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A LIFE OF THE IMMEASURABLE MIND

MARK KAC, *Enigmas of Chance, An Autobiography*. Harper and Row, New York, 1985, *xxvii* + 163 pages, $18.95.

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Live with the pleasure of the immeasurable mind.
Ralph Waldo Emerson, 1838

What worlds Mark Kac traversed! Born in August 1914, at the outbreak of World War I, he died in October 1984, somewhere between Mutual Assured Destruction and Star Wars. Born in Krzemieniec, Poland, a town that experienced Czarist, Polish, German, and Soviet control within his lifetime, he died in Los Angeles, California, near lots formerly used to make Hollywood movies, in a county of orange groves. Born where no teaching positions were open to his father, as a Jew, despite his father's advanced degrees from the universities of Leipzig and Moscow, Kac died in the fullness of honors the United States of America awards to those, of any origin, who create well. Born when the mathematical nature of probability theory was a mystery and statistical independence was an enigma within a mystery, he died during the spring flowering of probability theory and statistical physics.

In his autobiography, which he did not live to see in print, Mark Kac (pronounced "cots") aims to give the reader "a glimpse into the making of a scientist as he is influenced by his family, his teachers and collaborators as well as by those intangibles which pertain to his development: the political conditions and social attitudes of his time and the cataclysmic events of history."(xxvii) [I give page numbers after quotations.] The book does not aim to be, and is not, a history of probability theory in the twentieth century. With many delightful anecdotes, and with some keen insights into scientific creativity, the book gives a wonderful glimpse of Kac's life and times, incredible as they may seem to a younger generation brought up in the security of post-war America. And yet, as the book's title suggests, the enigmas remain.

Early years and education in Poland. In 1915, Kac's family moved from Krzemieniec east to Berdichev to get away from the fighting on the Eastern Front. With the Russian Revolution of 1917, civil war came to Berdichev. One of Kac's earliest memories is being bundled off to the cellar as a bombardment began. Early in the war he saw a man shot dead in the street.

Kac learned to read at home. "Except for an unusual memory [for poetry], I was not precocious in any respect... I was inordinately slow learning the

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multiplication tables."(11) When Krzemieniec reverted to Poland in 1921, Kac and his family returned. He and his mother fell ill with typhus. Kac overheard his father tutoring students in plane geometry and pestered him for explanations. Given the son's difficulty learning multiplication, the father was reluctant, and remarked to his wife, "I don't think he'll ever be a mathematician."

Kac did learn geometry from his father, and took special pleasure in solving difficult geometrical problems. On his own, he read Faraday's "wonderful" *Lectures on the Chemical History of a Candle* (in Russian translation). In the local lycée, he learned about evolution, cell division, and geometry from a liberal teacher whom the Germans promptly executed when they captured the town in 1941. Many of Kac's later professors at the University of Lwów shared the same fate.

Because secondary school students were not shown how to solve cubic equations, Kac picked for his reading in the summer of 1930 a book with a section on cubic equations. The method of solution given in the book began by anticipating the form of the solution. Kac rebelled: "I was not going to accept a formal derivation without understanding the motivation behind it."(2) So he spent the summer seeking his own method of solving cubic equations. "I had been stricken by an acute attack of a disease which at irregular intervals afflicts all mathematicians and, for that matter, all scientists: I became obsessed by a problem."(1)

Though he calls this chapter "How I Became a Mathematician," Kac does not explain why he was not, as other students were, deflected by his school's "This is too advanced for you," or why he picked for his summer's reading a book with a section on cubic equations, or why he rejected that book's derivation because it anticipated the form of the solution. His obsession with learning and understanding shows that he was already a mathematician. It merely took time for him and for the rest of the world to recognize that. How did he *become* a mathematician? An enigma.

After collecting on a bet with his skeptical father, who doubted that young Kac could find his own way to Cardano's formulas, Kac submitted his derivation to his school mathematics teacher, who in turn submitted it to a Polish journal, *The Young Mathematician*. Nothing further was heard until May 1931, when Kac was visited personally by the Counselor of the Ministry of Education, who was also the chief editor of the journal. Counselor Rusiecki informed Kac that his paper would be published, now that the editorial staff was convinced that Kac's method was new.

