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CENTRAL LIMIT THEOREM FOR A RANDOM WALK WITH RANDOM OBSTACLES IN $\mathbf{R}^d$

BY HIDEKI TANEMURA[1]

Chiba University

A random walk with obstacles in $\mathbf{R}^d$, $d \geq 2$, is considered. A probability measure is put on a space of obstacles, giving a random walk with random obstacles. A central limit theorem is then proven for this process when the obstacles are distributed by a Gibbs state with sufficiently low activity. The same problem is treated for a tagged particle of an infinite hard core particle system.

0. Introduction. Let $\mathfrak{M}$ be the set of all countable subsets η of $\mathbf{R}^d$, $d \geq 2$, satisfying $\#(\eta \cap K) < \infty$ for any compact set K, and let us consider a class of processes $(x(t), P_\eta)$, $\eta \in \mathfrak{M}$, each of which is an $\mathbf{R}^d$-valued continuous time Markov process of jump type starting from 0 with generator

$$L_\eta \varphi(x) = \int_{\mathbf{R}^d} dy\{\varphi(y) - \varphi(x)\}p(|x-y|)\exp\left\{-\sum_{\substack{u \in \eta \\ v = x, y}} \Psi(|u-v|)\right\},$$

where $p(\cdot)$ is a nonnegative function on $[0, \infty)$ such that $\int_{\mathbf{R}^d} dx\, p(|x|) = 1$ and Ψ is a measurable function on $[0, \infty)$ which is bounded from below and satisfies the following properties:

$$(\Psi.1) \qquad \Psi(\alpha) = \infty \quad \text{if and only if } \alpha \in [0, r),$$

$$(\Psi.2) \qquad \Psi(\alpha) = 0 \quad \text{if } \alpha \in [r_0, \infty),$$

for some positive constants r and r_0 with $r \leq r_0$.

For a probability measure ν on the space $\mathfrak{M}$ we write $P_\nu = \int_{\mathfrak{M}} \nu(d\eta) P_\eta$. We call the process $(x(t), P_\nu)$ a random walk with random obstacles. Let $\mathfrak{M}^*$ be the set of all $\eta \in \mathfrak{M}$ such that $P_\eta(x(1) \in \cdot)$ has an unbounded support and let μ be a Gibbs state associated with a pair potential Φ which is translation invariant. One of the main results of this paper is the central limit theorem for $(x(t), P_{\mu^*})$, where μ^* is a conditional probability measure of the Gibbs state μ given the event $\mathfrak{M}^*$, that is, $\mu^*(\cdot) = \mu(\cdot | \mathfrak{M}^*)$.

In a previous paper [10] we considered a system of infinitely many hard balls with the same diameter $\mathbf{r}$ moving discontinuously in $\mathbf{R}^d$. We constructed the Markov process ξ_t which describes the system. This process has a Gibbs state as a stationary measure. We showed the ergodicity of the stationary Markov

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process in the case where the density of balls is sufficiently small. Another main result of the present paper is the central limit theorem for a tagged particle.

We describe $x(t)$ as an additive functional of the environment process η_t. The process η_t is a Markov process which has the probability measure μ^* as a reversible measure. We prove that (η_t, P_{μ^*}) is ergodic by using Proposition 2.1, in which we show the uniqueness of the unbounded cluster of a percolation model defined on $\mathbf{R}^d$. This percolation model should be viewed as a continuum analogue of the discrete site percolation. Instead of sites being vacant we have points of $\mathbf{R}^d$ with each point being the center of a ball of radius r. We consider clusters associated with our process, whose definition is described in detail in Section 1. The idea of the proof of this proposition is similar to the idea in Burton and Keane [1], but we have to make suitable modifications to take account of the structure of the continuum model.

