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# ESTIMATION OF A COMMON MEAN AND RECOVERY OF INTERBLOCK INFORMATION

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Consider the problem of combining two unbiased estimators of a parameter when the estimators are known to be independent normal variables with unknown variances possibly unequal. The two one parameter families of estimators studied in Brown and Cohen, and Khatri and Shah, are accommodated in a single two parameter family studied in this paper and the results in the two papers are unified. For the type of estimators considered by Brown and Cohen, this paper not only offers a generalization but also a significant improvement. This improvement concerns the main result in Theorem 2.1 of Brown and Cohen and has bearing on their entire paper except the last section on interval estimation. Extensions of Brown and Cohen's Theorem 4.1 concerning the point estimation of the common mean of $K$-populations and Theorem 5.1 concerning interval estimation of the common mean of two populations are also presented.

**1. Introduction.** The problem of estimating the common mean of two normal distributions and the related problem of recovery of interblock information have been studied in several papers. Since Yates (1939, 1940) initiated the whole subject and his work was extended by Nair (1944) and Rao (1947, 1956) many have turned their attention to the problem of constructing a combined unbiased estimator with a variance uniformly smaller than that of the intrablock estimator (first of the two sample means in the common mean problem). Of these, the earlier works by Graybill and Deal (1959), Seshadri (1963 a, b), Shah (1964) and Stein (1966) are applicable only to some special designs. Recent works by Brown and Cohen (1974) and Khatri and Shah (1974), with which this paper is really concerned, are applicable to any incomplete block design with more than 3 blocks. For the sake of brevity, these two papers will henceforth be referred to as BC and KS, respectively. For the same reason a statement like '$T$ is uniformly better than $x$' will be written as '$T$ ub $x$'.

Both BC and KS, in fact considered a whole family of estimators depending on a single parameter and while the former required an upper bound on their parameter, the latter required a lower bound. The two families of estimators mentioned above can be treated as particular cases of a two parameter family and the desired property could be guaranteed by a single condition on the two parameters. The upper bound set on their parameter by BC was somewhat crude and involved a complicated expression which called for a table given in their paper. Thus, the

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present paper unifies these two classes of estimators and improves the result of BC for the subclass which corresponds to their estimator. The improved upper bound of the BC parameter, as given in this paper, is not only more precise but also is a simple expression, for which no table is required. For the estimator considered by BC, the improved upper bound of their parameter is, in fact, shown to be the best possible. Incidentally, it was noticed that Theorem 4.1 and Theorem 5.1 of BC can be easily extended to the corresponding two parameter family and these have been presented in Remark 2.4 after the main result in Section 2. Finally, Section 3 contains application of the results of Section 2 to the two main problems, namely, estimation of a common mean and recovery of inter-block information, which motivated most of the studies in this field.

**2. Main results.** Let $x, y, S, T, W_i, i = 1, 2, \cdots, q$ be independent random variables where $x \sim N(\mu, \alpha_0\sigma^2)$, $y \sim N(\mu, \beta_0\eta^2)$, $S/\sigma^2 \sim \chi^2_m$, $T/\eta^2 \sim \chi^2_n$, and $W_i/(\alpha_i\sigma^2 + \beta_i\eta^2) \sim \chi^2_1$, $i = 1, 2, \cdots, q$. Let

$$\hat{\mu} = x + \phi(y - x) \tag{2.1}$$

with

$$\phi = aS/\left[S + c\left\{T + (y-x)^2/\beta_0 + \Sigma_{i=1}^q W_i/\beta_i\right\}\right] \tag{2.2}$$

and $a$ and $c$ are constants to be suitably chosen.

