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INADMISSIBILITY RESULTS FOR THE SELECTED SCALE PARAMETERS

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Let $X_1, X_2, \ldots, X_k$ be k independent gamma random variables with different scale parameters but with a common known shape parameter. Suppose the population corresponding to the largest $X_{(1)}$ [or the smallest $X_{(k)}$] observation is selected. The problem of estimating the scale parameter $\theta_{(1)}$ [or $\theta_{(k)}$] of the selected population is considered. We derive, using the method of differential inequalities, explicit estimators that dominate the natural or the existing estimators. The improved estimators of $\theta_{(1)}$ are similar to that of DasGupta estimators for the usual simultaneous estimation problem. An implication of this result for the simultaneous estimation of the selected subset is also considered.

1. Introduction. Estimation of a characteristic of the selected population arises in various practical situations. Suppose, as an example, a doctor has experimented k types of drugs for a particular disease and chooses the best one. He might naturally be interested in obtaining an estimate of the effectiveness of the selected drug for the successful diagnosis of the disease. These types of problems of estimation after selection have been, of late, studied for various probability models. Some of the recent references in this area are Cohen and Sackrowitz (1982, 1989), Sackrowitz and Samuel-Cahn (1984, 1987) and Venter (1988).

Let $X_1, X_2, \ldots, X_k$ be k independent random variables, where the density of the X_i is

$$f_i(x|\theta_i, p) = \theta_i^{-p} e^{-(x/\theta_i)} x^{p-1}/\Gamma(p). \tag{1.1}$$

Assume the scale parameters θ_i's, $0 < \theta_i < \infty$, are unknown and the shape parameter p is known. Let $X_{(1)} \geq X_{(2)} \geq \cdots \geq X_{(k)}$ denote the order statistics of the X_i's. We shall call the population with the largest (or the smallest) scale parameter the best population.

Suppose we employ the natural rule, to select the best population, according to which the population corresponding to $X_{(1)}$ [or $X_{(k)}$] is selected. Let $\theta_{(1)}$ [or $\theta_{(k)}$] denote the scale parameter associated with the selected population. Observe that, for example, $\theta_{(1)}$ is a random quantity, and is given by

$$\theta_{(1)} = \begin{cases} \theta_i, & \text{if } X_i = X_{(1)}, \\ 0, & \text{otherwise}. \end{cases} \tag{1.2}$$

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In this paper we consider the problem of estimating $\theta_{(1)}$ and $\theta_{(k)}$ under the squared error loss (unless stated otherwise) defined, for instance, by

$$L(t, \theta_{(1)}) = (t - \theta_{(1)})^2. \tag{1.3}$$

We first consider the problem of estimating $\theta_{(k)}$. A natural estimator of $\theta_{(k)}$ is $X_{(k)}/p$. Also a natural competitor of $X_{(k)}/p$ is $X_{(k)}/(p+1)$, the analog of the best scale invariant estimator (of θ_1) $X_1/(p+1)$ of the component problem (i.e., the case $k = 1$). We have not been able to compare these two estimators analytically. However, we derive a class of estimators for $p > (k-1)^{-1}$ that dominate the natural estimator $X_{(k)}/p$. For the estimation of $\theta_{(1)}$, it is known that the estimator $X_{(1)}/(p+1)$ is better than the natural estimator $X_{(1)}/p$ [Vellaisamy and Sharma (1989)]. In this paper we obtain a class of estimators that are better than $X_{(1)}/(p+1)$, and are similar to DasGupta [(1986) Theorem 4] estimators for the usual simultaneous estimation of gamma scale parameters.

We exploit the technique of constructing improved estimators by solving certain differential inequalities on the sample space. This technique is well known and has been successfully employed by many statisticians, prominantly by Berger (1980), Hwang (1982) and DasGupta (1984, 1986), among others, for the simultaneous estimation problems where no selection is involved. Our method of proof is along the lines similar to that of Berger (1980) and that of DasGupta (1986). We also apply the UV method of Robbins (1988) to obtain the unbiased estimators of the risk difference.