Once we obtain the ergodicity, by applying an invariance principle for additive functionals of ergodic reversible Markov processes in De Masi, Ferrari, Goldstein and Wick [3], we obtain the central limit theorem for $(x(t), P_{\mu^*})$ except for the nondegeneracy of the diffusion matrix D^*. To prove the nondegeneracy of the diffusion matrix, it is important to study random electrical networks. A simple random electrical network is one in which the bonds of the hypercubic lattice $\mathbf{Z}^d$ are taken to be occupied independently by unit conductors with probability θ and vacant with probability $1 - \theta$. In the case of $d = 2$, Grimmett and Kesten [7] proved that the effective conductivity in the electrical network is bounded away from zero, if $\theta > \frac{1}{2}$. The case of general dimensions was studied by Chayes and Chayes [2]. We introduce an effective conductivity $\mathscr{I}^l(\eta)$ associated with our process. Then the nondegeneracy of D^* follows from the two inequalities

$$\mu(\mathfrak{M}^*)|eD^*|^2 \geq \limsup_{l\to\infty} \int \mu(d\eta)(2l)^{2-d}\mathscr{I}^l(\eta),$$

for $e \in \mathbf{R}^d$ with $|e| = 1$, and

$$\liminf_{l\to\infty} (2l)^{2-d}\mathscr{I}^l(\eta) > 0, \qquad \mu\text{-a.e.}$$

The first inequality is established by generalizing a technique in [3] in the case of a random walk in the infinite cluster of a percolation model on $\mathbf{Z}^2$. The second, which is stated in Proposition 3.1, is obtained by using a comparison argument to reduce the problem to the results of Grimmett and Kesten [7] and Chayes and Chayes [2].

In [10] we discussed the central limit theorem for the tagged particle of the infinite hard core particle system in $\mathbf{R}^d$ and showed that the tagged particle is described as an additive functional of an ergodic reversible Markov process. But we could not prove the nondegeneracy of the diffusion matrix. In this paper we show the nondegeneracy of the matrix using the lower bounds of the effective conductivity in the random electrical network associated with the random walk with random obstacles.

In Section 1 we state the main theorems precisely. The proof of the theorems is given in Section 4 using Proposition 2.1 and Lemma 4.3, which are shown in Sections 2 and 5, respectively. In Section 3 we give the proof of Proposition 3.1, a key part of the proof of Lemma 4.3.

1. Statement of results. Let $\mathfrak{M}$ be the set of all countable subsets η of $\mathbf{R}^d$ satisfying $N_K(\eta) < \infty$ for any compact subset K, where $N_A(\eta)$ is the number of points of η in $A \subset \mathbf{R}^d$, $d \geq 2$. We regard $\eta \in \mathfrak{M}$ as a nonnegative integer-valued Radon measure on $\mathbf{R}^d$: $\eta(\cdot) = \sum_{x \in \eta} \delta_x(\cdot)$, and accordingly equip $\mathfrak{M}$ with the vague topology, where δ_x denotes the δ-measure at x. We define σ-fields $\mathscr{B}(\mathfrak{M})$ and $\mathscr{B}_K(\mathfrak{M})$ by

$$\mathscr{B}(\mathfrak{M}) = \sigma\big(N_A;\ A \in \mathscr{B}(\mathbf{R}^d)\big)$$

and

$$\mathscr{B}_K(\mathfrak{M}) = \sigma\big(N_A;\ A \in \mathscr{B}(\mathbf{R}^d),\ A \subset K\big).$$

The σ-field $\mathscr{B}(\mathfrak{M})$ coincides with the topological Borel field of $\mathfrak{M}$.

For any $\eta \in \mathfrak{M}$ we define a measurable kernel $q_\eta(x, dy)$ on $\mathbf{R}^d \times \mathscr{B}(\mathbf{R}^d)$ by

$$q_\eta(x, dy) = p(|x - y|)\chi(x|\eta)\chi(y|\eta)\, dy,$$

where $p(\cdot)$ is a nonnegative Borel function on $[0, \infty)$ satisfying

$$(p.1) \qquad \int_{\mathbf{R}^d} dx\, p(|x|) = 1,$$

$$(p.2) \qquad \int_{\mathbf{R}^d} dx |x|^2 p(|x|) < \infty,$$

$$(p.3) \qquad \{\alpha \in [0, \infty) \colon p(\alpha) > 0\} = [0, h) \quad \text{for some } h \in (0, \infty],$$

$$(p.4) \qquad \operatorname{ess\,inf}\{p(\alpha) \colon \alpha \in [0, c)\} > 0 \quad \text{for any } c \in (0, h),$$

and, for any $\eta \in \mathfrak{M}$ and $x \in \mathbf{R}^d$,

$$(1.1) \qquad \chi(x|\eta) = \exp\Big\{ - \sum_{y \in \eta} \Psi(|x - y|)\Big\}.$$

