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RESISTANT AND SUSCEPTIBLE BIB DESIGNS[1]

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Let D be a BIB (v, b, r, k, λ) design on Ω. Let also $L \subset \Omega$ with cardinality of L being $n \leqq v - 2$. We define D to be locally resistant of degree n if upon deletion of all the experimental units in D assigned to the treatments in L the remaining structure is variance balanced in the sense that under the usual homoscedastic additive linear model every normalized estimable linear function of the treatment effects are estimable with the same variance. D is defined to be globally resistant of degree n if it has the above property with respect to any subset $L \subset \Omega$ as long as its cardinality is n. D is said to be susceptible if it is not resistant to any nonempty set L. Application of these concepts in various branches of sciences and engineering has been indicated. In this paper we have characterized all locally and globally resistant designs of degree one in two different ways. Through one of these characterizations we have been able to relate our theory to the theory of t-designs or tactical configurations. Methods for constructing some families of locally and globally resistant designs of degree one are provided. We have also shown that the property of being resistant depends not only on the parameters of D but also depends on the way D has been constructed. To illustrate this we have given three BIB (10, 30, 12, 4, 4) designs; the first design is susceptible, the second is locally resistant to the deletion of a single treatment and the third design is globally resistant. We have also indicated that every BIB (v, b, r, k, λ) design is locally resistant of degree k if $b = v$. Several miscellaneous results are also given, among which a locally resistant design of degree two is included. Several unsolved problems are indicated in the final section.

1. Background materials. Let $\Omega = \{a_1, a_2, \cdots, a_v\}$ be a set of v treatments. Next let D be a block design consisting of b blocks of sizes $k_1, k_2, \cdots, k_b$ such that the ith treatment has been assigned into r_i experimental units. We allow any repetition of any treatment in any block subject to the above restrictions. We also associate the following matrices with D.

$\mathbf{N} = [n_{ij}]$ where n_{ij} denotes the number of times the ith treatment appears in the jth block, $\mathbf{R} = \operatorname{diag}[r_1, r_2, \cdots, r_v]$, $\mathbf{K} = \operatorname{diag}[k_1, k_2, \cdots, k_b]$ and $\mathbf{C} = \mathbf{R} - \mathbf{N}\mathbf{K}^{-1}\mathbf{N}'$.

DEFINITION 1.1. D is said to be pairwise balanced if $\mathbf{NN}' = \boldsymbol{\delta} + \lambda\mathbf{J}$ where δ is a diagonal matrix, J is a matrix, of ones and λ is a scalar.

Two treatments a_i and a_j, $i \neq j$ in Ω are said to be connected in D provided that it is possible to construct a chain of treatments beginning with a_i and ending

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with a_j such that every consecutive pair of treatments in the chain occurs together in a block. The relationship a_i connected to b_j defines an equivalence relation on D which induces disjoint equivalence classes.

Definition 1.2. D is said to be connected if there is only one equivalence class in D under the connected relationship, i.e., D is connected if every pair of treatments in Ω is connected in D.

Lemma 1.1. *D is connected if and only if rank of $C = v - 1$.*

Let $\mathbf{Y}$ be the response vector associated with D. We assume throughout this paper the homoscedastic additive linear model for $E(\mathbf{Y})$ i.e., if y_{ijt} is the response associated with the tth replication of the treatment i in the jth block, then we assume $E(y_{ijt}) = \theta_0 + \theta_i + \xi_j$ with $\operatorname{Var} y_{ijt} = \sigma^2$ and the covariance between any two observations being zero. θ_0, θ_i, ξ_j and σ^2 are the parameters of the model and assumed to be constant.

Definition 1.3. D is said to be variance balanced if every normalized estimable linear function of the treatment effects can be estimated with the same variance.

Lemma 1.2. *D is variance balanced if and only if the nonzero characteristic roots of C are all equal.*

If D is connected then it is easy to characterize D whenever it is variance balanced (see e.g., Atiqullah (1961) and Rao (1958)).

Lemma 1.3.. *If D is connected then it is variance balanced if and only if $\mathbf{C}$ is of the form $\mathbf{C} = c_1\mathbf{I} + c_2\mathbf{J}$, c_i is a scalar and I is the identity matrix.*

It can be shown by counterexamples that pairwise balancedness is neither necessary nor sufficient for a design D to be variance balanced (for more details see Hedayat and Federer (1971)).