When Rusiecki asked Kac about his future, Kac said his family thought he should study engineering. Rusiecki advised him to study mathematics instead. This advice saved Kac's life. As a mathematician, Kac was good enough to win a fellowship that brought him to Johns Hopkins University in December 1938. Had he remained in engineering, and therefore in Europe, he would have been killed like millions of others.

Reflecting on his early schooling, Kac writes: "I wish that school mathematics had been more challenging. There was nothing about the mathematics curriculum or the way it was taught that particularly stimulated my interest." For how many future mathematicians does that remain true today?

Kac arrived in September 1931 at the university in Lwów, one hundred miles west of Krzemieniec. He had his father's assurance of support for one year, but no more. It was Kac's good luck that Lwów was one of Poland's two great centers of mathematics at the time (the other being Warsaw).

In a proseminar on algebra and number theory, Kac made the acquaintance of Marceli Stark, then a junior assistant at the Mathematical Institute. Stark helped Kac in numerous ways, including translating into English Kac's first "serious mathematical note" published in 1934. Stark spent World War II in a German concentration camp. He survived, and after the war became responsible for all advanced mathematical publications in Poland.

Kac gives this account of his oral examination in mechanics: "...he [the examiner] looked up from some work he was engaged in and said: 'Oh, you. You have an A.' When I said: 'But won't you ask me at least one question?' he said: 'Very well, what is the Schroedinger equation?' I said: 'I don't know', and then he said: 'All right. You still have an A.'... I am quite certain that [the examiner] himself didn't know the Schroedinger equation."(32)

In Lwów, Kac became first the student, then the assistant and the collaborator of Hugo Steinhaus, who brought mathematics to life in Lwów. It was Stark who suggested that Steinhaus look after Kac. Steinhaus hired Kac as his assistant, as a means of putting some money into Kac's hands. (When his fellowship funds were delayed for three months, Kac came "very close to starvation."(40)) As Steinhaus's assistant, Kac supervised the photography for the famous book *Mathematical Snapshots* and fell in love with a young lady whose hand he borrowed for one of the photographs.

Starting in 1935, Steinhaus asked Kac to explore the properties of what Steinhaus defined as "stochastically independent functions." Kac found them to be related to what A. A. Markov, in a book Kac had just been reading, called "independent random quantities." Steinhaus became excited and the two collaborated closely until Kac left Poland. For Kac, his mathematical life began with his collaboration with Steinhaus. Kac shared a perspective he attributed to Steinhaus, a "fundamental belief that real insights are gained from contemplating the simplest and most elementary things."(53)

In the spring of 1936, Kac discovered that the density of an appropriately standardized sum of cosines with linearly independent frequencies asymptotically approaches the normal density. Kac gives here an explanation of this discovery, with graphs of cosines and tracings of a stationary Gaussian process, that will strike most readers of this journal as naive and excessively verbose. I know that one brilliant lawyer, an amateur and active supporter of science and mathematics, found this "popularization" too condensed to understand, and felt the need for much amplification. Who is served by such "popular" writing? A bright high school student, lured into the book by its autobiographical narrative, will sense that this (thoroughly tenderized) mathematical meat offers real intellectual nourishment, even if he or she cannot absorb it immediately and completely. The most important message is that here is new mathematics being made by someone who admits to once having been 16 and obsessed with a problem.

Resolved to escape from Poland, Kac formed the plan of going to the United States, establishing himself, and then arranging for his lady love, and all his family, to follow. In June 1937, he received his doctorate from Lwów. A few days later, he learned from the Parnas Foundation that his application for a foreign fellowship had been rejected. This surprised even Steinhaus, who was on the board of the foundation, and shocked Kac. Unemployed, Kac took a temporary job as an actuary for an insurance company in receivership and reapplied to the Parnas Foundation for the following year. The next year, Kac won the fellowship. After bidding what he hoped was a temporary farewell to his family and friends, Kac set sail for the United States in November 1938, aged 24.

Had he been awarded the foreign fellowship the first time he applied to the Parnas Foundation, it is most likely that he would have come to the United States in November 1937 and would have returned to Poland a year later. As it was, his lady love and his family were killed by the Germans.