Let $W_0$ be independent of $S$, $T$, and $W_i$, $i = 1, 2, \cdots, q$ such that $W_0/(\alpha_0\sigma^2 + \beta_0\eta^2) \sim \chi^2_3$ and let $\phi^*$ be the expression obtained by replacing $(y-x)^2$ in $\phi$ by $W_0$. Both BC and KS have shown that $V(\hat{\mu}) \leqslant V(x)$ for all values of $\tau$ iff

$$(1+\tau)E(\phi^{*2}) \leqslant 2E\phi^* \quad \text{for every} \quad \tau \tag{2.3}$$

where $\tau = \beta_0\eta^2/(\alpha_0\sigma^2)$. Let $r = (1+\tau)\phi^*/a$. It is easy to verify that (2.3) is equivalent to $a \leqslant 2E(r)/E(r^2)$ for every $\tau$. Thus,

THEOREM 2.1. *$\hat{\mu}$ is uniformly better than $x$* iff *$a \leqslant 2\theta$ where $\theta = \inf_\tau E(r)/E(r^2)$.*

Since it is not easy to evaluate $\theta$ in all cases attempts will be made to obtain some nontrivial lower bounds for $\theta$. Let

$Z_1 = S/\sigma^2, Z_2 = T/\eta^2 + \Sigma_{i=0}^q W_i/(\alpha_i\sigma^2 + \beta_i\eta^2), \quad u_j = W_j/\left[(\alpha_j\sigma^2 + \beta_j\eta^2)Z_2\right]$, $j = 0, 1, \cdots, q$ and let $u = \beta_0(\Sigma\alpha_j u_j/\beta_j)/\alpha_0$. It is easy to see that $Z_1 \sim \chi^2_m$, $Z_2 \sim \chi^2_{n+q+3}$, and that $u, Z_1, Z_2$ are all independently distributed. It is also not difficult to see that $r$ can be written in the form

$$r = Z_1/\left[(1-\gamma)Z_1 + dZ_2 h(u,\gamma)\right] \tag{2.4}$$

where $d = c\alpha_0/\beta_0$, $\gamma = \tau/(1+\tau)$, $h(u,\gamma) = u(1-\gamma) + \gamma$. Let $\delta = \inf_{u,\gamma} f(u,\gamma)$ where $f(u,\gamma) = E(r|u)/E(r^2|u)$. Clearly, $\delta \leqslant \theta$ and hence it would be sufficient to have $a \leqslant 2\delta$ to ensure $v(\hat{\mu}) \leqslant V(x)$. Let primes denote derivations with respect to $\gamma$. Then direct computation from (2.4) shows that

$$r' = \left[r^2 - (1-u)r\right]/h(u,\gamma). \tag{2.5}$$

Also $f' = -g(u)/[h(u, \gamma)E^2(r^2|u)]$ where

$$g(u) = 2E(r|u)E(r^3|u) - (1 - u)E(r|u)E(r^2|u) - E^2(r^2|u).$$

The following lemma can now be proved easily.

LEMMA 2.1. *If* $E(r'|u) \geqslant 0$, *then* $f' \leqslant 0$.

PROOF. If $E(r'|u) \geqslant 0$, then $E(r^2|u) \geqslant (1 - u)E(r|u)$ and hence $g(u) \geqslant 2E(r|u)E(r^3|u) - 2E^2(r^2|u) \geqslant 0$ and hence $f' \leqslant 0$. To evaluate $\delta$, first consider the case where $u \geqslant 1$. Here $r' \geqslant 0$ and hence $E(r'|u) \geqslant 0$ for $0 \leqslant \gamma < 1$. Thus, $f' \leqslant 0$ for $u \geqslant 1$. This gives $\inf_\gamma f(u, \gamma) = \lim_{\gamma\to 1} f(u, \gamma) = da_0$ where $a_0 = (n + q - 1)/(m + 2)$.

Since $r'' \geqslant 0$ for $u > 0$, it follows that $r'$ is a nondecreasing function of $\gamma$ and hence either (i) $E(r'|u) \geqslant 0$ for $\gamma \in (0, 1)$ or (ii) $E(r'|u) < 0$ for $\gamma \in (0, 1)$ or (iii) $E(r'|u) < 0$ for $0 < \gamma < \lambda(u)$ and $E(r'|u) \geqslant 0$ for $\lambda(u) \leqslant \gamma < 1$.

