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BAHADUR EFFICIENCIES OF THE STUDENT'S t-TESTS

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Dedicated to the Memory of Jaroslav Hájek

An extremal problem in large deviation is solved for the one and two sample t-statistic. Let $T_n = \bar{X}/s_X$ be the t-statistic, except for normalizing constants, based on n independent observations from F where $\bar{X}$ and s_X are the sample mean and standard deviation. The statistic for the two sample case is $T_n = (\bar{X} - \bar{Y})/[(s_X^2 + s_Y^2)/2]^{\frac{1}{2}}$ based on two independent samples of size n. Let a be positive. We find the rate of convergence to 0 of $\sup_F P_F(T_n \geqq a)$ where the sup is taken over F's symmetric at 0 in the one sample case and all F's in the two sample case. The results are applied to obtain the Bahadur exact slopes of the t-statistic for the full nonparametric hypothesis testing problems. We include a table giving the slopes of the t-statistics, the slopes of standard rank statistics and the maximum possible slopes. This table shows that, in both the one and two sample situations, the t-statistic has, as expected, efficiency strictly less than one when compared with the normal scores statistic, and the slope of the normal scores test is very close to the maximum possible slope, at normal alternatives.

1. Introduction. Efron (1969) discusses the size of the one-sample t-statistic under weak symmetry conditions on the underlying distributions. He gives convincing evidence that a normal approximation to the size leads to a conservative test of zero location under most underlying distributions. Working with the t-statistic in the form $S_n = \sum_{i=1}^n X_i/(\sum_{i=1}^n X_i^2)^{\frac{1}{2}}$, he shows whenever $X_1, \cdots, X_n$ are independent and symmetric about zero, $ES_n^{\,\nu}$ ($\nu = 4, 6, 8, \cdots$) are largest if the X's are symmetric Bernoulli trials ($P(X_i = 1) = P(X_i = -1) = \frac{1}{2}$), so that S_n is less dispersed about zero than the standardized binomial distribution. Using an extension of the above moments result from Eaton (1970), we establish in Theorem 2.4 the minimal rate that $P(S_n \geqq n^{\frac{1}{2}}t)$ converges to zero as n goes to infinity and show that it is attained under symmetric Bernoulli trials. We establish in Theorem 2.5 a similar result for the two-sample t-statistic under a common df for both samples.

Using our large deviation results, we obtain the Bahadur exact slope of the t-statistic for testing the full one-sample and two-sample nonparametric

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hypothesis. These exact slopes are compared to standard rank tests in Section 3. At any specific alternative, the maximum possible slope is attained by a linear signed-rank test in the one-sample problem and by a linear rank test in the two-sample problem in view of Ho (1973) and Hájek (1976), respectively. The slopes of all these tests (calculated at normal alternatives) are given in Table 1 of Section 4. In view of existing results on maximum slope, the permutation t-test also has maximum slope at normal alternatives. Table 1 shows that at normal alternatives, the unpermuted t-test (for which, in view of Efron's results, a normal approximation to the size tends to give a conservative test) is less efficient than the normal scores test and that the slope of the normal scores test is very close to the maximum slope, in both the one-sample and two-sample problems. These results indicate that the unconditional t-test is inefficient for the one- and two-sample problems as we expected. However, this investigation leaves unknown the efficiency of an adaptive version based on the ordered absolute values for the one-sample or the combined sample for the two-sample problems as Efron proposes.

2. Extremal problems connected with large deviations of the t-statistic. Let

$$\bar{X} = \sum_{i=1}^{n} X_i/n\,, \qquad s_X{}^2 = \sum_{i=1}^{n} X_i^2/n - \bar{X}^2\,,$$

where $X_1, X_2, \cdots, X_n$ are independent and identically distributed random variables. The one-sample t-statistic T_n is defined as follows, except for certain normalizing constants.

$$\begin{aligned} T_n &= \bar{X}/s_X && \text{if } s_X > 0 \\ &= +\infty && \text{if } \bar{X} > 0\,,\ s_X = 0 \\ &= -\infty && \text{if } \bar{X} < 0\,,\ s_X = 0 \\ &= 0 && \text{if } \bar{X} = 0\,,\ s_X = 0\,. \end{aligned}$$

We will use the longer form of the t-statistic as opposed to the short form of Efron (1969). The proof of the results about the two-sample t-statistic will be analogous to that of the one-sample t-statistic. By a large deviation of T_n we mean the event $\{T_n \geqq a\}$ for $a > 0$. By an extremal problem for the probability of large deviations of T_n we mean the finding of the rate of convergence to 0 of $\sup_F P_F(T_n \geqq a)$, where the sup is taken over a large class.