2. Introduction and the problem. The classical BIB (v, b, r, k, λ) designs are known to be balanced in the two different senses which we discussed in Section 1. However, these two interesting features of BIB designs can be destroyed when these designs are used in actual experimentation. This destruction may come about by loss of some or all of the experimental units assigned to one or more treatments. In this paper we shall explore the case where all of the experimental units assigned to one or more treatments have been lost. In this case the remaining structure is still pairwise balanced but it is not in general variance balanced. Our purpose in this paper is to explore the cases where the variance balancedness of the design remains invariant under loss or deletion of one or more treatments. More formally, let D be a BIB (v, b, r, k, λ) design on a set of v treatments Ω. Let $L \subset \Omega$ with cardinality $n \leqq v - 2$. Delete all the experimental units in D assigned to the treatments in L. Call the remaining structure $\bar{D}$.

DEFINITION 2.1. D is said to be globally resistant of degree n if $\bar{D}$ is variance balanced when any subset L of cardinality n is deleted.

DEFINITION 2.2. D is said to be locally resistant of degree n if $\bar{D}$ is variance balanced only with respect to some subsets of cardinality n.

DEFINITION 2.3. D is said to be susceptible if there exist no subsets, L, such that $\bar{D}$ is variance balanced.

3. Application of resistant designs. Suppose an experiment is going to be conducted on a set of v treatments using a BIB design, because the experimenter wants to have an equal precision in comparing any two treatments. Now consider the following two possibilities:

(A) The experimenter wants to stop the continuation of the experiment on any treatment or a specific set of treatments if he is not satisfied with the condition of the experiment.

(B) The experimenter has doubt about the final outcome of the experiment with regard to some treatments. For example, (i) in an experiment involving drugs, some of the drugs may be expected to be lethal and thus kill some or all of the assigned experimental units to them; (ii) in an agricultural experiment, a high amount of a fertilizer or a pest control may totally or partially destroy the assigned plants or trees; (iii) in an experiment testing and comparing different brands of tires, certain road and/or speed and/or weather conditions may render some brands of tires unusable; (iv) or, in a chemical experiment, certain chemical combinations may be expected to be explosive, or to fail to support reaction.

If the experimenter selects his design in an arbitrary manner, then he may lose the equal precision under one or more of the above conditions. However, he can guard his goal if he can arrange to have a proper locally or globally resistant design. He simply assigns those treatments about which he has doubt to the subset of treatments to which the design is resistant.

4. Characterization of resistant designs. In this section we shall give a necessary and sufficient condition under which a given BIB design is locally resistant with respect to a fixed treatment. Then as a generalization a necessary and sufficient condition will be given for globally resistant designs of degree one. It is also shown that the property of being resistant (locally or globally) depends not only on the parameters of the design but also depends on the way the design has been constructed. To support this we give three BIB designs with the same set of parameters of which the first is susceptible, the second is locally resistant and the third is globally resistant. Finally, we shall give a second characterization of globally resistant designs. This latter characterization shows that globally resistant designs are intimately tied up with t-designs.

Let D be a BIB (v, b, r, k, λ) design on Ω and let $L = \{x\} \subset \Omega$. Divide D into two parts, D_x and $D_{\bar{x}}$. D_x consists of all the blocks which do not contain x; $D_{\bar{x}}$

is the set of blocks which contain x. Next, let $D_{\bar{x}}'$ be the design obtained by deleting x from the blocks of $D_{\bar{x}}$.

LEMMA 4.1. *D_x is a* BIB *design if and only if $D_{\bar{x}}'$ is a* BIB *design.*

PROOF. Note that D_x is a block design with parameters $v_1 = v - 1$, $b_1 = b - r$, $r_1 = r - \lambda$ and $k_1 = k$. $D_{\bar{x}}'$ is also a block design with parameters $v_2 = v - 1$, $b_2 = r$, $r_2 = \lambda$ and $k_2 = k - 1$. Now if x_1, x_2 are in $\Omega - \{x\}$, then the pair (x_1, x_2) appears λ_{12} times in D_x if and only if it appears $\lambda - \lambda_{12}$ times in $D_{\bar{x}}'$. Thus D_x is a BIB design if and only if $D_{\bar{x}}'$ is a BIB design.