2. The main results.

We first state a lemma which is essentially Corollary 2 of Berger (1980), and is useful in finding unbiased estimators of certain functions of $\mathbf{X} = (X_1, \dots, X_k)$ and $\boldsymbol{\theta} = (\theta_1, \dots, \theta_k)$.

LEMMA 2.1 [Berger (1980)]. *Suppose $X_1, X_2, \dots, X_k$ are k independent random variables with densities defined in* (1.1). *Let $u(\mathbf{x})$ be any real valued function defined on R^k such that* (i) $E_{\boldsymbol{\theta}}|u(\mathbf{X})| < \infty$ *and* (ii) *the indefinite integral*

$$h_i(\mathbf{x}) = \int_0^{x_i} u(x_1, \dots, x_{i-1}, t, x_{i+1}, \dots, x_k) t^{p-1}\, dt$$

exists for all $x_i \in R^1$. Then

$$v(\mathbf{x}) = x_i^{1-p} h_i(\mathbf{x}) \tag{2.1}$$

satisfies the condition

$$E_{\boldsymbol{\theta}}(v(\mathbf{X})) = \theta_i E_{\boldsymbol{\theta}}(u(\mathbf{X})) \quad \text{for all } \boldsymbol{\theta}.$$

We are now ready to state and prove the main results of the paper.

THEOREM 2.1. *Consider the estimation of $\theta_{(k)}$ under squared error loss. Let $p > (k-1)^{-1}$ and φ: $R^{(k-1)} \to R^1$ be any real valued function satisfying the* *following conditions*:

(i) $0 < \varphi \leq 2(kp - p - 1)/p(p+1)^2 = M$, *say*.
(ii) φ *is nonincreasing in each of the* $(k-1)$ *arguments*.

Then any estimator

$$\delta_\varphi(\mathbf{X}) = \frac{X_{(k)}}{p}\left[1 + p(p+1-X_{(k)})\varphi(X_{(1)},\dots,X_{(k-1)})e^{-X_{(k)}}\right]$$

dominates $\delta(\mathbf{X}) = X_{(k)}/p$.

PROOF. Let $\delta_1(\mathbf{X}) = \delta_1(X_{(1)}, X_{(2)},\dots, X_{(k)})$ be any estimator of $\theta_{(k)}$. Then the risk of δ_1 under squared error loss is

$$\begin{aligned} R(\delta_1, \boldsymbol{\theta}) = \sum_{i=1}^{k} E_{\boldsymbol{\theta}}\big[&\delta_1^2(X_{(1)i},\dots,X_{(k-1)i},X_i) \\ &-2\delta_1(X_{(1)i},\dots,X_{(k-1)i},X_i)\theta_i + \theta_i^2\big] I(X_i < X_{(k-1)i}), \end{aligned} \tag{2.2}$$

where $X_{(1)i} \geq X_{(2)i} \geq \cdots \geq X_{(k-1)i}$ denote the ordered values of $X_1, X_2,\dots, X_{i-1}, X_{i+1},\dots, X_k$. Applying Lemma 2.1, an unbiased estimator of

$$D_1(\boldsymbol{\theta}) = E_{\boldsymbol{\theta}}\left[\theta_i \delta_1(X_{(1)i},\dots,X_{(k-1)i},X_i) I(X_i < X_{(k-1)i})\right]$$

can be seen to be

$$\begin{aligned} \eta_1(\mathbf{X}) = X_i^{1-p}\big[&g_1(X_{(1)i},\dots,X_{(k-1)i},X_i) I(X_i < X_{(k-1)i}) \\ &+g_1(X_{(1)i},\dots,X_{(k-1)i},X_{(k-1)i}) I(X_{(k-1)i} < X_i)\big], \end{aligned} \tag{2.3}$$

where

$$g_1(x_1, x_2,\dots, x_k) = \int_0^{x_k} \delta_1(x_1, x_2,\dots, x_{k-1}, t) t^{p-1}\, dt. \tag{2.4}$$