We begin with one df under which the large deviation rates for T_n are known. Let B stand for the symmetric Bernoulli df, i.e., under B, $P_B(X_1 = 1) = P_B(X_1 = -1) = \frac{1}{2}$.

LEMMA 2.1. *Let* $0 < a < 1$. *Then*

$$\text{(2.1)} \qquad \lim_n \frac{1}{n} \log P_B(\bar{X} > a) = \inf_{\lambda>0} \log (e^{-\lambda a} \cosh \lambda) = -I\left(\tfrac{1}{2} + \frac{a}{2}, \tfrac{1}{2}\right)$$

where for $0 < p_1, p_2 < 1$,

$$I(p_1, p_2) = p_1 \log \frac{p_1}{p_2} + (1 - p_1) \log \frac{(1 - p_1)}{(1 - p_2)} .$$

PROOF. The proof follows by a direct application of a theorem of Chernoff ((1952), Theorem 1) and noting that $E_B(e^{\lambda X_1}) = \cosh \lambda$. The equality of the last two expressions in (2.1) is straightforward and will be used again in the proof of Theorem 2.4. □

LEMMA 2.2. *Let* $a > 0$. *Then*

$$\lim_n \frac{1}{n} \log P_B(T_n > a) = -I\left(\tfrac{1}{2} + \frac{a}{2(1 + a^2)^{\frac{1}{2}}}, \tfrac{1}{2}\right). \tag{2.2}$$

PROOF. Since $s_X{}^2 = 1 - \bar{X}^2$ with probability one under B,

$$P_B(T_n > a) = P_B\left(\bar{X} > \frac{a}{(1 + a^2)^{\frac{1}{2}}}\right).$$

Lemma 2.2 now follows from (2.1). □

Let $\Delta_1, \Delta_2, \cdots, \Delta_n$ be i.i.d. with common df B. The following lemma contains an inequality between expectations of functions of heterogeneous linear functions of $\Delta_1, \cdots, \Delta_n$ with those of homogeneous linear functions of $\Delta_1, \cdots, \Delta_n$, and it plays an important role in the Theorems 2.4 and 2.5.

LEMMA 2.3. *Let* $\Delta_1, \cdots, \Delta_n$ *be* i.i.d. *with common* df B. *Let* $\xi_1, \cdots, \xi_n$ *be constants. Then for all* λ,

$$\begin{aligned} E(\exp\{\lambda \textstyle\sum_{i=1}^n \xi_i \Delta_i\}) &\leqq E(\exp\{\lambda \textstyle\sum_i \Delta_i \cdot (\sum_i \xi_i^2/n)^{\frac{1}{2}}\}) \\ &= [\cosh \lambda (\textstyle\sum \xi_i^2/n)^{\frac{1}{2}}]^n . \end{aligned} \tag{2.3}$$

PROOF. This result is a special case of a theorem of Eaton ((1970), Example 2 of Theorem 1) and can also be deduced directly from the inequalities of Efron ((1969), moments theorem) connecting the moments of $\sum_i \xi_i \Delta_i$ with those of $(\sum_i \Delta_i)(\sum_i \xi_i^2/n)^{\frac{1}{2}}$. □

THEOREM 2.4. *Let* $\mathscr{S}$ *be any class of* df'*s symmetric about* 0 *for which the weak closure contains* B (*or any other symmetric two-point distribution*). *Then for* $a > 0$,

$$\lim_n \frac{1}{n} \log \sup_{F \in \mathscr{S}} P_F(T_n \geqq a) = -I\left(\tfrac{1}{2} + \frac{a}{2(1 + a^2)^{\frac{1}{2}}}, \tfrac{1}{2}\right). \tag{2.4}$$

PROOF. Let $F \in \mathscr{S}$. Throughout the proof $\frac{0}{0} = 0$ as before. Let $\xi_1, \cdots, \xi_n$, $\Delta_1, \cdots, \Delta_N$ be independent and, for $i = 1, \cdots, n$, let ξ_i and Δ_i have df's F and B respectively. The df of T_n under F equals the df of

$$\textstyle\sum \Delta_i \xi_i \{n \sum \xi_i^2 - (\sum \Delta_i \xi_i)^2\}^{-\frac{1}{2}} \tag{2.5}$$

with all sums ranging over $i = 1, \cdots, n$. A little algebra reveals that for $S^2 = n^{-1} \sum \xi_i^2$ and $b = a(1 + a^2)^{-\frac{1}{2}}$