THEOREM 4.1. *D is locally resistant with respect to x if and only if D_x is a* BIB *design.*

PROOF. Clearly $\bar{D} = D_x \cup D_{\bar{x}}'$ is a connected design. Then $\bar{D}$ is variance balanced if and only if its **C** matrix is of the form $c_1\mathbf{I} + c_2\mathbf{J}$. Now suppose the hth and ith treatments appear $\lambda_{hi}^{(1)}$ times in D_x and $\lambda_{hi}^{(2)}$ times in $D_{\bar{x}}'$. Then the diagonal and off-diagonal elements of **C** are given by

$$c_{ii} = r - \frac{\lambda}{k-1} - \frac{r-\lambda}{k}\,,$$

$$\begin{aligned} c_{hi} &= \frac{-\lambda_{hi}^{(2)}}{k-1} - \frac{\lambda_{hi}^{(1)}}{k} \\ &= -\lambda_{hi}^{(2)}\{(k-1)^{-1} - (k)^{-1}\} - \lambda k^{-1}\,. \end{aligned}$$

Therefore, $\bar{D}$ is variance balanced if and only if $\lambda_{hi}^{(1)}$ does not depend on h and i.

COROLLARY 4.1. *D is globally resistant of degree one if and only if D_x is a* BIB *design for all x in Ω.*

COROLLARY 4.2. *If D is locally resistant with respect to any treatment then its parameters must satisfy the following conditions*:

(N_1) $r \geqq v - 1$,
(N_2) $\lambda(k-2)/(v-2) =$ *integer* ,
(N_3) $\lambda > 1$.

PROOF. If D is resistant with respect to $x \in \Omega$ then $D_{\bar{x}}'$ is a BIB design which implies (N_1) and (N_2). Condition (N_3) follows from (N_2) and the fact that $k < v$.

There are not very many sets of parameters for D satisfying the conditions of Corollary 4.2; although one can satisfy them in a trivial sense by letting D be repeated until $\lambda \geqq v - 2$, the conditions of Corollary 4.2 are far from being sufficient as the next corollary shows.

COROLLARY 4.3. *The property of being resistant depends not only on the parameters of the design but also depends on the way the design has been constructed.*

PROOF. By example. We give three BIB (10, 30, 12, 4, 4) designs of three different types.

(1) Let $D_s = D_1 \cup D_1$ where D_1 is the residual of a BIB (16, 16, 6, 6, 2) design. Then D_s is a susceptible design. D_1 is a BIB (10, 15, 6, 4, 2) given below:

0	1	2	6	3	4	5	6	3	4	5	4	7	8	9
7	8	9	8	9	9	7	1	2	2	0	3	0	1	2
6	6	5	7	0	1	8	0	7	8	1	6	5	5	3
3	4	6	9	8	7	2	2	1	0	9	5	4	3	4.

(2) Let $D_l = D_2 \cup D_3$ where D_2 is the BIB (9, 18, 8, 4, 3) design given by Fisher and Yates (1953) and let D_3 be the complete lattice, the BIB (9, 12, 4, 3, 1) design with an extra plot containing a new treatment x added to each block. Then D_l is locally resistant with respect to the treatment x only.

0	1	2	3	4	5	6	7	8		0	0	1	2	3	6
1	2	3	4	5	6	7	8	0		1	3	4	5	4	7
2	3	4	5	6	7	8	0	1		2	6	7	8	5	8
4	5	6	7	8	0	1	2	3		x	x	x	x	x	x,

0	1	2	3	4	5	6	7	8		0	0	1	1	2	2
3	4	5	6	7	8	0	1	2		4	5	5	3	3	4
4	5	6	7	8	0	1	2	3		8	7	6	8	7	6
7	8	0	1	2	3	4	5	6		x	x	x	x	x	x.

(3) $D_g = D_2' \cup D_3$ where D_3 is the same design as in (2) and D_2' is the following BIB (9, 18, 8, 4, 3) design given by Sprott (1956):

3	4	5	6	7	8	0	1	2		4	5	3	7	8	6	1	2	0
2	0	1	5	3	4	8	6	7		5	3	4	8	6	7	2	0	1
6	7	8	0	1	2	3	4	5		8	6	7	2	0	1	5	3	4
1	2	0	4	5	3	7	8	6,		7	8	6	1	2	0	4	5	3.

