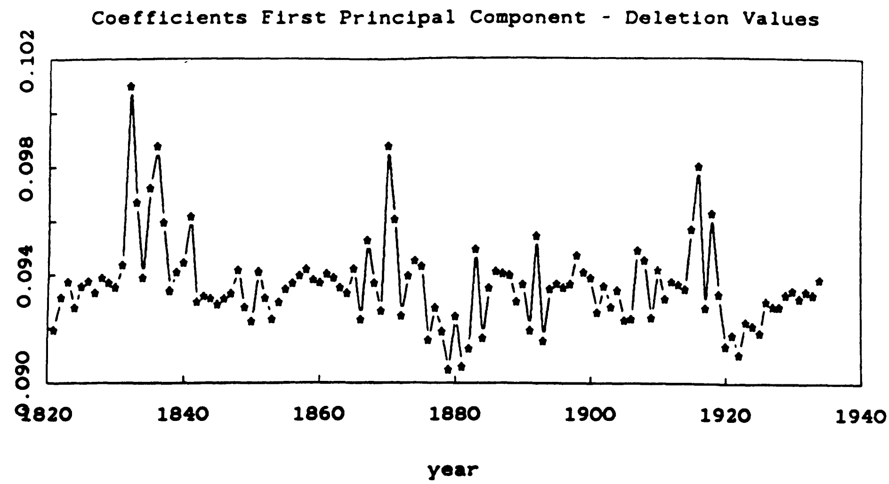

FIG. 3.

the innovation variance were estimated each time. (A program for fitting ARMA's with missing data was run each time. It is clear that an algorithm could be developed to reduce the computations involved, as is the situation in the i.i.d. case.) The coefficient estimates obtained were highly correlated. Rather than presenting six pictures of them, a principal component analysis of them was carried out. Figure 3 provides an (index) plot of the first principal component value versus the year of the deleted observation. A number of cases are seen to stand out, i.e., be apparently influential. In part these cases seem to correspond to "kinks" in the original series.

Professors Martin and Yohai have discussed bounding the influence of individual cases. This is a natural next step in the lynx data analysis, so I end by asking the authors how they would recommend fitting an ARMA in a bounded influence manner for the individual point case?

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This paper by Martin and Yohai will stimulate much future research. The authors are to be congratulated for that and for the presentation of the paper, which stresses statistical intuition and avoids technical detail, where possible.

The central point of their approach to the generalisation of the concept of influence function to a time series setting is the explicit dependence of that function on the arc along which the measure of the observed process approaches that of the nominal process. This emphasis on a specific model for the contamination is necessary because of the great range of possibilities for contamination in a time series setting. However, one can, consequently, ask how strongly the conclusions with respect to robustness and relative performance drawn from this influence function depend on the contamination model. The model (2.2) is very general but the major part of the paper and the examples in Section 5, in particular, deal only with z_i^γ given by (2.4). Consider, for example

$$(1) \qquad y_i = x_i + \eta_i, \qquad \eta_i = \sum_0^{k-1} \beta_j \varepsilon_{i-j},$$

where the ε_i are i.i.d. with distribution $(1-p)\delta_0 + pH$. Here the contamination is generated by impulses which excite a linear system whose effect is imposed on the nominal process. The model (1) is included in the general model (2.2) and both (1) and (2.4) could generate similar patterns of outlier patches so that it would be difficult to distinguish between them from the data. Of course it can be hoped that conclusions from the influence function, based on (2.4), will not differ substantially from those that would have been derived via (1), for example, since essential aspects of the influence function, such as gross error sensitivities, are essentially qualitative in nature and small numerical differences will be of no consequence. However, basically different types of outlier, e.g., isolated outliers compared to those occurring in patches, appear to lead to large differences, as