Thus for $0 < u < 1$, if $E(r'|u) \geqslant 0$, then $\inf_\gamma f(u, \gamma) = \lim_{\gamma\to 1} f(u, \gamma) = da_0$. On the other hand if for $0 < u < 1$, $E(r'|u) < 0$, then (2.5) gives $f(u, \gamma) = E(r|u)/E(r^2|u) > 1/(1 - u)$. Thus, $\delta = \inf_\gamma f(u, \gamma) = da_0$ if $da_0 \leqslant 1$ and $\delta \in (1, da_0)$ otherwise. For $da_0 \leqslant 1$, $da_0 = \delta \leqslant \theta$ and hence $da_0 = \theta$. In view of Theorem 2.1 we have thus proved

THEOREM 2.2. *If* $n + q \geqslant 2$ *and* $a_0 = (n + q - 1)/(m + 2)$, *then*
(a) *for* $a \leqslant 2 \min[1, da_0]$, $\hat{\mu}$ *is uniformly better than* $x$;
(b) *if* $da_0 \leqslant 1$, $\hat{\mu}$ *is uniformly better than* $x$ iff $a \leqslant 2da_0$;
(c) *for fixed* $a \leqslant 2$, $\hat{\mu}$ *is uniformly better than* $x$ iff $d \geqslant a/(2a_0)$.

REMARK 2.1. Taking $a = 1$, Theorem 2.2(c) gives the following result proved by KS in a different way: if $n + q \geqslant 2$ and $a_0 = (n + q - 1)/(m + 2)$, then the estimator $\hat{\mu}$ with $a = 1$ is uniformly better than $x$ iff $d \geqslant (m + 2)/[2(n + q - 1)]$.

The following useful modifications of $\hat{\mu}$ can be dealt in the same way as $\hat{\mu}$. Let $\hat{\mu}_i = x + \phi_i(y - x)$, $i = 1, 2, 3$ where

$$\phi_1 = aS/\left[S + c\left\{T + (y - x)^2/\beta_0\right\}\right]$$

$$\phi_2 = aS/\left[S + c\left\{(y - x)^2/\beta_0 + \Sigma_{j=1}^k W_j/\beta_j\right\}\right]$$

$$\phi_3 = aS/\left[S + cT\right].$$

Thus,

THEOREM 2.3. *Theorem 2.2 holds word by word for each* $\hat{\mu}_i$ *provided the expression for* $a_0$ *in that theorem is replaced by*

$$a_0 = f_i/(m + 2) \quad \textit{where} \quad f_1 = n - 1, \quad f_2 = q - 1, \quad f_3 = n - 4.$$

REMARK 2.2. The estimators $T_a$ and $T_a(1)$ of BC are particular cases of $\hat{\mu}_1$ and $\hat{\mu}_3$, respectively and application of Theorem 2.3 to these estimators leads to improvement of Theorem 2.1 and Theorem 2.2 of their paper. For details refer to Section 3 on application.

REMARK 2.3. Taking $a = 1$, Theorem 2.3 yields the following:

$$\hat{\mu}_3 \text{ ub } x \text{ iff } d \geqslant (m + 2)/2(n - 4), \qquad \text{provided} \quad n \geqslant 5.$$

In view of symmetry considerations, it follows that, $\hat{\mu}_3$ is uniformly better than both $x$ and $y$ iff $(m + 2)/[2(n - 4)] \leqslant d \leqslant 2(m - 4)/(n + 2)$, which is possible whenever $(m - 6)/(n - 6) \geqslant 16$. This result, which is an improvement of a similar result by Graybill and Deal (1959), was obtained by KS in a quite different way.

