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WEAK CONVERGENCE RESULTS FOR EXTREMAL PROCESSES GENERATED BY DEPENDENT RANDOM VARIABLES[1]

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In this paper we consider a stationary sequence $\{X_n, n \geqq 1\}$ satisfying weak dependence restrictions similar to those recently introduced by Leadbetter. Suppose a_n and $b_n > 0$ are norming constants for which $\max\{X_{n1}, \cdots, X_{nn}\}$ converges in distribution, where $X_{nk} = (X_k - b_n)/a_n$. Define a sequence of planar processes $I_n(B) = \#\{j : (j/n, X_{nj}) \in B, j = 1, 2, \cdots, n\}$, where B is a Borel subset of $(0, \infty) \times (-\infty, \infty)$. Then the I_n converge weakly to a nonhomogeneous two-dimensional Poisson process possessing the same distribution as for independent X_j. Applying the continuous mapping theorem to this result generates a variety of further results, including, for example, weak convergence of the order statistics of the X_n sequence. The dependence conditions are weak enough to include the Gaussian sequences considered by Berman.

1. Introduction. The extreme value theory of sequences of independent and identically distributed (i.i.d.) random variables has often been generalised to include the situation when the variables are no longer independent. These generalisations have been aimed in essentially two directions. Watson (1954), Loynes (1965) and Welsch (1971) consider sequences of "m-dependent" or "strong mixing" random variables, while Berman (1964, 1971) and others consider stationary Gaussian sequences in which the correlation between distant points of the sequence tends to zero as the distance between them tends to infinity. In two recent papers, Leadbetter, (1974b, 1976), these directions have been merged, and a new type of "asymptotic independence" condition introduced which is significantly weaker than those used previously, and also wide enough to include the Gaussian case. Leadbetter's main results include the distribution of the (normalised) maximum of the sequence, as well as a Poisson weak convergence result for the exceedances by the sequence of increasingly high levels.

In this paper we shall slightly strengthen Leadbetter's condition to obtain much fuller results. These include convergence of the joint finite-dimensional (fidi) laws of the kth sample extremal processes (defined below), as well as weak convergence and Poisson results. The form of the main result, given in Section 4, is such as to permit us to immediately apply known results for corresponding

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sequences of independent random variables to the dependent case. This is discussed in Section 5.

We obtain our results by applying an idea due to Weissman (1975) of getting all the limit processes we require in terms of a certain two-dimensional non-homogeneous Poisson process. Section 2 sets up the necessary notation, and considers some implications of our new dependence condition, which are further explored for the Gaussian case in Section 3.

2. Notation and the structure of dependence. Let $\{X_j, j = 1, 2, \cdots\}$ be a stationary sequence of random variables. Throughout the remainder of this paper we shall assume that there exist fixed sequences of normalising constants a_n and b_n ($b_n > 0$), and a distribution function $G(x)$, (which in all the cases we shall consider will necessarily be one of three classical extreme value distributions) such that

$$P\{\max_{1\leq j\leq n} (X_j - b_n)/a_n \leq x\} \to G(x) \quad \text{as} \quad n \to \infty . \tag{2.1}$$

We write $X_{nj} = (X_j - b_n)/a_n$, and define a sequence of two-dimensional processes I_n by

$$I_n(B) = \#\{j : (j/n, X_{nj}) \in B, j = 1, 2, \cdots, n\} , \tag{2.2}$$

where B is any Borel subset of $(0, 1] \times (-\infty, \infty)$, and $\#A$ is the number of elements in the set A. These processes may be regarded as random elements in the space $\mathfrak{N}$ of integer-valued Borel measures on $(0, 1] \times (-\infty, \infty)$. This space is metric under the "vague topology" (e.g., generated in $\mathfrak{N}$ by the functions $\mu \to \int f\, d\mu$ for continuous f with bounded support (cf. Jagers (1974), Kallenberg (1973)) and thus it makes sense to talk about convergence in distribution, denoted by $\Rightarrow$, of the I_n to some limit process I, which we now define.