$$\{\textstyle\sum \Delta_i \xi_i [n \sum \xi_i^2 - (\sum \Delta_i \xi_i)^2]^{-\frac{1}{2}} \geqq a\} = \{\sum \Delta_i \xi_i S^{-1} \geqq nb\} \tag{2.6}$$

application of Markov's inequality and Lemma 2.3 yields

$$\begin{aligned} P_F(T_n \geqq a) &= P(\sum \Delta_i \xi_i S^{-1} \geqq nb) \\ &\leqq \inf_{\lambda>0} \exp\{-n\lambda b\} \cosh^n \lambda \\ &= \exp\{-nI(\tfrac{1}{2} + b/2, \tfrac{1}{2})\}, \end{aligned} \tag{2.7}$$

with the last step following from Lemma 2.1. We can find a sequence $\{F_k\}$ in $\mathscr{S}$ such that $F_k \to B$ weakly. Since $\{T_n > a\}$ is an open set of R_n,

$$\sup_{F\in\mathscr{S}} P_F(T_n \geqq a) \geqq \liminf_k P_{F_k}(T_n > a) \geqq P_B(T_n > a), \tag{2.8}$$

for each n. In view of Lemma 2.2, relations (2.7) and (2.8) establish Theorem 2.4. □

REMARK. The inequality in (2.7) holds for all n and a and is quite remarkable. For instance, it implies that $T_n \to 0$ with probability one under any df symmetric about zero.

Together with $\bar{X}$, s_X^2 defined above, let $\bar{Y} = \sum_1^n Y_i/n$, $s_Y^2 = \sum_1^n Y_i^2/n - \bar{Y}^2$, $N = 2n$. The two-sample t-statistic T_N^*, except for certain normalizing constants, is defined as follows:

$$\begin{aligned} T_N^* &= (\bar{Y} - \bar{X})/[(s_Y^2 + s^2)/2]^{\frac{1}{2}} && \text{if } s_Y^2 + s_X^2 > 0 \\ &= \infty && \text{if } \bar{Y} - \bar{X} > 0, \quad s_Y^2 + s_X^2 = 0 \\ &= -\infty && \text{if } \bar{Y} - \bar{X} < 0, \quad s_Y^2 + s_X^2 = 0 \\ &= 0 && \text{if } \bar{Y} - \bar{X} = 0, \quad s_Y^2 + s_X^2 = 0. \end{aligned}$$

THEOREM 2.5. *Let $\mathscr{F}$ be any class of* df's *for which the weak closure contains B (or any other two-point distribution with equal probabilities). Then for $a > 0$,*

$$\lim_n \frac{1}{N} \log \sup_{F\in\mathscr{F}} P_{F,F}(T_N^* \geqq 2a) = -I\left(\tfrac{1}{2} + \frac{a}{2(1+a^2)^{\frac{1}{2}}}, \tfrac{1}{2}\right). \tag{2.9}$$

PROOF. Let $F \in \mathscr{F}$. Let $Z_1, \cdots, Z_N, \Delta_1, \cdots, \Delta_N$ be independent and, for $i = 1, \cdots, N$, let Z_i and Δ_i have df's F and B respectively. The df of T_N^* under (F, F) equals the conditional df given $\sum \Delta_i = 0$ of

$$2 \sum \Delta_i Z_i \{N \sum (Z_i - \bar{Z})^2 - (\sum \Delta_i Z_i)^2\}^{-\frac{1}{2}} \tag{2.10}$$

with $\bar{Z} = N^{-1} \sum Z_i$ and all sums ranging over $i = 1, \cdots, N$. A little algebra reveals that for $S^2 = N^{-1} \sum (Z_i - \bar{Z})^2$ and $b = a(1 + a^2)^{-\frac{1}{2}}$

$$\begin{aligned} P_{F,F}(T_N^* \geqq 2a) &= P\left(\frac{\sum \Delta_i(Z_i - \bar{Z})}{S} \geqq bN \mid \sum \Delta_i = 0\right) \\ &\leqq \frac{P\left(\frac{\sum \Delta_i(Z_i - \bar{Z})}{S} \geqq bN\right)}{P(\sum \Delta_i = 0)}. \end{aligned}$$

Following the proof of Theorem 2.4 the theorem can now be established by using (2.10) and the fact that $N^{-1} \log P(\sum \Delta_i = 0) \to 0$ as $N \to \infty$. □

REMARK. An inequality analogous to (2.7) but for the two-sample t-statistic can be established. It implies that $T_N^* \to 0$ with probability one under any df symmetric about zero.