In his first trimester at the University of Lwów, Kac was knocked down in the reading room of the Mathematical Institute during a riot against Jewish students. The anti-Semitism intensified steadily. In the fall of 1937, the Minister of Education ordered Jewish students throughout Poland to sit on the left side of the classroom. This order had to be accepted by the rectors of each university, and all but one accepted it. The one who resisted was Lwów's rector, Stanislaw Kulczynski, a politically conservative plant scientist, who resigned instead. With his resignation he sent a message: "If one destroys a power plant, it is dark at once; if one destroys a university, it is dark fifty years hence."

The story of Kac's life up to his arrival in the United States is a dark tapestry of war, plague, prejudice, anti-Semitism, and organized murder that seems more appropriate to the Middle Ages than to the twentieth century. Into this tapestry, Kac weaves small bright vignettes of personal friendship, integrity, honesty, respect for learning, and love of truth. Even in that darkness, there were men and women of honor.

Academic life in America. After sharing numerous Polish chocolate bars, which he thought were going to be confiscated, with the Hoboken customs inspector, Kac arrived in Baltimore in December 1938. He was met by Aurel Wintner, Wintner's wife, and E. R. van Kampen. They helped Kac find an apartment ($14 a month). Having command of no English, Kac had misadventures that are hilarious in retrospect.

"As an introduction to America," Kac writes, "my ten months in Baltimore were superb. I find it difficult to find words to convey the feeling of decompression, of freedom, of being caught in a sweep of unimagined and unimaginable grandeur.... And there was the friendliness and warmth from all sides, the ease, the naturalness of social contacts."(85)

Within a short time, unfortunately, the "personal relationship [between Kac and Wintner, whose work was closely related to his] deteriorated to the point that we were practically not on speaking terms.... To this day I do not know the real reasons for the break between us."(76) Knowing the geniality of Mark Kac, I can hardly imagine what happened, and do not find out here. Another

enigma. Nevertheless, with van Kampen as intermediary, Kac and Wintner coauthored four papers—a tribute to the triumph of the pursuit of truth over personality.

When Kac's money ran out in the summer of 1939, van Kampen, unasked, loaned him enough to survive till fall came and a new job began. The job had been offered by the department chairman, F. D. Murnaghan, because he felt that the situation in Europe had become too dangerous for Kac to return. Both the loan and the job are tributes to the triumph of generous friendship over bestiality. Kac's only alternative was to use his return ticket to Warsaw.

"As I look back on my life," Kac writes, "I marvel at the improbable assortment of people who, independently of each other, cooperated to keep me from being incinerated in the ovens of Auschwitz or Belsen:"(86)—Rusiecki, Stark, Steinhaus, Murnaghan, and van Kampen.

On the same day he read the Germans bombed Krzemieniec, Kac received a telegram offering a one-year appointment as an instructor in mathematics at Cornell. He had no idea where it was. "Friends from whom I inquired could only assure me that Cornell had a good football team."(94) Kac accepted, and Ithaca became his home for twenty-two years, longer than he lived anywhere else.

In January 1939, before he went to Cornell, Kac met Norbert Wiener (of whom more below) and Paul Erdös, with whom he shortly initiated a famous collaboration that created the field of probabilistic number theory. They proved, roughly speaking, that positive integers have a normally distributed number of prime divisors. "It took what looks now like a miraculous confluence of circumstances to produce our result.... It would not have been enough, certainly not in 1939, to bring a number theorist and a probabilist together. It had to be Erdös and me: Erdös because he was almost unique in his knowledge and understanding of the number theoretic method of Viggo Brun,... and me because I could see independence and the normal law through the eyes of Steinhaus."(91)

At Cornell, Kac located the mathematics department in White Hall (after first looking for a white hall), under the chairmanship of R. P. Agnew. "Coming from a farming family..., he had many of the traits associated with this hardy breed and in particular he was absolutely unflappable. When one must preside over mathematicians, who tend to flap more readily than the rest of the human race, this is an invaluable asset."(96) There, Kac came to know Agnew, W. B. Carver, and W. A. Hurwitz, and through Hurwitz, W. Strunk, whose famous book *Elements of Style* Kac greatly admired. Kac was reappointed two months after he came to Cornell and was told to "keep publishing."