Therefore the risk is equal to

$$\begin{aligned} R(\delta_1, \boldsymbol{\theta}) &= \sum_{i=1}^{k} E_{\boldsymbol{\theta}}\Big[\big\{\delta_1^2(X_{(1)i},\dots,X_{(k-1)i},X_i) \\ &\qquad -2X_i^{1-p} g_1(X_{(1)i},\dots,X_{(k-1)i},X_i) + \theta_i^2\big\} I(X_i < X_{(k-1)i}) \\ &\qquad -2X_i^{1-p} g_1(X_{(1)i},\dots,X_{(k-1)i},X_{(k-1)i}) I(X_i > X_{(k-1)i})\Big] \\ &= E_{\boldsymbol{\theta}}\big[\delta_1^2(X_{(1)},\dots,X_{(k)}) - 2X_{(k)}^{1-p} g_1(X_{(1)},\dots,X_{(k)}) \\ &\qquad -2G_1(X_{(1)},\dots,X_{(k)}) + \theta_{(k)}^2\big], \end{aligned} \tag{2.5}$$

where

$$G_1(X_{(1)},\dots,X_{(k)}) = X_{(1)}^{1-p} g_1(X_{(2)},\dots,X_{(k)},X_{(k)}) + \sum_{i=2}^{k-1} X_{(i)}^{1-p} g_1(X_{(1)},\dots,X_{(i-1)},X_{(i+1)},\dots,X_{(k)},X_{(k)}).$$

Using the fact

$$g_1^{k(1)}(x_1,\dots,x_k) = \frac{\partial}{\partial x_k} g_1(x_1,\dots,x_k) = \delta_1(x_1,\dots,x_k) x_k^{p-1},$$

we can write

$$R(\delta_1,\boldsymbol{\theta}) = E_{\boldsymbol{\theta}}\Big[X_{(k)}^{2(1-p)} \big(g_1^{k(1)}(X_{(1)},\dots,X_{(k)})\big)^2 \tag{2.6}$$
$$- 2X_{(k)}^{1-p} g_1(X_{(1)},\dots,X_{(k)}) - 2G_1(X_{(1)},\dots,X_{(k)}) + \theta_{(k)}^2 \Big].$$

Let $\delta_2(\mathbf{X}) = \delta_2(X_{(1)},\dots,X_{(k)})$ be any other competing estimator of $\theta_{(k)}$ such that $h(x_1,\dots,x_k) = g_2(x_1,\dots,x_k) - g_1(x_1,\dots,x_k)$ is a nonincreasing function in each of first $(k-1)$ arguments, and $g_2(x_1,\dots,x_k)$ be defined as in (2.4) with δ_1 replaced by δ_2. Then an unbiased estimator of the risk difference $R(\delta_2,\boldsymbol{\theta}) - R(\delta_1,\boldsymbol{\theta})$ is given by

$$\Delta_1(\mathbf{X}) = X_{(k)}^{2(1-p)}\Big\{ \big(g_2^{k(1)}(X_{(1)},\dots,X_{(k)})\big)^2 - \big(g_1^{k(1)}(X_{(1)},\dots,X_{(k)})\big)^2 \Big\} \tag{2.7}$$
$$- 2X_{(k)}^{1-p} h(X_{(1)},\dots,X_{(k)})$$
$$- 2\big\{G_2(X_{(1)},\dots,X_{(k)}) - G_1(X_{(1)},\dots,X_{(k)})\big\}.$$

Note that

$$\begin{aligned} G_2(X_{(1)},\dots,X_{(k)}) &- G_1(X_{(1)},\dots,X_{(k)}) \\ &= X_{(1)}^{1-p} h(X_{(2)},\dots,X_{(k)},X_{(k)}) \\ &\quad + \sum_{i=2}^{k-1} X_{(i)}^{1-p} h(X_{(1)},\dots,X_{(i-1)},X_{(i+1)},\dots,X_{(k)},X_{(k)}) \\ &\geq (k-1) X_{(k)}^{1-p} h(X_{(1)},\dots,X_{(k-2)},X_{(k-1)},X_{(k)}), \end{aligned}$$

since $h(x_1,\dots,x_k)$ is nonincreasing in the first $(k-1)$ arguments. Therefore,

$$\Delta_1(\mathbf{X}) \leq \Delta_2(\mathbf{X}) = X_{(k)}^{2(1-p)}\Big\{ \big(g_2^{k(1)}(X_{(1)},\dots,X_{(k)})\big)^2 - \big(g_1^{k(1)}(X_{(1)},\dots,X_{(k)})\big)^2 \Big\}$$
$$- 2kX_{(k)}^{(1-p)} h(X_{(1)},\dots,X_{(k)}).$$