Write G_* for the left end (possibly $-\infty$) of the support of G, and let $T \subset R^2$ denote the set $(0, 1] \times (G_*, \infty)$. Let μ be the Lebesgue-Stieltjes measure on (G_*, ∞) corresponding to $\log G(x)$; i.e., for $x < y$

$$\mu(x, y] = \log [G(y)/G(x)] . \tag{2.3}$$

Then we define $I(\cdot)$ to be the two-dimensional Poisson process on T with intensity (parameter) measure simply the product of Lebesgue measure and μ. Thus, if λ is the measure determined by

$$\lambda(A) = (t_2 - t_1)\mu(x, y]$$

for rectangles $A = (t_1, t_2] \times (x, y] \subset T$, then for any Borel $B_1, B_2 \subset T$ we have

$$I(B_i) \quad \text{is a Poisson variable with parameter} \quad \lambda(B_i) , \tag{2.4}$$

$$B_1 \cap B_2 = \emptyset \quad \text{implies} \quad I(B_1) \quad \text{and} \quad I(B_2) \quad \text{are independent.} \tag{2.5}$$

We shall establish that $I_n \Rightarrow I$, but first we need some conditions on the X_j. Let M be finite, and $\{A_j\}$ be a sequence of subsets of the real line, where each A_j is the union of at most M (possibly infinite) intervals. Furthermore, suppose that there are only a finite number, N, of different types of A_i. Write

$F^{(n)}_{i_1\cdots i_r}(A_1, \cdots, A_r)$ to denote the probability $P\{X_{ni_1} \in A_1, \cdots, X_{ni_r} \in A_r\}$. Then we shall say that the stationary sequence $\{X_j\}$ satisfies *condition D* if for any integers

$$1 \leqq i_1 < i_2 < \cdots < i_p < j_1 < \cdots < j_q \leqq n\,, \qquad j_1 - i_p > m$$

and any sets $A_1, \cdots, A_p, B_1, \cdots, B_q$ of the above form there exists a (double) sequence α_{nm} (perhaps dependent on M, N, as well as the A_i and B_i) for which

$$(2.6) \qquad |F^{(n)}_{i_1\cdots i_p j_1\cdots j_q}(A_1 \cdots A_p\, B_1 \cdots B_q) - F^{(n)}_{i_1\cdots i_p}(A_1 \cdots A_p) F^{(n)}_{j_1\cdots j_q}(B_1 \cdots B_q)| \leqq \alpha_{nm}\,,$$

where α_{nm} is nonincreasing in m and $\lim_{n\to\infty} \alpha_{n,q_n} = 0$ for some sequence $q_n \to \infty$ for which $q_n/n \to 0$.

Given such a sequence q_n it is easy to show, using methods such as those used to establish Theorem 1.3 of Ibragimov (1962), that there exist further sequences p_n and k_n, with $q_n = [(n - k_n p_n)/k_n]$, for which $p_n \to \infty$, $k_n \to \infty$, $k_n p_n/n \to 1$, and $k_n \alpha_{nq_n} \to 0$. (Note that we have just diverged from the notation of Leadbetter (1976), in favour of the longer-standing notation of Ibragimov (1962) and Welsch (1971)). Then, if condition D is satisfied, we shall say that the sequence $\{X_j\}$ also satisfies *condition D′* if for any sequences $\{p_n\}$, $\{q_n\}$, and $\{k_n\}$ satisfying the above conditions we have

$$(2.7) \qquad \lim_{n\to\infty} k_n \sum_{j=1}^{p_n-1} (p_n - j) P\{X_{n1} \geqq x, X_{nj} \geqq x\} = 0$$

for all x such that $0 < G(x) < 1$.