At his twenty-seventh birthday party on August 16, 1941, he met his future wife Kitty Mayberry, who came as the date of his apartment mate. "For the first time in my life I was truly smitten. This was nothing like the low-keyed affair"(104) he left behind in Poland. They married in Baltimore in 1942, and stayed married for more than 42 years, until his death. Kitty edited the text of his autobiography and brought it to publication.

During the war, Kac was promoted to assistant professor (having amassed by that time twenty-five publications: "How times have changed!"(105)) and

became a consultant to the Radiation Laboratory at MIT. His and Kitty's first child, a son, was born. Their second, a daughter, arrived in 1947.

After the war, Kac learned of the extermination of the Jewish population of Krzemieniec, including his brother, aged 19, and all the rest of his family. "The meaning of the tragedy for me cannot be captured in words. Language is a human instrument and it is simply not capable of coping with the acts of inhumanity which the Germans invented and perfected.... When I first learned of the tragedy it seemed that life would never be the same but being alive proved stronger than my grief. Much stronger. Gradually, layers of mist interposed between the present and the past and the grief receded. But it is there and it always will be."(106)

In July 1946, Kac went on leave from Cornell as a Guggenheim Fellow to study problems arising from his work at the Radiation Lab. While on leave, he attracted several job offers and received two promotions from Cornell. He returned in the fall of 1947 as a full professor at more than twice his previous salary. At the Rad Lab, Kac had come into close contact with G. E. Uhlenbeck, then head of the theory group of the Fundamental Research Division. Kac worked on problems of noise in radar, and learned from Uhlenbeck about the theory of Brownian motion and the work of Smoluchowski, who had created a major center for theoretical physics in Lwów at the beginning of the century. He then studied the results of Wiener and of Cameron and Martin. He worked on invariance principles, formulated in generality by M. D. Donsker. He found the solution to the Ehrenfest urn model. He began a lifelong romance with the mathematics of phase transitions.

In the spring of 1947, Richard Feynman talked to the Cornell physics department about his wartime thesis work. Kac heard the talk, and a few days later had his version of Feynman's path integral or sum over histories. The Feynman–Kac formula was born. Kac remarks, "It is only fair to say that I had Wiener's shoulders to stand on. Feynman, as in everything else he has done, stood on his own, a trick of intellectual contortion that he alone is capable of."(116)

During a sabbatical year, 1951–52, Kac visited Princeton, where he met or got to know better J. R. Oppenheimer, J. von Neumann, J. C. Ward, T. D. Lee, and C. N. Yang.

In 1961, Kac joined his friends and collaborators Uhlenbeck and Ted Berlin as the founders of mathematics and physics at the Rockefeller University. Around these stars formed a brilliant galaxy of mathematicians and physicists, under the generous despotism of Detlev W. Bronk, president of the university. As the optimism of the time suggested, "A unique experiment in the history of American education was on its way to becoming a spectacular success.... Most important, there was Bronk's deep conviction that science is a great adventure of the human mind, that it is indivisible and that it is a part of the humanistic tradition on which our civilization rests."(133–134)

But deficits soon drowned the dreams. Some of the mathematicians and mathematical physicists drifted away, and there was no prospect of a growing program to retain them. As the university's finances grew worse, first the

philosophers and then the psychologists departed from the campus. "By the time I left the Rockefeller in 1981 after twenty happy, productive years all signs indicated that... the institution was moving in the direction of becoming again primarily an institute for bio-medical research."(139) Kac's departure for the University of Southern California, "more in sorrow than in anger,"(149) left mathematics at Rockefeller reduced to a mere remnant.

Looking back over his Rockefeller years, Kac reviews with pleasure the scientific achievements of his "old age" (47–67), notably his work on condensation and his famous lecture and paper, "Can one hear the shape of a drum?" He welcomes frankly but without inflation the recognition that came to him in those years.