REMARK 2.4. As $\hat{\mu}_3$ generalizes $T_a(1)$ of BC the following generalize Theorem 4.1 and Theorem 5.1 of their paper:

(a) Let $x_i$, $S_i$, $i = 1, 2, \cdots, k$ be independent random variables such that $x_i \sim N(\mu, \delta_i \sigma_i^2)$ and $S_i/\sigma_i^2 \sim \chi^2(m_i)$. Let $b_i$, $c_i$, $i = 1, 2, \cdots, k - 1$ be arbitrary sequences of positive numbers such that

$$(2.6) \qquad b_i < \min[1, 2d_i(m_{i+1} - 4)/(m_i + 2)], \qquad \text{where} \quad d_i = c_i\delta_i/\delta_{i+1}.$$

Let $a_1 = b_1$ and $a_i = b_i(1 - \Sigma_{j=1}^{i-1} a_j)$, $i = 2, 3, \cdots, k - 1$. Then $\hat{\mu}_3(k) = x_1 + \Sigma_{i=1}^{k-1}(x_{i+1} - x_i)a_iS_1/(S_1 + c_iS_{i+1})$, is uniformly better than $x_1$, provided $m_i \geqslant 5$, $i = 2, 3, \cdots, k$. The result can be proved in the same way as in BC once it is noted that condition (2.6) ensures that for each $i$, $b_i < 1$ and the estimator $l_i = x_1 + (x_{i+1} - x_1)b_i/(S_1 + c_iS_{i+1})$ub $x_1$.

(b) Consider the random intervals

$$(2.7) \qquad x \pm t_m(\alpha)(\alpha_0 S/m)^{\frac{1}{2}} \cdots \text{ and,}$$

$$(2.8) \qquad \hat{\mu}_3 \pm t_m(\alpha)(\alpha_0 S/m)^{\frac{1}{2}} \cdots$$

where $t_m(\alpha)$ stands for the two-tailed $\alpha$ critical value determined from Student's $t$-distribution with $m$ degrees of freedom and the rest of the symbols are as defined in the beginning of this section. Then there exists an $\varepsilon(\alpha) \in (0, 1)$ such that for $a \leqslant \varepsilon(\alpha)$ the confidence interval (2.8) is better than the confidence interval (2.7) in the sense that both intervals have the same length and the probability of coverage of (2.8) is uniformly greater than that of (2.7), provided $n \geqslant 5$.

The proof of the result is analogous to that in BC and is omitted.

## 3. Applications.

(a) *Estimation of a common mean.* Let $(x_1, x_2, \cdots, x_m)$ and $(y_1, y_2, \cdots, y_m)$ be independent random samples from two normal populations having a common unknown mean and unknown variances $\sigma_x^2$ and $\sigma_y^2$ respectively. Consider the problem of estimating $\mu$. Let $\bar{x} = \Sigma x_i/m$, $s_{\bar{x}}^2 = \Sigma(x_i - \bar{x})^2/\{m(m - 1)\}$ and let $\bar{y}$, $s_y^2$ be defined similarly. Let,

$$T_1(a, c) = \bar{x} + (\bar{y} - \bar{x})as_{\bar{x}}^2 \Big/ \left[s_{\bar{x}}^2 + c\left\{s_{\bar{y}}^2 + (\bar{y} - \bar{x})^2/(n - 1)\right\}\right]$$

$$T_2(a, c) = \bar{x} + (\bar{y} - \bar{x})as_x^2 \Big/ \left(s_{\bar{x}}^2 + cs_{\bar{y}}^2\right).$$

These include as particular cases the estimators $T_a$, $T_a(1)$ of BC and $\mu^*$, $\mu^{**}$ of KS. In fact,

$$T_a = T_1(a, (n-1)/(n+2)), \qquad T_a(1) = T_2(a, 1)$$
$$\mu^* = T_1(1, c(n-1)/(m-1)), \qquad \mu^{**} = T_2(1, c(n-1)/(m-1)).$$

