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NOTE

CORRECTION TO

MINIMAX TESTS AND THE NEYMAN-PEARSON LEMMA FOR CAPACITIES

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The proof of Theorem 4.1 (*Annals of Statistics* **1** 251-263) contains an error, which was brought to our attention by Werner Goller. The set functions F and G, as defined there (page 258), do not in general satisfy (4′) and (4) respectively. The definition of F has to be modified as follows (continuity of G is irrelevant).

Define for $t < s$ and *compact* B:

$$F(t, s; B) = u_1((B \cap A_t) \cup A_s) - u_1(A_s) . \tag{*}$$

For general B, put $F(t, s; B) = \sup F(t, s; C)$, where C ranges over the compacts $C \subset B$. Then F satisfies (2), (3′), (4′), (5′); of these, (2), (4′) and (5′) are immediate, and (3′) follows from a well-known inequality for 2-monotone functions: if $K_i \subset A_i$, $1 \leqq i \leqq n$, then $u(\bigcap A_i) - u(\bigcap K_i) \leqq \sum (u(A_i) - u(K_i))$.

LEMMA. *If either* $B = A_z$ *or* $B = C \cap A_z$, *with* C *compact, then* (*) *still holds.*

PROOF. We only give the proof for one typical case: $B = C \cap A_z$, $t \leqq z \leqq s$. Let $K \subset A_z$ be compact, then

$$\begin{aligned} F(t, s; B) &\geqq \sup_K u_1((C \cap K) \cup A_s) - u_1(A_s) \\ &\geqq \sup_K u_1((C \cup A_s) \cap K) - u_1(A_s) \\ &= u_1((C \cup A_s) \cap A_z) - u_1(A_s) , \end{aligned}$$

by u_1-capacitability of Borel sets, and this lower bound happens to coincide with the upper bound furnished by (*).

The proof now proceeds as in the paper until the middle of page 259, where continuity of F is used to show that $\tilde{Q}_0 \geqq \tilde{u}_1{}^* \geqq \tilde{u}_1{}^J$ implies $Q_0 \geqq u_1{}^J$; the first part of the above Lemma now gives $Q_0(A_t) \geqq u_1{}^J(A_t) \geqq \alpha[v_0(A_t) - v_0(A_{t_n})]$, hence $Q_0(A_t) = v_0(A_t)$.

The calculations after the definition of Q_1 (page 259 lower third) have to be changed as follows.

Let B be any compact set and $J = (t_0, \cdots, t_n)$ be such that $(t_{k-1} - t_0)/t_k \geqq \alpha$ for $k > 1$. Then

$$\begin{aligned} &Q_1(B \cap A_0) - Q_1(B \cap A_\infty) \\ &\quad \geqq \textstyle\sum_1^n [Q_1(B \cap A_{t_{i-1}}) - Q_1(B \cap A_{t_i})] + [Q_1(B \cap A_{t_n}) - Q_1(B \cap A_\infty)] \\ &\quad \geqq \textstyle\sum_1^n t_{i-1}[Q_0(B \cap A_{t_{i-1}}) - Q_0(B \cap A_{t_i})] + t_n Q_0(B \cap A_{t_n}) \\ &\quad \geqq \textstyle\sum_1^n (t_i - t_{i-1}) Q_0(B \cap A_{t_i}) \end{aligned}$$

$$\begin{aligned}
&\geqq \sum_1^n (t_i - t_{i-1}) u_1^J(B \cap A_{t_i}) \\
&= \sum_{1 \leqq i < k \leqq n} \frac{t_i - t_{i-1}}{t_k} F(t_{k-1}, t_k; B \cap A_{t_i}) \\
&= \sum_1^n \frac{t_{k-1} - t_0}{t_k} F(t_{k-1}, t_k; B) \\
&\geqq \alpha \sum_{k=2}^n F(t_{k-1}, t_k; B) \\
&\geqq \alpha F(t_1, t_n; B) \\
&\geqq \alpha [u_1(B \cap A_{t_1}) - u_1(B \cap A_{t_n})] \,.
\end{aligned}$$

The remainder of the proof (from the bottom line of page 259 on) is the same.

The remark after the proof of Lemma 2.2 (page 253) should read: "... if u satisfies (1), (2), (3′), (4′) and $1 + u(A \cap B) \geqq u(A) + u(B)$. ..."