In an eloquent postscript, Kac invokes the dual sources of mathematical inspiration: unfettered imagination and the understanding of nature, both of which contributed so much to his work. "The progress of mathematics and its vigor depend on the abstract helping the concrete and on the concrete feeding the abstract.... The two great streams of mathematical creativity are a tribute to the universality of human genius. Each carries its own dreams and its own passions. Together they generate new dreams and new passions."(153)

Kac in the mirror of others' eyes. Kac knew personally many major figures in the mathematics and science of his time. Cameos of some of these appear in Kac's autobiography, and some of those who do appear have written their own autobiographies. It is interesting to compare how Kac views them, and they him. Perhaps the most interesting comparison will have to wait, for while Kac's full portrait of Hugo Steinhaus is reprinted in Kac's autobiography, only a small portion of Steinhaus's memoirs has been published.

Here is Kac's initial sketch of Norbert Wiener. At a sectional meeting of the American Mathematical Society in New York in January 1939, "I gave a ten-minute talk and, luckily for me, Norbert Wiener was in the audience. In the corridor, between sessions, he complimented me on my presentation three different times and each time he also introduced himself to me.... We became good though not close friends and once we even went skiing together in the White Mountains of New Hampshire. That this event actually took place is usually greeted with a measure of disbelief by all who knew either one of us."(89) Kac does not appear in Wiener's two volumes of autobiography, *Ex-Prodigy* and *I Am a Mathematician*.

Kac cites Richard Feynman to illustrate one half of a theory of genius. "In science," Kac writes, "as well as in other fields of human endeavor, there are two kinds of geniuses: the 'ordinary' and the 'magicians.' An ordinary genius is a fellow that you and I would be just as good as, if we were only many times better. There is no mystery as to how his mind works. Once we understand what he has done, we feel certain that we, too, could have done it. It is different with the magicians... the working of their minds is for all intents and purposes incomprehensible. Even after we understand what they have done, the process by which they have done it is completely dark.... Richard Feynman is a magician of the highest caliber."(xxv) Kac recalls his efforts to simplify Onsager's

solution of the Ising model (in statistical mechanics), and how "it took several years and the efforts of several people before the gap in the derivation was filled. Even Feynman got into the act. He attended two lectures I gave at Cal Tech in November 1952 and came up with the clearest and sharpest formulation of what was needed to fill the gap." Kac adds, with pride: "The only time I have ever seen Feynman take notes was during these two lectures. Usually, he is miles ahead of the speaker but following combinatorial arguments is difficult for all mortals."(125)

Feynman, in *Surely You're Joking, Mr. Feynman!*, accords Kac the respect due a senior citizen in recounting his search for people to talk to at Cornell. "There were quite a few people I did enjoy talking to, of course. In the math department there was [sic] Professor Kac and Professor Feller; and a great guy in the zoology department, Dr. Griffin, who found out that bats navigate by making echoes. But it was hard to find enough of these guys to talk to..."(232)

Stan Ulam's autobiography *Adventures of a Mathematician* is reviewed at length in Kac's introduction to his own autobiography. "Ulam did have an excellent sense of humor," Kac writes, "...as when he told Francoise, who later became his wife, that he had infinitely many faults but modesty precluded him from enumerating them all.... Among mathematicians, who as a rule tend to be loners, Ulam was unique in having worked almost entirely in collaboration with others. Except for his earliest brilliant papers on set theory and on the problem of measure, almost everything he published was joint work with others. The way Stan did mathematics was by talking, a work style which goes back to his young days in Lwów, which were spent largely in coffee houses...endlessly discussing problems, ideas and conjectures.... Stan's breadth of interests and achievements was astounding."(xx–xxi)

Ulam returns the compliments. "Mark Kac had also studied in Lwów, but since he was several years younger than I (and I had left when only twenty-six myself), I knew him then only slightly.... I remembered him in Poland as very slim and slight, but here he became rather rotund. I asked him, a couple of years after his arrival, how it had happened. With his characteristic good humor he replied: 'Prosperity!' His ready wit and almost constant joviality make him extremely congenial. After the war he visited Los Alamos, and we developed our scientific collaboration and friendship.... Mark is one of the very few mathematicians who possess a tremendous sense of what the real applications of pure mathematics are and can be; in this respect he is comparable to von Neumann. He was one of Steinhaus's best students.... In a way, Kac, with his superior common sense, as a mathematician is comparable to Weisskopf and Gamow as physicists in their ability to select topics of scientific research which lie at the heart of the matter and are at the same time of conceptual simplicity. In addition—and this is perhaps related—they have the ability to present to a wider scientific audience the most recent and modern results and techniques in an understandable and often very exciting manner. Kac is a wonderful lecturer, clear, intelligent, full of sense and avoidance of trivia."(269–270)