It can be seen that $T_1(a, c)$ and $T_2(a, c)$ are of the same forms as $\hat{\mu}_1$ and $\hat{\mu}_3$, respectively. Note that $d = c(m-1)/(n-1)$ in each case and the values of $a_0$ are $(n-2)/(m+1)$ and $(n-5)/(m+1)$, respectively. It follows from Theorem 2.3 that $T_1(a, c)$ub $\bar{x}$ for all $a \leqslant 2\min[1, d(n-2)/(m+1)]$ provided $m \geqslant 2$, $n \geqslant 3$. Similarly $T_2(a, c)$ub $\bar{x}$ for $a \leqslant 2\min[1, d(n-5)/(m+1)]$, provided $m \geqslant 2$, $n \geqslant 6$. Given $a \leqslant 2$, as is the case with KS estimators it further follows that $T_1(a, c)$ub $\bar{x}$ iff $d \geqslant (\frac{1}{2})a(m+1)/(n-2)$ and $T_2(a, c)$ub $\bar{x}$ iff $d \geqslant (\frac{1}{2})a(m+1)/(n-5)$. From these, the results concerning KS estimators are immediate. Given $c$ such that $da_0 \leqslant 1$, as is the case with BC estimators, it also follows that $T_1(a, c)$ub $\bar{x}$ iff $a \leqslant 2d(n-2)/(m+1)$ and $T_2(a, c)$ub $\bar{x}$ iff $a \leqslant 2d(n-5)/(m+1)$. In particular, $T_a$ ub $\bar{x}$ iff $a \leqslant 2(m-1)(n-2)/[(m+1)(n+2)]$ and $T_a(1)$ub $\bar{x}$ iff $a \leqslant 2(m-1)(n-5)/\{(m+1)(n-1)\}$. These results are readily seen to be improvements of Theorems 2.1 and 2.2 of BC.

(b) *Recovery of inter-block information.* Consider a connected binary equireplicate incomplete block design. Let $b$ = number of blocks, $k$ = number of plots per block. Let $N(v \times b)$ denote the incidence matrix of the design and let rank $(N) = t$. Let $\phi_1, \cdots, \phi_{v-1}$ denote the $v-1$ characteristic roots of $NN'$ other than $rk$ and assume, without loss of generality that $\phi, \cdots, \phi_{t-1}$ are the nonzero ones among these. Consider the following canonical reduction of the plot yields, given by Roy and Shah (1962) under an Eisenhart model III (Eisenhart (1947)): $x_i \sim N(\xi_i, a_i\sigma_1^2)$, $i = 1, \cdots, v-1$; $y_i \sim N(\xi_i, b_i\sigma_2^2)$, $i = 1, \cdots, t-1$; $S_1^2/\sigma_1^2 \sim \chi^2(e_1)$ and $S_2^2/\sigma_2^2 \sim \chi^2(e_2)$ where $a_i = k/(rk - \phi_i)$, $b_i = k/\phi_i$, $e_i = bk - b - v + 1$, $e_2 = b - t$ and $\xi_1, \cdots, \xi_{v-1}$, $\sigma_1^2, \sigma_2^2$ are unknown. The statistics $x_1, \cdots, x_{v-1}, y_1, \cdots, y_{t-1}, S_1^2, S_2^2$ are mutually independent and constitute a set of minimal sufficient statistics for the treatment effects. Consider the problems of estimating $\xi_i$(a canonical contrast) for some $i \leqslant t-1$. Let

$$T_3(a, c) = x_i + (y_i - x_i)aS_1^2 \big/ \left[S_1^2 + c\left\{S_2^2 + \Sigma_{j=1}^{t-1}(y_j - x_j)^2/b_j\right\}\right]$$
$$T_4(a, c) = x_i + (y_i - x_i)aS_1^2 \big/ \left[S_1^2 + c\left\{S_2^2 + (y_i - x_i)^2/b_i\right\}\right]$$
$$T_5(a, c) = x_i + (y_i - x_i)aS_1^2 \big/ \left[S_1^2 + cS_2^2\right].$$