Before we proceed to our main results, it will be worthwhile to take time off to discuss D and D'. Condition D' is the same in spirit as the corresponding condition $D'(u_n)$ of Leadbetter (1974b), except that it holds for arbitrary rather than fixed x. Condition D, however, is somewhat stronger than the corresponding condition of Leadbetter, in which the sets A_i and B_i are all $(-\infty, x]$, for some fixed x. It is, however, significantly weaker than a strong mixing assumption, in that it is only a condition on the tails of the joint distribution functions of the X_j. The real demonstration of the weakness of D and D', however, lies in the fact that in the case when the X_j are Gaussian these conditions can be translated into a covariance condition, which is known to be rather tight for the type of results we are considering.

3. The Gaussian case.

THEOREM 3.1. *Suppose $\{X_j\}$ is a stationary Gaussian sequence with zero means and unit variances, and put $r_n = E\{X_1 X_{n+1}\}$. Let $\{a_n\}$ and $\{b_n\}$ be the "usual" norming sequences for the Gaussian case,* i.e.,

$$a_n = (2 \log n)^{-\frac{1}{2}}\,, \qquad b_n = (2 \log n)^{\frac{1}{2}} - \tfrac{1}{2}(2 \log n)^{-\frac{1}{2}}(\log \log n + \log 4\pi)\,.$$

Then conditions D and D' hold for the $\{X_j\}$ with these norming sequences if either $r_n \log n \to 0$ or $\sum r_n^2 < \infty$. Furthermore, the bound α_{nm} in (2.6) *can be chosen in such a way that $\lim_{n\to\infty} \alpha_{nm} = 0$ for each m, so that the sequence q_n can be chosen with complete freedom.*

We shall prove the theorem in a reasonably standard fashion, via two lemmas. The following lemma is proved in Berman (1964).

LEMMA 3.1. *Suppose that either* $r_n \log n \to 0$ *or* $\sum_{n=1}^{\infty} r_n^2 < \infty$. *Then for any fixed* x

$$n \sum_{j=1}^{n} |r_j| \exp\{-(a_n x + b_n)^2/(1 + |r_j|)\} \to 0 \qquad as \quad n \to \infty .$$

For the following lemma let $M, N < \infty$ be fixed, and let $A_1, A_2, \cdots$ be a sequence of subsets of R with the properties given before (2.6). Furthermore, let $\{x_i\}$ be the sequence of numbers comprising the various finite endpoints of the intervals that make up the A_i. There are at most $2NM$ such x_i's. Then we have

LEMMA 3.2. *Let* $\{X_j\}$ *be as in the statement of Theorem* 3.1, *and let* $1 \leqq l_1 \leqq l_2 \leqq \cdots \leqq l_s$. *Then with the* $\{A_i\}$ *as described above*

$$|F^{(n)}_{l_1 \cdots l_s}(A_{l_1}, \cdots, A_{l_s}) - \prod_{j=1}^{s} F^{(n)}_{l_j}(A_{l_j})| \\ \leqq K \sum_{1 \leqq i < j \leqq s} \sum |\rho_{ij}| \exp\{-(a_n x_k + b_n)^2/(1 + |\rho_{ij}|)\}$$

where $\rho_{ij} = r_{l_i - l_j}$ *and the second summation is over the* (*no more than* $2NM$) x_k*'s.*

PROOF. This result is well known for the case in which all the A_i are of the form $(-\infty, x]$, (see, for example, Berman (1964, 1971) and Leadbetter (1974a)) and since the proof for the current situation is virtually the same we leave it to the reader.

It is now a simple matter to combine the above two lemmas to establish the theorem (cf. Lemma 4.3 of Leadbetter (1974b)). As a referee of an earlier version of this paper intimated, Theorem 3.1 is true only because in defining the condition D we are careful to insist on the bounds N and M, and allow the sequence α_{nm} to depend on the sets we are considering.