D_g is globally resistant of degree one.

COROLLARY 4.4. *D is locally resistant with respect to the treatment x in Ω if and only if every triple containing x appears the same number of times in D.*

PROOF. Recall that every block of $D_{\bar{x}}$ contains x. Moreover, upon removal of x from the blocks of $D_{\bar{x}}$ the remaining structure viz., $D_{\bar{x}}'$ is a BIB design by Lemma 4.1 and Theorem 4.1; thus the proof.

Before proceeding further we need the following definition:

DEFINITION 4.1. Given a set Ω of v elements, and given positive integers, k, t ($t \leqq k < v$) and μ, we denote by a *tactical configuration* $C[k, t, \mu, v]$ a system of blocks (subsets of Ω), having k elements each and such that every subset of Ω having t elements is included in exactly μ blocks (see also Carmichael (1956)).

Tactical configurations $C[k, 2, \mu, v]$ are simply balanced incomplete block designs and tactical configurations $C[k, 3, \mu, v]$ are referred to as doubly balanced incomplete block designs. Recently, they have been referred to as t-designs and many mathematicians are now working on t-designs for $t \geqq 3$ (see e.g., Alltop

(1969, 1971). Assmus and Mathson (1966, 1969), Hanani (1963, 1971), Hughes (1965), Lane (1971) and Pless (1969, 1972), Raghavarao and Tharthare (1967, 1970)).

THEOREM 4.2. *A necessary condition for the existence of a tactical configuration* $C[k, t, \mu, v]$ *is that*

$$(*) \qquad \mu\binom{v-h}{t-h}\Big/\binom{k-h}{t-h} = \text{integer}\,, \qquad h = 0, 1, \cdots, t-1\,.$$

PROOF. The left side of (*) is the number of blocks of $C[k, t, \mu, v]$ that contain h fixed elements of Ω.

COROLLARY 4.5. *A tactical configuration* $C[k, t, \mu, v]$ *is necessarily a tactical configuration* $C[k, t' \mu, v]$ *for all* $t' < t$.

Now we give a second characterization of globally resistant designs of degree one.

THEOREM 4.3. *D is globally resistant of degree one if and only if it is a doubly balanced incomplete block design, i.e., if and only if it is a* 3*-design.*

The proof follows directly from Corollary 4.4 and Definition 4.1.

COROLLARY 4.6. *Any t-design,* $t \geqq 3$ *is at least globally resistant of degree one.*

This follows from Theorem 4.3 and Corollary 4.5.

This latter characterization of globally resistant designs allows us to utilize all the available theory related to tactical configurations or t-designs for our present theory. This also shows the practical usefulness of t-designs which are currently of great interest in combinatorial analysis.

5. Existence and construction of resistant designs.

THEOREM 5.1. *The existence of a* BIB (v, b, r, k, λ) *design on* Ω *such that* $b + 2\lambda = 3r$ *implies the existence of a globally resistant* BIB $(v+1, 2b, b, (v+1)/2, r)$ *design of degree one.*

Hereafter, $\mathbf{I}_n$ and $\mathbf{J}_{a,b}$ indicate the identity matrix of order n and an $a \times b$ matrix with unit entries everywhere respectively. We also denote an $m \times n$ matrix of zeros by $\mathbf{0}_{m,n}$.

PROOF. Let D_1 be the given BIB (v, b, r, k, λ) design. Augment every block of D_1 with a new treatment, say ϕ. Call the resulting design $\bar{D}_1$. Next, let D_2 be the complementary design associated with D_1. Then $D = \bar{D}_1 \cup D_2$ is a BIB design with the given parameters, and moreover it is globally resistant of degree one. The fact that D is a BIB design can be easily established. Here we prove that it is globally resistant. To do so, let $x \neq \phi$ be an arbitrary treatment in Ω. Then with no loss of generality the incidence matrix of $\bar{D}_1$ can be written as

$$\mathbf{N} = \begin{bmatrix} \mathbf{J}_{1,r} & \mathbf{0}_{1,b-r} \\ \mathbf{N}_1 & \mathbf{N}_2 \\ \mathbf{J}_{1,r} & \mathbf{J}_{1,b-r} \end{bmatrix} \begin{matrix} \leftarrow \text{related to } x \\ \\ \leftarrow \text{related to } \phi\,. \end{matrix}$$