These include as particular cases the estimators $\hat{\xi}_i$ of KS and $\hat{\mu}_a$, $\hat{\mu}_a(1)$, $\hat{\mu}_a^*$ of BC. In fact, $\hat{\xi}_i = T_3(1, cb_i/a_i)$,

$$\hat{\mu}_a = T_4(a, e_1(b_i/a_i)/(b-t-3)), \qquad \hat{\mu}_a(1) = T_5(a, e_1(b_i/a_i)/(b-t))$$
$$\hat{\mu}_a^* = T_3(a, e_1(b_i/a_i)/(b+1)).$$

Note that BC actually concerned themselves only with a BIBD. It can be seen that $T_3(a, c)$, $T_4(a, c)$ and $T_5(a, c)$ are of the same forms as $\hat{\mu}$, $\hat{\mu}_1$ and $\hat{\mu}_3$, respectively.

Note that $d = ca_i/b_i$ in each case and the values of $a_0$ are $(b - 3)/(e_1 + 2)$, $(b - t - 1)/(e_1 + 2)$ and $(b - t - 4)/(e_1 + 2)$, respectively. It follows from Theorems 2.2 and 2.3 that $T_3(a, c)$ ub $x_i$ for all $a \leqslant 2 \min[1, d(b - 3)/(e_1 + 2)]$ provided $b \geqslant 4$, $T_4(a, c)$ ub $x_i$ for all $a \leqslant 2 \min[1, d(b - t - 1)/(e_1 + 2)]$ provided $b - t \geqslant 2$, and $T_5(a, c)$ ub $x_i$ for all $a \leqslant 2 \min[1, d(b - t - 4)/(e_1 + 2)]$ provided $b - t \geqslant 5$. Given $a \leqslant 2$, as is the case with the KS estimator, it further follows that $T_3(a, c)$ ub $x_i$ iff $d \geqslant (\frac{1}{2})a(e_1 + 2)/(b - 3)$, $T_4(a, c)$ ub $x_i$ iff $d \geqslant (\frac{1}{2})a(e_1 + 2)/(b - t - 1)$ and $T_5(a, c)$ ub $x_i$ iff $d \geqslant (\frac{1}{2})a(e_1 + 2)/(b - t - 4)$. Results concerning $\hat{\xi}_i$ of KS is immediate from that concerning $T_3(a, c)$ above.

Given $c$ such that $da_0 \leqslant 1$, as is the case with BC estimators, $T_3(a, c)$ ub $x_i$ iff $a \leqslant 2d(b - 3)/(e_1 + 2)$, $T_4(a, c)$ ub $x_i$ iff $a \leqslant 2d(b - t - 1)/(e_1 + 2)$ $T_5(a, c)$ ub $x_i$ iff $a \leqslant 2d(b - t - 4)/(e_1 + 2)$. In particular, $\hat{\mu}_a$ ub $x_i$ iff $a \leqslant 2e_1(b - t - 1)/\{(e_1 + 2)(b - t - 3)\}$, $\hat{\mu}_a(1)$ ub $x_i$ iff $a \leqslant 2e_1(b - t - 4)/(e_1 + 2)(b - t))$, and $\mu_a^*$ ub $x_i$ iff $a \leqslant 2e_1(b - 3)/\{(e_1 + 2)(b + 1)\}$.

Note that for a BIBD, $v(x_i) \leqslant v(y_i)$, as shown in BC and hence the estimators considered above are, in fact, uniformly better than both $x_i$ and $y_i$ under the stated conditions. With $t = v$, as is the case for a BIBD, the results above concerning the BC estimators are readily seen to be improvements of those in Section 3 of BC where the knowledge that $\tau = v(y_i)/v(x_i) > 1$ is used to improve the upper limit of $a$ from $a_{\max}$ to $a''_{\max}$. Note that $a''_{\max}$ is even more difficult to compute. (There is a misprint of the expression for $a''_{\max}$ given there and the correct expression should be $a''_{\max}(\cdot\cdot\cdot) = 2Ev^{-1}/E \max[2/\{v(1 + v)\}, 1/v^2]$.

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