3. Bahadur efficiency of the t-test. We will consider the one-sample t-statistic first. Let $\mathscr{S}_c$ denote the class of all continuous df's symmetric around 0, i.e., $F(x) = 1 - F(-x - 0)$ for all x. Consider testing $\mathscr{S}_c$: $F \in \mathscr{S}_c$ against the alternative G: $F = G$ where G is a nonsymmetric df. We shall further assume that G admits a density function g and

$$(3.1) \qquad \int xg(x)\,dx = \mu\,, \qquad \int x^2 g(x)\,dx = \mu^2 + \sigma^2\,, \quad \mu, \sigma^2 > 0\,,$$

and

$$(3.2) \qquad \int \left|\log \frac{g(x)}{g(-x)}\right|^3 (g(x) + g(-x))\,dx < \infty$$

and $\log g(x)/g(-x)$ is of bounded variation in every closed bounded interval. Put

$$a = \mu/\sigma$$

where μ and σ are as defined in (3.1). For the one-sample t-statistic we have

$$T_n \to a$$

with probability one under G, and from Theorem 2.4 and Theorem 7.2 of Bahadur (1971), the Bahadur slope of $\mathbf{T} = \{T_n\}$ under the null hypothesis $\mathscr{S}_c$ at the alternative G is

$$(3.3) \qquad c(\mathbf{T}, \mathscr{S}_c, G) = 2I\left(\tfrac{1}{2} + \frac{a}{2(1 + a^2)^{\frac{1}{2}}}\,, \tfrac{1}{2}\right).$$

REMARK. In view of the hypothesis of Theorem 2.4, the null hypothesis could be any class of df's symmetric about 0 for which the weak closure contains B.

The slopes of the sign statistic, Wilcoxon signed-rank statistic, the normal scores signed-rank statistic (also called the Fraser statistic) for the one-sample problem present no new problems since these rank statistics have null distributions. These slopes are the same as given by Bahadur (1960a) and Klotz ((1965), Section 2). Section 4 presents these three slopes together with $c(\mathbf{T}, \mathscr{S}_c, G)$, when $G = \Phi_{a,1}$, in columns 5, 4, 6 and 2 of Table 1 respectively. This table clearly shows that the efficiency of the t-test is less than one when compared with the normal scores signed-rank test, at all normal alternatives.

By appealing to Raghavachari ((1970), Theorem 1) and Ho ((1973), Theorem 7) it is easily seen that the maximum slope for this problem is attained by a linear signed-rank statistic and this maximum slope is $2K(G, G^*)$ where

$$G^*(x) = (G(x) + 1 - G(-x))/2$$

and for any two df's U and V if $dV = f(x)\,dU$, let

$$K(U, V) = E_V[\log f(X)]\,;$$

otherwise let $K = \infty$.

Putting $I(a) = K(G, G^*)$ when $G = \Phi_{a,1}$, we have

$$(3.4) \qquad I(a) = a^2 - \frac{1}{(2\pi)^{\frac{1}{2}}} \int_{-\infty}^{\infty} \log\cosh(xa) \exp\{-(x-a)^2/2\}\, dx\,.$$

Values of $I(a)$, obtained by numerical integration, are tabulated in column 7 of Table 1. It can be seen that the slope of the normal scores signed-rank test falls short of this maximum slope by less than .002.

Next we will consider the two-sample t-test. Let $\mathscr{F}_c$ be the class of all continuous df's. Consider the two-sample problem which consists of testing the null hypothesis $\mathscr{F}_c^*: F = G \in \mathscr{F}_c$ against the alternative $(F_1, F_2): F = F_1$, $G = F_2$ where F_1, F_2 are df's possessing density functions which satisfy

$$(3.5) \qquad \int x f_i(x) = \mu_i\,, \qquad \int x^2 f_i(x) = \mu_i^2 + \sigma_i^2\,,$$
$$i = 1, 2\,, \quad \mu_2 - \mu_1 > 0\,, \quad \sigma_1^2 + \sigma_2^2 > 0\,,$$

and

$$(3.6) \qquad \int \log \frac{f_1(x)}{f_2(x)} (f_1(x) + f_2(x))\, dx < \infty$$

and $\log f_1(x)/f_2(x)$ is of bounded variation in every bounded closed interval.

