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W. DOEBLIN 1915–1940

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We commemorate W. Doeblin with a brief recollection of his work, influence and life.

1. Introduction. With the dramatic progress of probability theory during the 1930s even a person with a minimal interest in the history of the discipline associates the names Khintchine, Kolmogorov, Lévy and Doeblin. However, it seems too few people today have a comprehensive view of Doeblin's contributions. One review of his work and life exists: Lévy (1955). That paper was meant for a wide audience, hence it is written in rather general terms. It gives no account of the influence of Doeblin's work, which certainly has been strong and continues to live. We feel that a recollection might be due, 35 years after Lévy (1955).

Readers without access to Doeblin's papers are referred to Doob (1953), Loève (1963), Chung (1960) and Feller (1966). Our presentation of his work and influence, in Section 2, consists of a guide to his innovations, some further references and a number of citations.

Due to the fact that Doeblin was a son of a famous writer, Alfred Döblin, we have unusual possibilities for obtaining a picture of his life, a chronicle of which is given in Section 3.

The only complete Doeblin bibliography I am aware of is in Lévy (1955). It is reprinted here, with a few corrections, in Section 4.

2. Work and influence. Not much was understood about Markov chains and processes in the mid 1930's when Doeblin was introduced to the field by Fréchet. With all his striking innovations, often emphasizing paths, it is safe to say that Kolmogorov and he are the creators of modern Markov theory. This is the main theme of Doeblin's work. We first consider his contributions to that. Most terms and notation, supposed to be standard or self-explanatory, are left undefined.

Doeblin studied four major topics, which may be presented in terms of a Markov chain $(X_n)_0^\infty$ with a state space E, say. They are:

1. Strong ergodicity, that is, the convergence of $P^n(x, A)$ as $n \to \infty$ to a limit independent of x, for all $x \in E$ and measurable sets $A \subset E$.

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2. Weak ergodicity: the same as the preceding topic, but w.r.t. the convergence of $n^{-1}\Sigma_1^n P^i(x, A)$.
3. Asymptotic normality of $n^{-1/2}(\Sigma_1^n(f(X_i) - E[f(X_i)]))$.
4. A law of the iterated logarithm for $\Sigma_1^n(f(X_i) - E[f(X_i)])$.

In topics 3 and 4, f is a bounded function.

In the remaining text, references cited in curly brackets (e.g., {1}, {CR2}) refer to Doeblin's works as cited in Section 4.

The papers {1}, {2} and {4} present steps in the early work by Doeblin that are summarized in {5}, the first chapter of which treats topics 1–4 exhaustively for E finite. But that paper is best known for the ideas introduced in order to handle these topics in a general state space E. That case is treated in the second chapter of {5}, with generalizations in {12}. The subtlety needed for classification of states in a general E is one thing, but the major novelty is to impose a condition on the transition probabilities of the following kind: There exists a $k_0 \geq 1$ such that the probability measures $P^{k_0}(x, \cdot)$ uniformly dominate a subprobability measure on E, for x in a set large enough (this was an attempt to catch the spirit; Doeblin's basic definition is more general, but less transparent), and then to exploit that in a masterful way. It is today fully understood that conditions of this type are actually recurrence conditions.

For an account that follows Doeblin rather closely, see Doob (1953), and for an account that includes further progress, see Orey (1971). On the basic chapter of his study, Orey remarks: "It is essentially Doblin's theory as completed during the quarter of a century following the publications of his papers that is presented here."

The work has continued during the 1970s and 1980s; see, for example, Revuz (1984).

In {5}, ergodicity is studied with analytical methods only. It is in {8} that Doeblin presents what we now call the coupling method, which is probabilistic, to prove strong ergodicity when the state space is finite (that paper was submitted for publication in the summer of 1936). Doeblin is very explicit in his presentation of the new device, and a first version of the basic coupling inequality is given. Why it took 30 years or so for the method to come into use again is puzzling.

In {7}, with E countably infinite, the first ratio limit theorem for Markov chains is proved, a deep result in the null-recurrent case. That paper should also be remembered for a central limit theorem for sums $\Sigma_1^n f(X_i)$, where Doeblin for the first time makes use of the fact that the segments of a recurrent Markov chain between successive visits to a reference state constitute i.i.d. random elements.

We now turn our attention to the continuous-time Markov processes. In {6}, with a finite state space, Doeblin carries out a thorough analytical investigation of the Chapman–Kolmogorov equation with only measurability assumptions on the transition probabilities $p_{ik}(t)$. These are proved to be continuous at all $t > 0$, and to converge as $t \to \infty$ with exponential speed. Doeblin also makes a brave attempt to describe the path behavior when $p_{ik}(0+) > 0$ for some states i and k, $i \neq k$.

