

BOOK REVIEW

D. REVUZ AND M. YOR, *Continuous Martingales and Brownian Motion*. Springer-Verlag, Berlin, 1991, 533 + ix pages, \$89.00.

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The book's title is a little misleading. As the following chapter titles indicate, a better name would be *The Desktop Guide to Stochastic Processes*:

0. Preliminaries
1. Introduction
2. Martingales
3. Markov Processes
4. Stochastic Integration
5. Representations of Martingales
6. Local Times
7. Generators and Time Reversal
8. Girsanov's Theorem
9. Stochastic Differential Equations
10. Additive Functionals of Brownian Motion
11. Bessel Processes and Ray–Knight Theorems
12. Excursions
13. Limit Theorems in Distribution

The reason for the title is that, as the authors explain in the preface, “after a first chapter where Brownian motion is introduced, each of the following ones is devoted to a new technique or notion and to some of its applications to Brownian motion.” Continuous martingales appear in the title since stochastic calculus in that level of generality figures prominently in the developments and much of the theory (e.g., local times) is developed not just for Brownian motion but also for continuous local martingales.

In the fifteen years or so since Halmos was book review editor at the *Bulletin of the American Math Society*, it has been traditional that a book review is an excuse for a mini-essay on the subject. That style, however, is not appropriate for a review of *Encyclopedia Britannica* or of the French encyclopedia under review, so I will adopt a more traditional style, which in this case means cheerleading for what I think is a really terrific book.

Revuz and Yor's book contains a wealth of interesting material. Even though it is “only” 520 pages long, it has about 2000 pages of material in it. This book is some sort of literary fractal. Chapters, like number 12 on excursion theory, are almost books by themselves, and sections contain chap-

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ters from other people's books. A randomly chosen example of the latter is the section on Infinitesimal Generators in Chapter 7. Inside you will find all the standard material covered in Chapter 1 of Dynkin's Markov processes (except for the Hille–Yosida theorem), a discussion of the extended infinitesimal generator, computations of the generators for processes with independent increments and Brownian motion (including the exact domain in one dimension), and the form of the generator for a Feller semigroup on R^d . All of this is crammed into 10 pages and followed up by 13 challenging exercises.

The first nine chapters provide enough material for a very substantial course on stochastic processes. The only drawback with using this book as a text is that it would be very difficult to adequately cover this material in a year and most of us only have the luxury of devoting one semester to such courses. The main use for the book then seems to be as a reference or “bedtime reading” for professional probabilists. In this capacity, I think it belongs in the library of anyone who deals with Markov processes or stochastic integration. Even the most serious student of these topics can open the book to almost any section and learn several new things. I won't embarrass myself by telling you what I didn't know but it is only a slight exaggeration to say that there were things on every page. Of course, I don't claim to have read every page, but those that I did read in my non-random sample delighted me with their crisp and clear proofs and comprehensive treatments of topics that I thought I knew fairly well.

The bulk of my review has concentrated on the first nine chapters since I feel that those will have the broadest appeal. The material on excursions in Chapter 12 is valuable for understanding Brownian paths, and it is nice to have a clear treatment of the limiting behavior of additive functionals of Brownian motion, and of the Ray–Knight theorems. As those of us who have followed Marc Yor's career should expect, there is throughout the book a fascination with concrete computations that makes the book lively and interesting to read. Likewise, those who have read Daniel Revuz's book on Markov chains will again find the writing clear and the treatment definitive. Like most yellow books in the 500 + page range, the purchase of this book will have a noticeable impact on your bank account, but when you divide the cost by the amount of material it contains, you have quite a good bargain.

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