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CORRECTION NOTE

CORRECTION TO
"A COUNTEREXAMPLE ON MEASURABLE PROCESSES"

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I am very grateful to Vaclav Fabian for pointing out that my paper [1] needs the following corrections and clarifications.

Generally, measures should be taken complete. In particular, in equation (2) take both μ and ν complete. Then the Bledsoe–Morse product measure $\mu \times_M \nu$ is the minimal extension of the usual product measure by the new sets of measure zero. Product measures should be taken complete throughout. In Proposition 1, the space $\mathscr{L}^0(H, \bar{R})$ of Borel measurable functions should be replaced by the larger space $\mathscr{L}^0(H, \bar{R}, \mu)$ of μ-measurable functions where μ, defined around (4), should be completed. (The Proposition is true in either case, but is trivially weak as stated for Borel measurability.)

Around equation (1), the hypotheses on X and the citation of Kolmogorov's theorem are unnecessary since P_x is just an image measure here.

The passage from equation (17) to (18) is incorrect as stated. To repair it one can make the following changes.

After the paragraph containing equation (3), note that $|\omega_n|/n$ is almost surely bounded and replace Ω by the subset on which it is bounded.

In the proof of the lemma, third paragraph, definition of $\mathscr{A}_n$, we now require that sets in $\mathscr{A}_n$ depend only on the first n co-ordinates $\omega_1, \cdots, \omega_n$, and replace the inequality with max by

$$\max(|\zeta_j| : 1 \leqq j \leqq n) \leqq n^2 .$$

In (15) and (17), ζ_j should be $(s_\alpha)_j$. In (18) replace ε by 2ε. Then to prove (18) for N large, write

$$|L(s_\alpha)(\omega) - \textstyle\sum_{1 \leqq j \leqq N} (s_\alpha)_j \zeta_j| \leqq S_1 + S_2 \qquad \text{where}$$

$$S_2 \equiv |\textstyle\sum_{j > N} (s_\alpha)_j \omega_j| < \varepsilon \qquad \text{by (17), and}$$

$$S_1 \equiv |\textstyle\sum_{1 \leqq j \leqq N} (s_\alpha)_j (\zeta_j - \omega_j)| \leqq \|s_\alpha\| [\sum_{1 \leqq j \leqq N} (\zeta_j - \omega_j)^2]^{\frac{1}{2}}$$

by the Schwarz inequality. $\|s_\alpha\|$ is fixed and for N large enough we have $|\zeta_j| \leqq N^2$ for $j = 1, \cdots, N$, so that since $\omega \in E_N(\zeta)$, $S_1 \leqq \|s_\alpha\|/N^{\frac{1}{2}} < \varepsilon$ for N large enough as desired.

After equation (12), the measurability of T_{FU} is perhaps not obvious. The following fact may be used.

LEMMA. *Let* $(\Omega, \mathscr{B}, P)$ *be the completed Cartesian product of probability spaces* $(\Omega_n, \mathscr{B}_n, P_n)$. *Let* $B \in \mathscr{B}$ *and* $P(B) > 0$. *Let* $B^\infty = \{\omega \in \Omega : \textit{for some } \zeta \in B, \zeta_n = \omega_n$ *for all large enough* $n\}$. *Then* $P(B^\infty) = 1$.

PROOF. Let C_n be the sub-σ-algebra of $\mathscr{B}$ generated by $\mathscr{B}_1 \times \cdots \times \mathscr{B}_n$. Then by martingale convergence $E(\chi_B \mid C_n) \to \chi_B$ P-almost surely. Now

$$E(\chi_B \mid C_n)(\{\omega_j\}_{j \leq n}) = (\prod_{m>n} P_m)(\{\omega_m\}_{m>n} : \{\omega_m\}_{m=1}^{\infty} \in B) \quad \text{a.s.}$$

Thus the right side is arbitrarily close to 1 for suitable $\omega_1, \cdots, \omega_n$. Hence $P(B^\infty) = 1$. □

REFERENCE

[1] DUDLEY, R. M. (1972). A counterexample on measurable processes. *Proc. Sixth Berkeley Symp. Math. Statist. Prob.* Univ. of California Press.