The paper {10} is dedicated to non-time-homogeneous Markov processes on a general state space with transition probabilities satisfying

$$\lim_{t \downarrow s} P_{s,t}(x, \{x\}) = 1$$

uniformly in $x \in E$ and $s \in$ (any finite interval). Doeblin proves that a process satisfying this condition has only a finite number of jumps in finite time, and several other results, including that in the time-homogeneous case $P_t(x, A)/t$ is convergent as $t \downarrow 0$ for $x \notin A$.

Doeblin's contribution to the theory of diffusions and related processes are four notes ({CR9}, {CR10}, {CR12}, and {CR13}) that contain announcements of results and brief discussions, but no proofs. Remarkable for a person without previous knowledge of the state of the art in the late 1930's is a law of the iterated logarithm, and the necessary and sufficient condition for a time-homogeneous diffusion never to hit a finite boundary point.

A vital part in probability theory in the 1930s may be described through the key words: central limit theorems, infinitely divisible distributions and domains of attraction. Fundamental work by Lévy was followed by significant contributions by Khintchine, Gnedenko and Feller, among others.

Doeblin's work in this area is his second theme. In {9}, he studies arrays of independent random variables satisfying a condition of uniform asymptotic negligibility, and establishes the first necessary and sufficient conditions for the existence of normalizing constants making the sequence of row sums in such an array convergent in distribution. Concerning {9}, Feller (1966) writes: "A new and simpler approach to the whole theory was made possible by the discovery of infinitely divisible distributions. (...) The interest in the theory was stimulated by Doblin's masterful analysis of the domains of attraction (1939). His criteria were the first to involve regularly varying functions. The modern theory still carries the imprint of this pioneer work(...)."

It is in {13} that Doeblin presents his amazing discovery of "universal laws," that is, distributions that belong to the domain of partial attraction of every infinitely divisible distribution. For a modern presentation, see Feller (1966).

Chains with complete connections, that is, random sequences where at each time point the entire history is allowed to influence the state taken in the next step, are studied in {3} by Doeblin and Fortet. The authors consider what may be called time-homogeneous chains with a finite state space. A metric measuring the distance between histories is introduced, and for chains satisfying a uniformity condition relative to that metric, ergodic theorems are proved. See also Norman (1972).

Ergodic properties of continued fractions excited several eminent mathematicians in the 1930s, among them Fréchet, Lévy and Khintchine. In {11}, Doeblin proves several results on the entries of such fractions, including the law of the iterated logarithm and a Poisson approximation of a number of "high level exceedances" in addition to the basic ergodic results. This work leans on {3} and {5}. For an account of continued fractions, see Billingsley (1965), which covers Doeblin's ergodic theorem.

3. Life. Wolfgang Doeblin came from a Jewish family. His father, Alfred Döblin (1878–1957), is considered one of the important German writers of our century. His best known work is the novel *Berlin Alexanderplatz*, published in 1929. He was a medical doctor, with speciality in psychiatry (doctoral degree in 1905). Many of his letters and notes are published; the chronicle that follows leans heavily on Döblin (1970, 1980) and Zeller (1978). For convenience we abbreviate Alfred Döblin's name AD.

1915 *March* 17. Wolfgang is born in Berlin as the second child of AD and his wife Erna (née Reiss). There were another three children in that marriage: Peter (b. 1912), Klaus (b. 1917) and Stefan (b. 1926).

1915 *June* 17. Erna Döblin and her two sons join AD in Saargemünd, where he volunteered in an army hospital during the First World War.

1918–1933. The Döblin family returns to Berlin in November 1918 and stays there until 1933. Throughout the period, AD is practicing medicine and writing. By the end of the 1920s, he has become a central figure in the intellectual world, not only through his books but also because of his participation in debates and other activities, where he often took radical standpoints. On January 10, 1928, he was elected a member of the Preussischen Akademie der Künste (Prussian Academy of Arts and Letters); rightwing members voted against him.

1933 *February* 28. This is the day after the Reichstag fire in Berlin, which signified the beginning of Nazi despotism. Warned by friends, AD leaves the city for Zürich, Switzerland, followed a few days later by his family. Wolfgang, however, stays in Berlin in order to complete his Gymnasium studies.

1933 *April*. Wolfgang passed his Abitur exam in Berlin, arrives in Zürich and commences his mathematical studies.

1933 *September*. The Döblin family settles in Paris and stays there until 1940. Wolfgang continues his studies at the Sorbonne. From Lévy we learn that "it became very quickly apparent that he was one of those who do not need any teacher." The library of l'Institut Henri Poincaré becomes his working place.

The Döblins got their French citizenship in October 1935. Wolfgang then adopted the name Vincent, but, except for a few cases, he prefers to use "W. Doeblin" or "W. Doblin" on his mathematical papers.