Now write the incidence matrix of D_2 denoted by $\bar{\mathbf{N}}$ taking into account the same ordering of the treatments upon which $\mathbf{N}$ has been formed. Then we have

$$\bar{\mathbf{N}} = \begin{bmatrix} \mathbf{0}_{1,r} & \mathbf{J}_{1,b-r} \\ \mathbf{J}_{v-1,r} - N_1 & \mathbf{J}_{v-1,b-r} - \mathbf{N}_2 \\ \mathbf{0}_{1,r} & \mathbf{0}_{1,b-r} \end{bmatrix} \begin{matrix} \leftarrow \text{related to } x \\ \\ \leftarrow \text{related to } \phi\,. \end{matrix}$$

Note that the incidence matrix of D will be $[\mathbf{N} \vdots \bar{\mathbf{N}}]$. Let $D_{\bar{x}}$ denote the blocks of D which contain x. The incidence matrix of $D_{\bar{x}}$ upon deletion of x is

$$\mathbf{M} = \begin{bmatrix} \mathbf{N}_1 & \mathbf{J}_{v-1,b-r} - \mathbf{N}_2 \\ \mathbf{J}_{1,r} & \mathbf{0}_{1,b-r} \end{bmatrix}.$$

Then by Lemma 4.1 and Corollary 4.1 the design D is globally resistant if and only if $\mathbf{M}$ is the incidence matrix of a BIB design, i.e., $\mathbf{MM}'$ has a fixed diagonal entry and a fixed off-diagonal entry. Computing $\mathbf{MM}'$ and setting $b = 3r - 2\lambda$ we obtain

$$\mathbf{MM}' = (r - \lambda)\mathbf{I}_v + \lambda \mathbf{J}_{v,v}\,.$$

The case $x = \emptyset$ is straightforward. □

COROLLARY 5.1. *The existence of a* BIB $(4t - 1, 4t - 1, 2t - 1, 2t - 1, t - 1)$ *design implies the existence of a globally resistant* BIB $(4t, 8t - 2, 4t - 1, 2t, 2t - 1)$ *design of degree one.* (See also Raghavarao and Tharthare (1970).)

COROLLARY 5.2. *If* $4t - 1 = p^\alpha$ *or* $4t - 1 = p^\alpha q^\beta$, *where* p, q *are primes and* $q^\beta = p^\alpha + 2$, *then the globally resistant design in the preceding corollary exists.*

For the case $4t - 1 = p^\alpha$ the symmetric designs are derived cyclically following Bose (1947); the designs D_1 when $4t - 1 = p^\alpha q^\beta$ were obtained by Stanton and Sprott (1958).

EXAMPLE. Let $4t - 1 = 7$. D_1 is obtained by letting $\Omega = \{0, 1, 2, 3, 4, 5, 6\}$ and developing cyclically the initial block (2, 4, 1) with arithmetic modulo 7. Then D is the following design:

$$\begin{matrix} \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \\ 2 & 3 & 4 & 5 & 6 & 0 & 1 \\ 4 & 5 & 6 & 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 & 5 & 6 & 0, \end{matrix} \qquad \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 0 & 1 & 2 \\ 5 & 6 & 0 & 1 & 2 & 3 & 4 \\ 6 & 0 & 1 & 2 & 3 & 4 & 5. \end{matrix}$$

(See also Bhat and Shrikhande (1970).)

Since Theorem 5.1 is an important theorem it would be very useful if we could characterize all BIB designs with $b + 2\lambda = 3r$, $v = 2k + 1$. We shall now do this.

Characterization of BIB $(v = 2k + 1, b = 3r - 2\lambda, r, k, \lambda)$. It is easily shown that $r = mk$, where m is an integer. We shall now explore all the possibilities for m.

(i) $m = 1$. This implies that the design must be a symmetric BIB $(4\lambda + 3, 4\lambda + 3, 2\lambda + 1, 2\lambda + 1, \lambda)$ design.

(ii) $m = 2$. We get the series BIB $(2\lambda + 3, 4\lambda + 6, 2\lambda + 2, \lambda + 1, \lambda)$ designs which have the parameters of the derived design of symmetric BIB $(4\lambda + 7, 4\lambda + 7, 2\lambda + 3, 2\lambda + 3, \lambda + 1)$ designs which is the same series as in (i).