Now let $\delta_1(X_{(1)},\dots,X_{(k)}) = X_{(k)}/p$. Then

$$h^{k(1)} = g_2^{k(1)} - g_1^{k(1)} = g_2^{k(1)} - x_k^p/p$$

and

$$\left(g_2^{k(1)}\right)^2 - \left(g_1^{k(1)}\right)^2 = \left(h^{k(1)}\right)^2 + 2x_k^p h^{k(1)}/p.$$

Hence,

$$\Delta_2(\mathbf{X}) = X_{(k)}^{2(1-p)}\left(h^{k(1)}\right)^2 + \frac{2}{p}X_{(k)}^{2-p}h^{k(1)} - 2kX_{(k)}^{1-p}h. \tag{2.8}$$

Therefore, we have to solve the differential inequality

$$\Delta_3(\mathbf{x}) < 0 \quad \text{for } x_1 \geq x_2 \geq \cdots \geq x_k > 0, \tag{2.9}$$

where

$$\Delta_3(\mathbf{x}) = x_k^{(1-p)}\left(h^{k(1)}\right)^2 + \frac{2}{p}x_k h^{k(1)} - 2kh.$$

Now let

$$h(x_1, \dots, x_k) = \varphi(x_1, \dots, x_{k-1})\psi(x_k),$$

where φ satisfies the assumptions of the theorem and $\psi(t)$ is positive and is differentiable at all $t \in (0, \infty)$. Then our problem reduces to that of solving the inequality

$$\Delta_4(x_k) = x_k^{1-p}\left(\psi'(x_k)\right)^2 + a_1 x_k \psi'(x_k) - a_2\psi(x_k) < 0, \tag{2.10}$$

where $a_1 = 2/pM$, $a_2 = 2k/M$, $\psi'(x) = (d/dx)\psi(x)$ and $x_k > 0$. It can be shown that $\psi(x_k) = x_k^{p+1}e^{-x_k}$ is a solution of the above inequality. Hence

$$h(x_1, \dots, x_k) = \varphi(x_1, \dots, x_{k-1})x_k^{p+1}e^{-x_k} \tag{2.11}$$

is a solution of the differential inequality given in (2.9).

Let us now take

$$\delta_2(x_1, \dots, x_k) = x_k s(x_1, \dots, x_k)/p, \tag{2.12}$$

where $s(x_1, \dots, x_k)$ is a suitable function to be determined later. Then, from (2.11) and (2.12),

$$\begin{aligned} h^{k(1)}(x_1, \dots, x_k) &= x_k^p[s(x_1, \dots, x_k) - 1]/p. \\ &= \varphi(x_1, \dots, x_{k-1})(p + 1 - x_k)x_k^p e^{-x_k} \end{aligned} \tag{2.13}$$

and hence,

$$s(x_1, \dots, x_k) = 1 + p\varphi(x_1, \dots, x_{k-1})(p + 1 - x_k)e^{-x_k}.$$

Therefore,

$$\delta_2(\mathbf{X}) = \frac{X_{(k)}}{p}\left[1 + p(p + 1 - X_{(k)})\varphi(X_{(1)}, \dots, X_{(k-1)})e^{-X_{(k)}}\right]$$

improves upon $\delta_1(\mathbf{X})$, which proves the theorem. □

REMARK 2.1. We have been able to find improved estimators when $p > (k-1)^{-1}$, by solving the differential inequality in a rather crude way. Although our result covers most of the practical cases, it will be interesting to obtain improved estimators for the case $p < (k-1)^{-1}$ also.

