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CORRECTION

PROBABILITY INEQUALITIES FOR EMPIRICAL PROCESSES AND A LAW OF THE ITERATED LOGARITHM

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All numbered statements and theorems mentioned but not included here come from Alexander (1984), unless otherwise stated.

In Corollary 2.2, the exponent on n is incorrect for $r > 2$; (2.7) should read

$$M \geq \begin{cases} K_1 n^{(r-2)/2(r+2)} \vee K_2 \alpha^{(2-r)/4} & \text{if } r < 2, \\ K_3 L n & \text{if } r = 2, \\ K_4 n^{(r-2)/2r} & \text{if } r > 2. \end{cases}$$

In Corollary 2.5, therefore, the exponent on n should be $(r-2)/2r$, not $(r-2)/2(r+2)$, for classes of functions with $r > 2$. Corollary 2.5 is correct as it stands for classes of sets. We wish to thank P. Massart and M. Talagrand, whose comments led to discovery of this error.

In the proof of Theorem 2.3, it is legitimate to assume $\delta_0 > s := (EM/16n^{1/2})^{1/2}$ when bounding $\mathbb{P}_2$, but not when bounding $\mathbb{P}_3$ as is done implicitly by the use of the word "similarly." Therefore, Theorem 2.3 is valid only under the additional assumption that $t_0 > s$. To handle the case $t_0 \leq s$, we need the following.

THEOREM 2.3a. *Let $M > 0$ and let $\mathscr{C}$, ψ, n, ε, α and t_0 be as in Theorem 2.3. Let $s := (\varepsilon M/16n^{1/2})^{1/2}$ and suppose $t_0 \leq s$. If*

(i) $$\psi(M, n, \alpha) \leq 2\psi_1(M, n, \alpha)$$

and

(ii) $$M \leq \varepsilon n^{1/2}\alpha/16$$

then (2.10) *holds.*

PROOF. Set $\delta_0 := t_0$, and $\eta_0 := \varepsilon M/8$. Taking $N = 0$, we have $\mathbb{P}_1$ [of (3.1)] bounded as in (3.2) and $\mathbb{P}_2 = 0$. Since $t_0 \leq s$, $\mathbb{P}_3$ satisfies (3.6), and

$$M/n^{1/2}\delta_0^2 \geq 16/\varepsilon > 4$$

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(assuming, as we may, that $\varepsilon < 1$). Therefore, by (1.6) and (ii),

$$\psi_1(\eta_0, n, \delta_0^2) \geq \tfrac{1}{2}\eta_0 n^{1/2} = \varepsilon M n^{1/2}/16 \geq M^2/\alpha$$
$$\geq 2\psi_1(M, n, \alpha) \geq \psi(M, n, \alpha),$$

while by the definition of t_0,

$$|\mathscr{F}_0| \leq \exp\left(\tfrac{1}{4}\varepsilon\psi(M, n, \alpha)\right)$$

and the theorem follows from (3.6). □

The assumption (i) is a very mild one, as it can be satisfied by taking $\psi = \psi_1$, and (ii) just says we are in the region of Gaussian-like tails [see (1.5) and (1.6)].

If $\psi = \psi_1$, (ii) holds and equality holds in (2.11), then $t_0 \leq s$ is equivalent to

$$M \geq K_0 \alpha^{2/(r+4)} n^{r/2(r+4)}$$

for some $K_0 = K_0(r, \varepsilon, A)$.

Since H is really only an upper bound for the metric entropy, and increasing H increases t_0, one could avoid having $t_0 \leq s$ by increasing H sufficiently. But this would make it harder to satisfy (2.8).

Combining Theorems 2.3 and 2.3a leads to the following corrected result, as in the proof of Corollary 2.2.

COROLLARY 2.4. *Let $\mathscr{C}$, n, α and ε be as in Theorem* 2.3. *There exist constants $K_i = K_i(r, \varepsilon, A)$ such that if*

$$H_2^B(u, \mathscr{C}, \overline{P}_{(n)}) \leq Au^{-r} \quad \textit{for all } u > 0, \tag{2.11}$$

$$M \leq \varepsilon n^{1/2}\alpha/16$$

and

$$M \geq \begin{cases} K_1 n^{(r-2)/2(r+2)} & (\textit{required for all } r), \\ K_2 \alpha^{(2-r)/r} & \textit{if } r < 2, \\ K_3 L n & \textit{if } r = 2, \end{cases}$$

then (2.10) *holds.*

The incorrect version of Corollary 2.4 was applied to prove Theorem 1.1 of Alexander (1985). The only correction needed there is that (1.10) of Alexander (1985) should read

$$2^{-5} b^{-1} \beta n^{1/2} \geq K b^{-1} n^{-(2-r)/2(r+2)}.$$

Observe that the analog of $t_0 \leq s$ is not a problem in Theorem 2.1 and Corollary 2.2, as it implies $N = \mathbb{P}_2 = \mathbb{P}_3 = 0$ in the proof of the former.

Some additional minor corrections: in (2.8), 64 should be 256; in (2.9), 4 should be 16; on page 1051, line 8, delete "been"; on page 1053, line 4, 2.11 should be 2.12; on line 2 of Remark 2.15, "Theorem 2.12" should be "the usual LIL"; on page 1055, line 5 up, the second f_N^U should be f_N^L; in (3.7), $4H(s)$ should be $16H(s)$.

REFERENCE

ALEXANDER, K. S. (1985). Rates of growth for weighted empirical processes. In *Proceedings of the Berkeley Conference in Honor of Jerzy Neyman and Jack Kiefer* (L. M. Le Cam and R. A. Olshen, eds.) **2** 475–493. Wadsworth, Monterey, Calif.

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