Starting in 1967, Kac served on the Committee of Selection of the John Simon Guggenheim Memorial Foundation, from which he had received a fellowship in

1946. He was chairman of the committee from 1980 until his death. The president of the foundation, Gordon N. Ray, wrote of Kac in his annual report for 1984: "American science in its wider bearings lay clearly mapped in his mind, and he had sage counsel concerning nearly every issue in that realm which the Committee had to face. Moreover, his views were presented with a captivating geniality and an ebullient wit which made our meetings a pleasure to attend."(xxiii–xxiv)

Enigmas. For all his ebullience, grace and wit, his wide circle of friends, and his endless store of anecdotes, Kac was a modest and a private man. This autobiography respects the modesty and privacy of Kac and those around him. The autobiography mentions Sheldon Glashow and Steve Weinberg, who as undergraduates took a graduate course of his, but does not name any of Kac's doctoral students, though among them are mathematicians of such distinction as Harry Kesten, Murray Rosenblatt, and Daniel Stroock. The adventures in the flesh pots that make such incredible reading in Feynman's *Surely You're Joking* form no part of Kac's tale. Neither does any description of Kac's long and happy domestic life, his cats, and bridge games. Though Kac expresses his pride in his discoveries at several points in the book, there are no stories in which Kac is the hero. He does not mention his devoted work on behalf of oppressed scientists through the Committee of Concerned Scientists. He does not mention his long service to the Guggenheim Foundation and his untiring insistence, there and in other advisory capacities, on intellectual excellence. One cannot find here one of the most famous Kac witticisms, which I verified with him the summer before he died.

Kac went to Pasadena to lecture at the California Institute of Technology. Richard Feynman was in the audience. After the lecture, Feynman got up and announced: "If all mathematics disappeared, it would set physics back precisely one week." Without a pause, Kac responded: "Precisely the week in which God created the world."

Details of personal and professional life aside, the chief enigma that remains at the end of this book is the nature of the creative world that appears at the beginning of the book: "Creative people live in two worlds. One is the ordinary world which they share with others and in which they are not in any special way set apart from their fellow men. The other is private and it is in this world that the creative acts take place. It is a world with its own passions, elations and despairs, and it is here that, if one is as great as Einstein, one may even hear the voice of God."(xv) That creative world is *private*; no travelogue, no autobiography can take you there, only your own explorations, and then only to *your* creative world. Perhaps it was Kac's modesty that led him only to hint in his autobiography at what was apparent in his life and conversations: that the joy of discovery, the thrill of the chase, the excitement of mathematics, which took command of his life at age 16 returned to him throughout his life again and again.

This autobiography then cannot tell all of Kac's story. Before Kac's introduction stands an epigraph by Boris Rybak to the effect that every creation and

invention of a man is autobiographical. The converse is also true: every autobiography is invention and creation. Alfred Kazin writes (in a book called *Telling Lives*, edited by Marc Pachter): "For the nonfiction writer, as I can testify, personal history is directly an effort to find salvation, to make one's own experience come out right.... By the time experience is distilled enough through our minds to set some particular thing down on paper, so much unconscious reordering has gone on that even the naive wish to be wholly 'truthful' fades before the intoxication of line, pattern, form."(79, 88–89) This autobiography stands in relation to Kac's life like a mathematical model of nature in relation to nature: it is a created work, a beautiful effort to illuminate and to comprehend the thing itself.

Kac's father, a scholar without a school, wrote his doctoral dissertation in Leipzig on an obscure eighteenth century Polish–German philosopher, Solomon Maimon. Kac quotes from Maimon's autobiography: "In the search for truth, I left my people, my country and my family. It should not therefore be assumed that I shall forsake truth for any lesser motives."(9) This declaration inspired Kac's father, and describes Kac's own life and standards.

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