We next consider the estimation of $\theta_{(1)}$. The following theorem, which is similar to Theorem 4 of DasGupta (1986), presents improved estimators over the existing estimator $X_{(1)}/(p+1)$ [Vellaisamy and Sharma (1989)].

THEOREM 2.2. *Let* $Y = (\prod_{i=1}^{k} X_{(i)})^{1/k}$, *and for* $t > 0$, $s(t)$ *be a real valued nondecreasing function such that* (i) $s(t)/t$ *is nonincreasing, and* (ii) $0 < s(t)/t \le 2k(k-1)/(p+k)^2$. *Then any estimator of the form*

$$(2.14)\quad \delta_s(X_{(1)},\dots,X_{(k)}) = \frac{X_{(1)}}{p+1} + \frac{1}{(p+1)}\left[ps(Y) + \frac{1}{k}Ys'(Y)\right]$$

dominates $X_{(1)}/(p+1)$.

PROOF. We present here only the important steps as most part of the proof is similar to that of Theorem 2.1.

Let $\nu_1(\mathbf{X}) = \nu_1(X_{(1)},\dots,X_{(k)})$ be an arbitrary estimator of $\theta_{(1)}$. Then an unbiased estimator of

$$D_2(\boldsymbol{\theta}) = E_{\boldsymbol{\theta}}\left[\theta_i \nu_1(X_i, X_{(1)i},\dots,X_{(k-1)i})\right]$$

can be seen to be

$$(2.15)\qquad \eta_2(\mathbf{X}) = X_i^{(1-p)} q_1(X_i, X_{(1)i},\dots,X_{(k-1)i}) I(X_i > X_{(1)i}),$$

where

$$q_1(x_1, x_2,\dots,x_k) = \int_{x_2}^{x_1} \nu_1(t, x_2,\dots,x_k) t^{p-1}\,dt.$$

Let $\nu_2(\mathbf{X}) = \nu_2(X_{(1)},\dots,X_{(k)})$ be any other estimator of $\theta_{(1)}$, and $q_2(x_1,\dots,x_k)$ be the corresponding function associated with ν_2. Then an unbiased estimator of the risk difference $R(\nu_2,\boldsymbol{\theta}) - R(\nu_1,\boldsymbol{\theta})$ is,

$$(2.16)\quad H(X_{(1)},\dots,X_{(k)}) = X_{(1)}^{2(1-p)}\left[\left(q_2^{1(1)}\right)^2 - \left(q_1^{1(1)}\right)^2\right] - 2X_{(1)}^{1-p}(q_2 - q_1),$$

where for example

$$q_1^{1(1)}(x_1,\dots,x_k) = \frac{\partial}{\partial x_1} q(x_1,\dots,x_k) = x_1^{p-1}\nu_1(x_1,\dots,x_k).$$

Let now $\nu_1(X_{(1)},\dots,X_{(k)}) = X_{(1)}/(p+1)$, and

$$\nu_2(X_{(1)},\dots,X_{(k)}) = \frac{X_{(1)}}{(p+1)} w(X_{(1)},\dots,X_{(k)}),$$

where w is some suitable function.

Following Berger (1980), let

$$F(x_1,\dots,x_k) = (p+1)(q_2(x_1,\dots,x_k) - q_1(x_1,\dots,x_k)),$$

where now $q_1(x_1, \ldots, x_k) = (x_1^{p+1} - x_2^{p+1})/(p+1)^2$. Then (2.16) reduces to

$$(2.17)\quad \begin{aligned} H(X_{(1)}, \ldots, X_{(k)}) = (p+1)^{-2}\Big[& X_{(1)}^{2(1-p)}\big(F^{1(1)}(X_{(1)}, \ldots, X_{(k)})\big)^2 \\ & + 2X_{(1)}^{(2-p)} F^{1(1)}(X_{(1)}, \ldots, X_{(k)})\Big] \\ & - 2(p+1)^{-1} F(X_{(1)}, \ldots, X_{(k)}) X_{(1)}^{1-p}. \end{aligned}$$