In fact, the result of this theorem cannot be proven in the usual fashion, and indeed may not be true, if we insist that the bound α_{nm} in (2.6) be uniform over all choices of N, M and the sets A_i and B_i. Such uniformity would in fact bring us very close to the assumption of strong mixing.

4. The main convergence theorem.

THEOREM 4.1. *Let* $\{X_j\}$ *be a stationary sequence of random variables, and assume two sequences of real numbers,* $\{a_n\}$, $\{b_n\}$, $b_n > 0$, *exist for which*

$$P^n\{X_1 \leqq a_n x + b_n\} \to G(x) . \tag{4.1}$$

Then, if conditions D *and* D' *are satisfied,* $I_n \Rightarrow I$, *where the* I_n *are defined by* (2.2) *and* I *is defined by* (2.4) *and* (2.5).

REMARKS. From (4.1) it is clear that G must be one of the three classical extreme value distributions of Gnedenko. What we are actually doing through (4.1) is choosing a_n and b_n sequences that are appropriate, in terms of deriving extremal results, for the independent sequence "associated" with $\{X_j\}$; i.e., a sequence of i.i.d. variables with the same marginal distribution as X_1. Under (4.1) that this is no restriction is a consequence of Theorem 3.2 of Leadbetter (1974b), which states that the normalising sequences and the limit distribution are the same for the X_j and the associated sequence of i.i.d. variables. However, without (4.1), it may be possible that a limit may exist in the dependent case, when no limit exists for the associated independent sequence.

Before we commence the proof of Theorem 4.1 we require the following result, which, like Theorem 3.1 of Leadbetter (1976), is a special case of Theorem 2.3 of Kallenberg (1973), modified according to a remark of Kurtz (1974), and then specialised to the Poisson case. The sets B in (4.2) are finite unions of disjoint, bounded, rectangles of the form $(x, y] \times (s, t]$.

THEOREM 4.2. *Let $I_1, I_2, \cdots$ be point processes on $[0, 1] \times (-\infty, \infty)$ and I a Poisson process with parameter measure $\lambda(\cdot)$. Then $I_n \Rightarrow I$ if for any sets B of the form described above the following two conditions hold*:

$$P\{I_n(B) = 0\} \to \exp\{-\lambda(B)\}, \tag{4.2}$$

$$\lim_{n\to\infty} \sup E\{I_n(B)\} \leqq \lambda(B). \tag{4.3}$$

PROOF OF THEOREM 4.1. We shall show that (4.2) and (4.3) are satisfied. Fix a set $B \subset T$ of the above form, and for each $n \geqq 1$ define a sequence of indicator random variables $\{Y_{nj}, j = 1, 2, \cdots, n\}$ by $Y_{nj} = 1$ if $(j/n, X_{nj}) \in B$ and $Y_{nj} = 0$ otherwise. Clearly $I_n(B) = \sum_{j=1}^n Y_{nj}$. Then it is a simple consequence of (4.1) and the definition of μ that if we set $A_{nj} = \{x : (j/n, x) \in B\}$ we have

$$E\{Y_{nj}\} = P\{(j/n, X_{nj}) \in B\} \sim \mu(A_{nj})/n. \tag{4.4}$$

It now follows simply from (4.4), the fact that $I_n(B) = \sum Y_{nj}$, and the form of λ, that (4.3) is satisfied. It remains to establish (4.2).

Let $E_n(F_n)$ be an event defined in terms of the values of $\{Y_{n1}, \cdots, Y_{nk}\}$ ($\{Y_{n,m+k}, \cdots, Y_{nn}\}$). Then by (2.6) there exists a double sequence α_{nm} such that

$$|P(E_n F_n) - P(E_n)P(F_n)| \leqq \alpha_{nm}, \tag{4.5}$$

and a sequence q_n with $q_n/n \to 0$ and $\alpha_{nq_n} \to 0$. Now choose sequences $\{p_n\}$ and $\{k_n\}$ as described in Section 2, so that (2.7) implies (by choosing $x = \inf\{t : (s, t) \in B, \forall s \in [0, 1]\}$)

$$\lim_{n\to\infty} k_n \sum_{j=1}^{p_n-1} (p_n - j) P\{Y_{n1} = 1, Y_{nj} = 1\} = 0. \tag{4.6}$$

We now follow a simplified version of the proof of the theorem of Meyer (1973), which in turn is closely related to one of Loynes (1965).