Here Ψ is a given measurable function on $[0, \infty)$ which is bounded from below and satisfies (Ψ.1) and (Ψ.2) in the introduction. Let $\mathbf{C}_\infty(\mathbf{R}^d)$ be the space of continuous functions φ on $\mathbf{R}^d$ such that $\varphi(x) \to 0$ as $|x| \to \infty$. We denote by $(\Omega, \mathscr{F}, P_\eta, x(t))$ the right-continuous Markov process starting from 0 with generator

$$L_\eta \varphi(x) = \int_{\mathbf{R}^d} q_\eta(x, dy)\{\varphi(y) - \varphi(x)\}, \qquad \varphi \in \mathbf{C}_\infty(\mathbf{R}^d).$$

Denote the open r-neighborhood of $A \subset \mathbf{R}^d$ by $U_r(A)$ and abbreviate $U_r(\{x\})$ to $U_r(x)$. If $U_r(\eta)$ does not contain 0, $x(t)$ is a jump-type Markov process whose state space is the complement of $U_r(\eta)$; otherwise $x(t) = 0$, $t \geq 0$. In the case where $\eta = \varnothing$, $x(t)$ is a spatially homogeneous Markov process of jump type, that is, a random walk in $\mathbf{R}^d$. For any probability measure ν on $\mathfrak{M}$ we write $P_\nu = \int_{\mathfrak{M}} \nu(d\eta) P_\eta$. We call the process $(\Omega, \mathscr{F}, P_\nu, x(t))$ a random walk with random obstacles.

For $x \in \mathbf{R}^d$ and $\eta \in \mathfrak{M}$ we denote by $A_{x,\eta}$ the connected component of $U_{h/2}(U_r(\eta)^c)$ containing x and put

$$C(x,\eta) = \begin{cases} A_{x,\eta} \setminus \overline{U_r(\eta)}, & x \notin \overline{U_r(\eta)}, \\ \varnothing, & x \in \overline{U_r(\eta)}. \end{cases} \tag{1.2}$$

We call the set $C(x,\eta)$ the cluster containing x for η. Define a measurable subset $\mathfrak{M}^*$ of $\mathfrak{M}$ by

$$\mathfrak{M}^* = \{\eta \in \mathfrak{M} : C(0,\eta) \text{ is unbounded}\}.$$

For a probability measure on $\mathfrak{M}$ satisfying $\mu(\mathfrak{M}^*) > 0$, we define

$$\mu^*(d\eta) = \frac{\mathbb{1}_{\mathfrak{M}^*}(\eta)}{\mu(\mathfrak{M}^*)} \mu(d\eta),$$

where $\mathbb{1}_A$ stands for the indicator function for a set A.

In this paper we study the asymptotic behavior of $(x(t), P_{\mu^*})$ in the case where μ is a Gibbs state. We introduce terminologies for Gibbs states. Let Φ be a real valued measurable function on $[0,\infty)$ which is bounded from below and satisfies the following condition $(\Phi.1)$, called the *regularity condition*:

$$\int_{\mathbf{R}^d} dx \left| \exp(-\Phi(|x|)) - 1 \right| < \infty. \tag{Φ.1}$$

Next we assume either one of the following conditions $(\Phi.2)$ and $(\Phi.2')$:

$$\Phi(\cdot) \geq 0, \tag{Φ.2}$$

$(\Phi.2')$ (i) There exists a positive number r' such that $\Phi(\alpha) = \infty$ if and only if $\alpha \in [0, r')$,

(ii) There exists a nonnegative number c_0 such that

$$\sum_{i=1}^{m} \Phi(|x_i|) \geq -c_0,$$

for all m and $x_1, x_2, \ldots, x_m \in \mathbf{R}^d$ with $|x_i - x_j| \geq r'$ for $i \neq j$.