Note here that (2.17) is of the same form as that of (2.7) of Berger (1980). Therefore, the differential inequality to be solved is

$$(2.18)\qquad H(x_1, \ldots, x_k) \le 0 \quad \text{for } x_1 \ge x_2 \ge \cdots \ge x_k > 0.$$

As in DasGupta (1986), choose now

$$F(x_1, \ldots, x_k) = x_1^p s(y),$$

where $y = (\prod_1^k x_i)^{1/k}$, and $s(\cdot)$ satisfies the assumptions of the theorem. Using the facts,

$$(2.19)\qquad F^{1(1)}(x_1, \ldots, x_k) = x_1^{p-1}\left[ps(y) + \frac{1}{k} ys'(y)\right],$$

$ys'(y) \le y$ and $x_1 \ge y$, it can be shown [see DasGupta (1986), Theorem 4] that the above choice of F is a solution of the inequality in (2.18). Also, it is easy to observe that

$$w(x_1, \ldots, x_k) = 1 + x_1^{-1}\left[ps(y) + \frac{1}{k} ys'(y)\right].$$

This establishes the theorem. □

REMARK 2.2. Note that one could also take Y as the geometric mean of the first r order statistics, that is, $Y = (\prod_{i=1}^r X_{(i)})^{1/r}$, $2 \le r \le k$. The theorem still holds although the upper bound for $s(t)/t$ is now $2r(r-1)/(p+r)^2$.

3. Estimation of the selected subset. Suppose a subset of the given k gamma populations is selected using following rule [Gupta (1963)]:

RULE. Select the population corresponding to X_i if and only if

$$X_i \ge cX_{(1)},$$

where c, $0 < c < 1$, is some suitable number.

It is known [Vellaisamy (1990)] that the estimator

$$U_1 = \frac{X_{(1)}}{(p+1)} I(X_{(2)} < cX_{(1)}) + \frac{1}{p}\sum_{r=2}^{k} X^{(r)} I(X_{(r+1)} < cX_{(1)} \le X_{(r)})$$

is better than the natural estimator

$$U = \frac{1}{p} \sum_{r=1}^{k} X^{(r)} I\big(X_{(r+1)} < cX_{(1)} \le X_{(r)}\big),$$

for the estimation of

$$Q = \sum_{r=1}^{k} \theta^{(r)} I\big(X_{(r+1)} < cX_{(1)} \le X_{(r)}\big)$$

under the squared error loss function defined, for example, by

$$L(Q, U) = \sum_{r=1}^{k} \left[\sum_{j=1}^{r} \left(\frac{X_{(j)}}{p} - \theta_{(j)} \right)^2 \right] I\big(X_{(r+1)} < cX_{(1)} \le X_{(r)}\big),$$

where $X^{(r)} = (X_{(1)}, \dots, X_{(r)})$, $\theta^{(r)} = (\theta_{(1)}, \dots, \theta_{(r)})$ and $\theta_{(i)}$ is the parameter associated with $X_{(i)}$, $i = 1, \dots, k$.

Let us now define

$$L = \sum_{r=1}^{k} L^{(r)} I\big(X_{(r+1)} < cX_{(1)} \le X_{(r)}\big)$$

and

$$M = \sum_{r=1}^{k} M^{(r)} I\big(X_{(r+1)} < cX_{(1)} \le X_{(r)}\big)$$

to be two estimators of Q, where for $1 \le r \le k$,

$$L^{(r)} = (L_1, L_2, \dots, L_r), \qquad M^{(r)} = (M_1, L_2, \dots, L_r)$$

with $L_1 = X_{(1)}/(p+1)$, $M_1 = \delta_s(X_{(1)}, \dots, X_{(k)})$ defined in (2.14), and $L_2, \dots, L_r$ are arbitrary estimators. An immediate consequence of Theorem 2.2 is that the estimator M dominates L.

It is now clear that the estimator

$$U_s = \delta_s(X_{(1)}, \dots, X_{(k)}) I\big(X_{(2)} < cX_{(1)}\big) + \frac{1}{p} \sum_{r=2}^{k} X^{(r)} I\big(X_{(r+1)} < cX_{(1)} \le X_{(r)}\big)$$

dominates the existing estimator U_1 of Q.

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