(iii) $m \geqq 3$. We get BIB $(2k + 1, m(2k + 1), mk, k, m(k - 1)/2)$ designs. Now consider 2 cases:

(a) k odd $= 2t + 1$. Then the design has parameters of an m-replicate of the BIB $(4t + 3, 4t + 3, 2t + 1, 2t + 1, t)$ designs; i.e., an m-replicate of the design in (i).

(b) k even $= 2t$. Then the design is an $m/2 = \mu$ replicate of a BIB $(4t + 1, 2(4t + 1), 4t, 2t, 2t - 1)$ design. Note that this BIB design is the same series as in (ii) when λ is odd. Thus designs in (iii) have parameters of μ replicates of designs in (i) or (ii).

Thus we have characterized the entire family. Whether or not all these designs exist is a major unsolved problem. This is so because the existence of a BIB $(4t + 3, 4t + 3, 2t + 1, 2t + 1, t)$ design is equivalent to the existence of a Hadamard matrix of order $4(t + 1)$. These probably exist for all t, but as of today this is an open problem for infinitely many t's.

In the first case of Corollary 5.2., D_1 was obtained from the initial block $(g^2, g^4, g^6, \cdots)$ where g is a primitive element of $G_F(p^\alpha)$; in our example $g = 3$. Then D_2 was the complement of D_1 and itself a cyclic design. Suppose, however, we replace D_2 by the cyclic design D_2^* which has initial block $(0, g^2, g^4, g^6, \cdots)$; we then have the following theorem.

THEOREM 5.2. *The design* $\bar{D}_h = D_1 \cup D_2^*$ *is locally resistant with respect to treatment* $\emptyset$ *only.*

EXAMPLE. For the case $4t - 1 = 7$ D_h is

∅	∅	∅	∅	∅	∅	∅		0	1	2	3	4	5	6
2	3	4	5	6	0	1		2	3	4	5	6	0	1
4	5	6	0	1	2	3		4	5	6	0	1	2	3
1	2	3	4	5	6	0,		1	2	3	4	5	6	0.

REMARK. Note that Corollary 5.2 and Theorem 5.2 show that the property of being locally or globally resistant depends both on the parameters of the design and also on the way in which it is constructed.

Sprott (1955) establishes the following theorem, which we state without proof.

THEOREM 5.3. *Let D be a* BIB *design with $v = 2k$ and $\bar{D}$ its complement. Then $D \cup \bar{D}$ is a 3-design.*

Application of this theorem leads to Sprott's (1956) Series 4 BIB designs which have parameters $(v = 2k, b = 4(2k - 1) = 2r, \lambda = 2k - 1)$. These designs exist whenever $2k - 1$ is a prime power. His Series 5 designs in the same paper are examples of our Theorem 4.3.

If s is a prime power, then Hanani (1971) has constructed for every positive integer d a family of $C[s+1, 3, 1, s^d+1]$. Therefore by Theorem 4.3 we have

THEOREM 5.4. *If s is a prime power, then there exists a globally resistant* BIB (v, b, r, k, λ) *design of degree one for any positive integer d and*

$$v = s^d + 1\,, \qquad b = (s^d+1)/(s+1) \textstyle\sum_{j=d-1}^{2(d-1)} s^j\,,$$
$$r = \textstyle\sum_{j=d-1}^{2(d-1)} s^j\,, \qquad k = s+1\,, \qquad \lambda = \textstyle\sum_{j=0}^{d-1} s^j\,.$$

Thus for example, if $s = 3$ and $d = 2$, one can construct a globally resistant BIB (10, 30, 12, 4, 4) design.

Preece (1967) lists non-isomorphic BIB designs with parameters $(2k, 4k-2, 2k-1, k, \lambda)$ for $\lambda = 1, 2, \cdots, 7$. He has four non-isomorphic designs for $v = 8$, five for $v = 10$, eight for $v = 12$, twelve for $v = 14$ and no fewer than thirty for $v = 16$. Each of these designs together with its complement gives a 3-design and thus or globally resistant design of degree one.

Corollary 4.5 and Theorem 4.3 enable us to utilize all the available results on t-designs for our theory. Unfortunately, no one, as yet, has found a nontrivial t-design for $t \geqq 6$. The selected references at the end of this paper can lead the interested reader to the available results on t-designs.