Put

$$2a = (\mu_2 - \mu_1)/[(\sigma_1^2 + \sigma_2^2)/2]^{\frac{1}{2}}\,.$$

For the two-sample t-statistic T_N^*, we have

$$T_N^* \to 2a$$

with probability one under (F_1, F_2). From Theorem 2.5 and Theorem 7.2 of Bahadur (1971), we have that the Bahadur slope of $\mathbf{T}^* = \{T_N^*\}$ under the null hypothesis $\mathscr{F}_c^*$ at the alternative (F_1, F_2) is

$$(3.7) \qquad c(\mathbf{T}^*, \mathscr{F}_c^*, (F_1, F_2)) = 2I\left(\tfrac{1}{2} + \frac{a}{2(1+a^2)^{\frac{1}{2}}}, \tfrac{1}{2}\right).$$

REMARK. In view of the hypothesis of Theorem 2.5, the null hypothesis could be any class of df's symmetric about 0 for which the weak closure contains a two point distribution with equal probabilities.

This slope is the same as the slope of the one-sample t-statistic under $\mathscr{S}_c$ against an alternative G leading to the same a. When $F_i = \Phi_{\mu_i,1}$, $i = 1, 2$ and $(\mu_2 - \mu_1)/2 = a$, the slope is tabulated in column 2 of Table 1. The slopes of the Wilcoxon rank sum statistic, median scores statistic, the normal scores rank statistic for this two-sample problem present no new problems since these rank statistics have null distributions. These slopes are the same as given by Woodworth ((1970), Section 4). We present these slopes for normal alternatives, namely $F_i = \Phi_{\mu_i,1}$, $i = 1, 2$, with $(\mu_2 - \mu_1)/2 = a$ in columns 4, 5, 6 of Table 1 and are exactly the same as the slopes of the one-sample Wilcoxon and normal scores signed-rank tests at a normal alternative $G = \Phi_{a,1}$. From this table we see that the slope of $\mathbf{T}^*$ is smaller than the slope of the normal scores test, thus once again confirming the fact that the t-test is less efficient in the Bahadur sense.

Hájek (1974) has obtained the maximum possible slope at the alternative (F_1, F_2) and has shown that this slope is attained at a simple linear rank test which depends on (F_1, F_2). When $F_i = \Phi_{\mu_i,1}$, $i = 1, 2$, $(\mu_2 - \mu_1)/2 = a$, it is easy to check from relation (15) and Corollary 2 of Hájek (1974) that this maximum slope is equal to $2I(a)$ where $I(a)$ is as given in (3.4). This maximum slope is given in column 7. It can be seen that the slope of the normal scores statistic falls short from this maximum by less than .002.

4. Table giving the various slopes at normal alternatives. Table 1 gives the various slopes, simultaneously for the one-sample problem at the alternative $\Phi_{a,1}$ and for the two-sample problem at the alternative $(\Phi_{\mu_1,1}, \Phi_{\mu_2,1})$ with $(\mu_2 - \mu_1)/2 = a$.

TABLE 1

Exact slopes ($\times \frac{1}{2}$) of one-sample test statistics at the alternative $\Phi_{a,1}$, and of two-sample test statistics at the alternative $(\Phi_{\mu_1,1}, \Phi_{\mu_2,1})$

	Shift	Statistic				
one-sample	a	t	Wilcoxon	Sign	normal scores	Maximum Possible
two-sample	$(\mu_1 - \mu_2)/2$	t	Wilcoxon	Median	normal scores	
	.125	.007712	.007416	.004956	.007752	.0077521
	.250	.02971	.02914	.01961	.03031	.0303113
	.375	.06298	.06368	.04336	.0657	.0657679
	.500	.1036	.1087	.07522	.1114	.1114215
	.625	.1479	.1615	.1139	.1642	.1642302
	.750	.1927	.2189	.1580	.2211	.2211709
	.875	.2361	.2779	.2058	.2793	.2794855
	1.000	.2767	.3360	.2557	.3365	.3368310
	1.125	.3139	.3910	.3062	.3908	.3913517
	1.250	.3475	.4416	.3552	.4409	.4416957
	1.375	.3778	.4867	.4034	.4859	.4869890
	1.500	.4048	.5262	.4478	.5254	.5267823
	1.625	.4289	.5600	.4885	.5593	.5609802
	1.750	.4503	.5884	.5250	.5880	.5897642
	1.875	.4694	.6120	.5570	.6117	.6135150
	2.000	.4865	.6311	.5846	.6309	.6327416
	2.125	.5018	.6464	.6079	.6463	.6480201
	2.250	.5155	.6585	.6272	.6584	.6599435
	2.375	.5278	.6678	.6429	.6677	.6690852
	2.500	.5389	.6749	.6554	.6747	.6759730
	2.625	.5490	.6802	.6652	.6800	.6810742
	2.750	.5580	.6841	.6841	.6839	.6847883
	2.875	.5663	.6869	.6867	.6867	.6874474
	3.000	.5738	.6889	.6887	.6887	.6893195
	∞	ln 2	ln 2	ln 2	ln 2	ln 2

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