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CORRECTION

BAD RATES OF CONVERGENCE FOR THE CENTRAL LIMIT THEOREM IN HILBERT SPACE

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The claim "(b) is a consequence of (3.2)" on page 849 is incorrect. If N^0 is an infinitely differentiable norm on l^2, the norm on the Hilbertian sum $\oplus_{p=1}^{\infty} l^2_{n(p)}$ given for $x = (x_p)$, $x_p \in l^2_{n(p)}$ by $N(x) = N^0(x^0)$, where $x^0 = (\|x_p\|)_p$ need not be infinitely differentiable.

This was pointed out to us by V. Bentkus (and later by others). However the norm we construct is infinitely differentiable, and this can be checked essentially by our argument. For $x \in \oplus_{p=1}^{\infty} l^2_{n(p)}$ we define $B(x) = A(x^0)$, where A is defined on page 845. Since, as shown in step 2, page 845, the various definitions of A patch well, B is infinitely differentiable and each differential is bounded on

$$V' = \left\{x;\ \tfrac{1}{2} \le \|x\| < 2\right\} = \{x;\ x^0 \in U'\}.$$

Exactly as in the 4th step, bottom of page 846, one sees that $D_x B(x) \ge 1/3$ for $x \in V'$.

We now mimick the 5th step, top of page 847. For $x \in V'$, $t \in \mathbb{R}$, let $h(x, t) = B(x/t)$. Since $N(x) = N^0(x^0)$, we have

$$A\left(\frac{x^0}{N^0(x^0)}\right) = 1 = B\left(\frac{x}{N(x)}\right),$$

and thus $h(x, N(x)) = 1$.

The implicit function theorem shows that $D_x N(y) = D_x B(y)/D_{\bar{x}} B(\bar{x})$, where $\bar{x} = x/N(x)$. This shows by induction that N is infinitely differentiable on V' and that the nth differential is bounded on V', hence on the unit sphere.

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