Write $P_{nk} = P\{I_n(B) = k\} = P\{\sum_{j=1}^n Y_{nj} = k\}$. We will eventually be interested only in P_{n0}. For each n, partition the integers $1, 2, \cdots, n$ into $2k_n$ consecutive blocks of size p_n and q_n alternately beginning with the initial block $\{1, 2, \cdots, p_n\}$. (The last such block may, of course, be incomplete.)

Let $P_n(Q_n)$ denote those positive integers falling into size $p_n(q_n)$ blocks. Define events $B_{nk}(C_{nk})$ by "for exactly k values of i, $i = 1, 2, \cdots, n$, $Y_{ni} = 1$, and all (some) such i's are in $P_n(Q_n)$." Then $P_{nk} = P\{B_{nk}\} + P\{C_{nk}\}$. However, by (4.4) $P(C_{nk}) \leqq \sum_{Q_n} P\{Y_{nk} = 1\} \sim (n^{-1}k_n q_n) \times \sup_j \mu(A_{nj})$. Since for a given $B \in T$ this supremum is bounded, it follows that $P(C_{nk}) \to 0$. Thus, if either P_{nk} or $P\{B_{nk}\}$ has a limit as $n \to \infty$, both have and the two are equal.

But this implies that $|P_{n0} - P\{G_{n1} \cdots G_{nk_n}\}| \to 0$, where G_{ni}, $i = 1, \cdots, k_n$, is the event "$Y_{nm} = 0$ for every m in the ith P_n block." Via (4.5) and a standard induction argument we have $|P\{G_{n1} \cdots G_{nk_n}\} - \prod_{i=1}^{k_n} P\{G_{ni}\}| \leqq k_n \alpha_{nq_n} \to 0$. Thus we need only estimate the product $\prod P\{G_{ni}\}$ to complete the proof. By the Bonferroni inequalities and (4.6) $P\{G_{ni}\} = 1 - n^{-1} \sum \mu(A_{nj}) + o(k_n^{-1})$, where the summation is over all j in the ith P_n block. Hence $|\prod_{i=1}^{k_n} P\{G_{ni}\} - \exp\{-n^{-1} \sum_{j \in P_n} \mu(A_{nj})\}| \to 0$, and since the exponent in this difference converges to $\lambda(B)$ (4.2), and thus the theorem, are established.

5. Some applications of Theorem 4.1. In this section we shall show that the way in which Theorem 4.1 has been set up enables us to obtain a variety of useful results about extreme values in dependent sequences. The main idea is that since Theorem 4.1 is effectively an invariance principle, and the limit process is the same as arises for independent sequences $\{X_j\}$, many results that were hitherto known in the independence situation can now be automatically carried over to the dependent case. We consider only two examples related to order statistics and record values. Firstly, however, we need to change topologies.

So far, we have been working with the vague topology on $\mathfrak{N}$, so that Theorem 4.1 holds for weak convergence relevant to this topology. There is, however, another "natural" topology for $\mathfrak{N}$, this being the extension to the plane of the Skorohod J_1-topology, as developed by Straf (1972) and Bickel and Wichura (1971). Weak convergence in this topology, which we shall denote by $\Rightarrow_s$, implies weak convergence in the vague topology, although in general the converse is not true. However, since vague convergence implies the convergence of all the fidi distributions, it follows from a remark of Straf (1972, page 212) that in certain special cases, which include the case of the limit process being Poisson, the converse does hold. Thus Theorem 4.1 holds with Skorohod convergence. We can exploit this fact to easily obtain further results.