1936 *June* 16. Doeblin's first note to l'Académie des Sciences is presented. ({CR1}, a joint work with Lévy.)

1938 *March* 26. Doeblin defends his thesis (virtually the paper {5}) at the Sorbonne.

1938 *November* 3. Doeblin is enrolled in the French army.

1939 *September* 2. The Second World War breaks out and Doeblin is called up by the order on general mobilization.

1940 *February* 14. AD reports in a letter to his son Peter, now living in USA: "Your brothers write. Wolf lives most of the time in a barn, but he is always in good mood, and occassionally even sends mathematical drafts."

1940 *April* 29. Doeblin's last note ({CR13}) is presented to l'Academie des Sciences by Émile Borel, who had fulfilled that role in connection with most of Doeblin's notes.

1940 *June* 21. Four days before the suspension of arms, Doeblin loses contact with his regiment on a mission to the small village Housseras in the Vosges. Because he had grown up in Berlin, was a Jew, the son of Alfred Döblin, and spoke French with a thick accent, Doeblin decided to die by his own hand rather than give himself up to the Nazi troops that were just a few minutes away.

Doeblin was decorated with two medals: *la croix de guerre avec palme* and *la médaille militaire*. He is buried in Housseras.

4. The Doeblin bibliography.

Articles.

{1} Le cas discontinu des probabilités en chaîne. *Publ. Fac. Sci. Univ. Masaryc* **236** 3–13, 592 (a correction) (1937).

{2} Sur le cas continu des probabilité en chaîne. *Rend. Accad. Lincei* **25** 170–176 (1937).

{3} Sur des chaînes à liaisons complétes (R. Fortet, coauthor). *Bull. Soc. Math. France* **65** 132–148 (1937).

{4} L'équation de Smoluchowsky. *Prakt. Akad. Athēnōn* **12** 116–119 (1937).

{5} Sur les propriétés asymptotiques de mouvements régis par certains types de chaînes simples. *Bull. Soc. Math. Roumaines des Sciences* **39**(1) 57–115; **39**(2) 3–61 (1937).

{6} Sur l'équation matricielle $A(t+s) = [A(t)A(s)]$ et ses applications aux probabilités en chaînes. *Bull. Sci. Math.* **62** 21–32 (1938) and **64** 35–37 (a correction) (1940).

{7} Sur deux problèmes de M. Kolmogoroff concernant les chaînes dénombrables. *Bull. Soc. Math. France* **66**, 210–220 (1938).

{8} Exposé de la Théorie des Chaînes simples constantes de Markoff à un nombre fini d'États. *Rev. Math. de l'Union Interbalkanique* **2** 77–105 (1938).

{9} Sur les sommes d'un grand nombre de variables indépendantes. *Bull. Sci. Math.* **63** 23–32, 35–64 (1939).

{10} Sur certains mouvements aléatoires discontinus. *Skand. Aktuarietidskr.* **22** 211–222 (1939).

{11} Remarques sur la théorie métrique des fractions continues. *Compositio Math.* **7** 353–371 (1940).

{12} Élément d'une théorie générale des chaînes simples constantes de Markoff. *Ann. École Norm. Sup.* **57** 61–111 (1940).

{13} Sur l'ensemble des puissances d'une loi de probabilité. *Studia Math.* **9** 71–96 (1940). Reprinted with a complement in *Ann. École Norm. Sup.* **63** 317–350 (1947).

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{CR1} Sur les sommes de variables aléatoires indépendantes à dispersions bornées inférieurement (P. Lévy, coauthor). **202** 2027–2029 (1936).

{CR2} Sur les chaînes discrètes de Markoff. **203** 24–26 (1936).

{CR3} Sur les chaînes de Markoff. **203** 1210–1211 (1936).

{CR4} Sur deux notes de MM. Kryloff et Bogoliouboff (R. Fortet, coauthor). **204** 1699–1701 (1937).

{CR5} Éléments d'une théorie générale des chaînes constantes simple de Markoff. **205** 7–9 (1937).

{CR6} Premiers éléments d'une étude systématique de l'ensemble de puissances d'une loi de probabilité. **206** 306–308 (1938).

{CR7} Étude de l'ensemble de puissances d'une loi de probabilité. **206** 718–720 (1938).

{CR8} Sur les sommes d'un grand nombre de vecteurs aléatoires. **207** 511–513 (1938).

{CR9} Sur l'équation de Kolmogoroff. **207** 705–707 (1938).

{CR10} Sur certains mouvements aléatoires. **208** 249–250 (1939).

{CR11} Sur un problème de calcul des probabilités. **209** 742–743 (1939).

{CR12} Sur l'équation de Kolmogoroff. **210** 365–367 (1940).

{CR13} Sur les mouvements mixtes. **210** 690–692 (1940).

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