The Annals of Probability
1976, Vol. 4, No. 1, 153–154

NOTE

CORRECTION TO "A COUNTEREXAMPLE TO PEREZ'S GENERALIZATION OF THE SHANNON-McMILLAN THEOREM"

BY JOHN C. KIEFFER
University of Missouri, Rolla

In [1] we presented an example which was stated to be a counterexample to Theorem 2.3 of [2]. This is not true since an assumption implicit from page 553 of [2] was overlooked. In this correction note, by modifying the technique of [1], we obtain a correct counterexample. We remark that Dr. Perez has recently repaired his Theorem 2.3 by adding an additional assumption; see [3].

Let $(\Omega, \mathscr{F})$ be the measurable space where Ω is the set of doubly infinite sequences of integers and $\mathscr{F}$ is the usual product sigma-field. For each integer i, let X_i be the projection from Ω onto the ith coordinate. If i, j are integers such that $i \leqq j$, let $\mathscr{F}_{i,j}$ be the sub-sigmafield of $\mathscr{F}$ generated by $X_i, X_{i+1}, \cdots, X_j$. If P is a probability measure on $\mathscr{F}$ let P_n be the restriction of P to $\mathscr{F}_{1,n}$, $n = 1, 2, \cdots$.

To provide a counterexample to Theorem 2.3 of [2] we construct probability measures P, Q on $\mathscr{F}$ such that

(a) P, Q are stationary;
(b) P_n is absolutely continuous with respect to Q_n, $n = 1, 2, \cdots$;
(c) $\lim_{n\to\infty} \int_\Omega n^{-1} \log (dP_n/dQ_n)\, dP$ exists and is finite;
(d) for each $E \in \mathscr{F}_{0,0}$, $\lim_{n\to\infty} Q(E \mid \mathscr{F}_{-n,-1})$ exists a.e. $[P]$, and there exists a probability measure P' on $\bigvee_{i\leqq 0} \mathscr{F}_{i,0}$ such that

$$P'(F \cap E) = \int_F \lim_{n\to\infty} Q(E \mid \mathscr{F}_{-n,-1})\, dP ,$$

for each $F \in \bigvee_{i<0} \mathscr{F}_{i,-1}$, $E \in \mathscr{F}_{0,0}$;
(e) $\lim_{n\to\infty} n^{-1} \log (dP_n/dQ_n)$ does not exist in $L^1(P)$.

(Condition (d) was overlooked in [1]. Also, all logarithms are to base 2.)

It is not hard to construct a double sequence $a_{n,j}$, $n, j = 1, 2, \cdots$, such that

(f) $a_{n,j} \geqq 2$, $n, j = 1, 2, \cdots$;
(g) $|a_{n+1,j} - a_{n,j}| \leqq [n \log (n+1)]^{-1}$, $n, j = 1, 2, \cdots$;
(h) $\lim_{n\to\infty} a_{n,j} = 2$ for each j;
(i) $\sum_{j=1}^{\infty} 2^{-j} a_{n,j} = 3$ for each n.

For each positive integer j, let P^j be the discrete probability measure on $\mathscr{F}$ which assigns probability one to the sequence which is identically $2j$. From [1] we can construct a stationary discrete probability measure Q^j on $\mathscr{F}$ with support contained in the set of all sequences each of whose entries are $2j$ or $2j-1$, such that

$$Q^j(X_1 = 2j, X_2 = 2j, \cdots, X_n = 2j) = 2^{-na_{n+1,j}}, \qquad n = 1, 2, \cdots.$$

Let $P = \sum_{j=1}^{\infty} 2^{-j} P^j$, $Q = \sum_{j=1}^{\infty} 2^{-j} Q^j$. Then (a)—(e) hold.

REFERENCES

[1] KIEFFER, J. C. (1973). A counterexample to Perez's generalization of the Shannon-McMillan Theorem. *Ann. Probability* **1** 362–364.

[2] PEREZ, A. (1964). Extensions of Shannon-McMillan's limit theorem to more general stochastic processes. *Trans. Third Prague Conf. on Information Theory: Statistical Decision Functions and Random Processes.* 545–574.

[3] PEREZ, A. (1974). Generalization of Chernoff's result on the asymptotic discernability of two random processes. *Colloquia Mathematica Societas Janos Bolyai* **9** 619–632.