Retaining our previous notation, for each pair of integers k, n define a stochastic process $m_n^k(t)$, $0 < t \leqq 1$, by

$$m_n^k(t) = k\text{th largest among} \quad \{X_{n1}, \cdots, X_{n[nt]}\}$$

if $1 \leqq k \leqq [nt]$, and $m_n^k(t) = -\infty$ if $k > [nt]$. Now write $I_n(t, x)$ to denote $I_n((0, t] \times (x, \infty))$, with a similar definition for $I(t, x)$. Then, as Weissman (1975) points out, m_n^k is simply related to the process I_n, since $m_n^k(t) \equiv \min\{x : I_n(t, x) \leqq k - 1\}$ and $\{m_n^k(t) \leqq x\} \equiv \{I_n(t, x) \leqq k - 1\}$. Thus we should be able to obtain the limit of m_n^k in terms of I. To this end, let us define the kth *extremal process* $m^k(t)$, $0 < t \leqq 1$, by $m^k(t) = \min\{x : I(t, x) \leqq k - 1\}$, and the *$k$-dimensional extremal process* by $Z^k = (m^1, \cdots, m^k)$. Similarly, write $Z_n^k = (m_n^1, \cdots, m_n^k)$. Then a brief check of the proof of Theorem 1.1 of Weissman (1976) shows that since he makes no use of the independence properties assumed in his paper that proof can be used verbatim in our current circumstances to prove the following result.

THEOREM 5.1. *Under the assumptions of Theorem* 4.1, Z_n^k *converges weakly to* Z^k *in terms of the Skorohod* J_1*-topology on* $D^k[a, 1]$ *for each fixed* k, *and* $0 < a < 1$. *(Here, as usual,* $D[a, b]$ *denotes the space of finite right-continuous functions on* $[a, b]$ *with left-hand limits.)*

Theorem 5.1 represents a generalisation of the main result of Welsch (1971) in two directions. Firstly, his requirement of strong mixing is relaxed to require only the satisfaction of condition D, and secondly, Theorem 4.1 contains an explicit representation of the distribution of Z^k for all k, not merely $k = 2$. As is noted in Weissman (1975), the properties of m^k and Z^k are easily derived from the properties of I. For example,

$$P\{m^k(t) \leqq x\} = G^t(x) \textstyle\sum_{i=0}^{k=1} (-t \log G(x))^i / i!\,,$$

$$\begin{aligned} P\{m^1(t) \leqq x, m^2(t) \leqq y\} &= G^t(x) && x \leqq y \\ &= G^t(y)\{1 - t \log (G(x)/G(y))\} && x > y\,. \end{aligned}$$

Similarly, it is a simple matter to write down transitional probabilities for the m^k and Z^k processes, and even such abstruse probabilities as $P\{m^3(t) \leqq x, m^7(t) \leqq y\}$ become trivial to compute. See Weissman (1975) for details.

We conclude with one more area of application of Theorem 4.1. To simplify notation, write Y_n for m_n^1 and Y for m^1. Let $x(t)$ be nondecreasing and $Nx(I)$ be the number of times x jumps in the time interval I. Suppose that the conditions of Theorem 4.1 are in force. Then, as in the independence case, reasonably straightforward applications of the continuous mapping theorem yield $Y_n \Rightarrow_s Y$, on $D[\delta, 1]$, $Y_n^{-1} \Rightarrow_s Y^{-1}$ on $D(0, \infty)$ or $D(-\infty, \infty)$, (depending on the support of G) and $NY_n \Rightarrow_s NY$ on $D[\delta, 1]$ for any $\delta > 0$. We refer the reader to Resnick (1975) for a discussion of these results, as well as applications of them to limit results for record values, record value times, and inter-record times.

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