REMARK. We warn the reader that some authors like H. J. Ryser, W. G. Bridges, E. S. Kramer and perhaps others, have different definitions for t-designs (also called H-design) which seem of no use in our theory.

Before closing this section we mention the following important theorem.

THEOREM 5.5. *Every symmetric* BIB (v, k, λ) *design is locally resistant of degree k (but not necessarily of degree less than k).*

The proof follows from the known fact that if we delete any k treatments which appear in the same block then the remaining structure is a BIB $(v-k, v-1, k, k-\lambda, \lambda)$ design.

6. Miscellaneous results. In this section we shall give some results which we have not, as yet, been able to generalize. However, they are interesting enough to be included in this report. Among these results, a locally resistant design of degree two is also included.

(a) A locally resistant design of degree one with respect to two treatments. The following BIB (8, 14, 7, 4, 3) is Preece's (1967) type (iv).

0 4 5 6	0 4 5 7	0 3 6 7	0 1 3 4	0 1 2 6
1 5 3 6	1 5 3 7	1 4 6 7	1 2 4 5	0 1 2 7
2 3 4 6	2 3 4 7	2 5 6 7	2 0 5 3	

This design is resistant if either 6 or 7 is deleted. However, it is not resistant if both 6 and 7 are deleted.

(b) A locally resistant design of degree two. Consider the following BIB (12, 66, 33, 6, 15) design constructed in the following way. The design consists of four parts. Part one is a BIB (10, 15, 9, 6, 5) design. Part two is a BIB (10, 18, 9, 5, 4) design with x added to each block. Part three is a BIB (10, 18, 9, 5, 4) design and y added to each block. Part four is a BIB (10, 15, 6, 4, 2) design and both x and y added to each block. This design is locally resistant of degree 2, i.e., we may delete either of the two treatments, x and y, or we may delete both x and y.

(c) Note that we can construct a globally resistant design of degree one with the same parameters as of the design in part (b). Let D be a three-fold BIB (11, 11, 5, 5, 2) design. Then D is a BIB (11, 33, 15, 5, 6) design. Since $b + 2\lambda = 3r$ in D we can construct a globally resistant BIB (12, 66, 33, 6, 15) design of degree one utilizing D by the method of Theorem 5.1.

(d) A susceptible design which satisfies (N_1), (N_2), and (N_3) of Corollary 4.2. We have already given an example of this type of design in (1) of Corollary 4.3. Here we give another example which is more interesting because it is a cyclic design. This design is the BIB (12, 22, 11, 6, 5) design derived from the initial block 1 2 4 8 9 0 and 1 3 4 9 x y:

```
1 2 3 4 5 6 7 8 9 0     1 2 3 4 5 6 7 8 9 0
2 3 4 5 6 7 8 9 0 1     3 4 5 6 7 8 9 0 1 2
4 5 6 7 8 9 0 1 2 3     4 5 6 7 8 9 0 1 2 3
8 9 0 1 2 3 4 5 6 7     9 0 1 2 3 4 5 6 7 8
9 0 1 2 3 4 5 6 7 8     x x x x x x x x x x
0 1 2 3 4 5 6 7 8 9,    y y y y y y y y y y.
```

Note that $11 = 4t - 1$ satisfies the condition of Corollary 5.2 and thus one can also construct a globally resistant BIB (12, 22, 11, 6, 5) design of degree one.

7. Discussion and some open problems. In this paper we have opened up a new and practically useful area for research. A practical utility of locally and globally resistant designs was pointed out in Section 3. We may mention that our theory is also a contribution to the theory of balanced incomplete block designs with unequal block sizes; from all locally and globally resistant designs one can derive balanced incomplete block designs with unequal block sizes simply by deleting those treatments to which the designs are resistant. We hope that our theory encourages further research by those who are interested to expand the theory and application of design of experiments. Unsolved problems in this area can be formulated as follows:

(a) Our theory depends heavily on the homoscedastic additive linear model. Thus a generalization will be for a more general model.

(b) We have mainly concentrated on locally and globally resistant designs of degree one. Hence, the development of the theory is open for other degrees. Even for degree one we have not solved all the problems.

(c) We have indicated that a sufficient condition for BIB (v, b, r, k, λ) design to be locally resistant of degree k is $v = b$. Is this condition also necessary?

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