Φ is regarded as a pair potential which is rotation invariant and translation invariant. For $x_1, x_2, \ldots, x_n \in \mathbf{R}^d$ and $\eta \in \mathfrak{M}$, we associate a potential energy

$$U(x_1, x_2, \ldots, x_n | \eta) = \sum_{1 \leq i < j \leq n} \Phi(|x_i - x_j|) + \sum_{i=1}^{n} \sum_{y \in \eta} \Phi(|x_i - y|).$$

For any compact subset $K \subset \mathbf{R}^d$, we denote by $\mathfrak{M}(K)$ and $\mathfrak{M}(K,n)$ the set of all finite subsets of K and the set of all subsets of K having n points, respectively. An alternative description of $\mathfrak{M}(K,n)$ is given by

$$\mathfrak{M}(K,n) = \begin{cases} \{\varnothing\}, & \text{if } n = 0, \\ (K^n)'/\mathbf{S}_n, & \text{if } n \geq 1, \end{cases} \tag{1.3}$$

where $(K^n)' = \{(x_1, x_2, \dots, x_n) \in K^n : x_i \neq x_j \text{ if } i \neq j\}$ and $\mathbf{S}_n$ is the symmetric group of degree n. By means of the factorization (1.3) we introduce a measure $\lambda_{K,z}$ on $\mathfrak{M}(K) = \bigcup_{n=0}^{\infty} \mathfrak{M}(K,n)$ (direct sum) such that

$$\lambda_{K,z}(\varnothing) = 1$$

and

$$\lambda_{K,z}(A) = \frac{z^n}{n!} \int_{\tilde{A}} dx_1\, dx_2 \cdots dx_n$$

for a Borel set A of $\mathfrak{M}(K,n)$, $n \geq 1$, where $z \geq 0$ and $\tilde{A}$ is a preimage of A by the factor mapping in the factorization (1.3).

Now we define a Gibbs state.

DEFINITION 1.1. A probability measure μ on $\mathfrak{M}$ is called a Gibbs state with respect to the activity $z \geq 0$ and the potential Φ, if μ satisfies the Dobrushin–Landford–Ruell (DLR) equation: for any compact subset K of $\mathbf{R}^d$,

$$\mu(\cdot \mid \mathscr{B}_{K^c}(\mathfrak{M}))(\eta) = \mu_{K,\eta,z}(\cdot), \qquad \mu\text{-a.s. } \eta,$$

where $\mu_{K,\eta,z}$ is the probability measure on $\mathfrak{M}(K)$ defined by

$$\mu_{K,\eta,z}(d\mathbf{x}) = \frac{1}{Z_{K,\eta,z}} \exp\{-U(\mathbf{x}|\eta \cap K^c)\} \lambda_{K,z}(d\mathbf{x}),$$

$$Z_{K,\eta,z} = \int_{\mathfrak{M}(K)} \lambda_{K,z}(d\mathbf{x}) \exp\{-U(\mathbf{x}|\eta \cap K^c)\}.$$

Denote by $\mathscr{G}(z,\Phi)$ the set of all Gibbs states with respect to the activity $z \geq 0$ and the potential Φ, and by $\mathscr{G}_\Theta(z,\Phi)$ the set of all elements of $\mathscr{G}(z,\Phi)$ which are translation invariant.

REMARK 1.1. (i) The set $\mathscr{G}_\Theta(z,\Phi)$ is convex and any element of $\mathscr{G}_\Theta(z,\Phi)$ is represented by the extremal points of $\mathscr{G}_\Theta(z,\Phi)$, which are characterized by their ergodicity under translation (see [5]). We denote the set of all extremal points of $\mathscr{G}_\Theta(z,\Phi)$ by ex $\mathscr{G}_\Theta(z,\Phi)$. If $\#\mathscr{G}(z,\Phi) = 1$ and $\mu \in \mathscr{G}(z,\Phi)$, then μ is rotation invariant, translation invariant and ergodic under translation.

(ii) There exists a positive constant $z_1 > 0$ such that if $z \in (0, z_1)$ and $\mu \in \mathscr{G}(z,\Phi)$, then $\mu(\mathfrak{M}^*) > 0$. In case $h = \infty$ we can take $z_1 = \infty$.

Now, we state our first main result.

THEOREM 1.1. *There exists $z_2 \in (0, z_1]$ such that if $z \in (0, z_2)$ and $\mu \in$ ex $\mathscr{G}(z,\Phi)$, then the process $\varepsilon x(t/\varepsilon^2)$ on $(\Omega, \mathscr{F}, P_{\mu^*})$ converges to $D^*B(t)$ as $\varepsilon \to 0$ in distribution with respect to the J_1-topology on Skorohod's function* *space* $\mathbf{D}[0,\infty)$, *where* $B(t)$ *is a d-dimensional Brownian motion and* D^* *is a positive definite* $d \times d$ *matrix. In the case* $h = \infty$ *we can take* $z_1 = \infty$.

In the previous paper [10] we studied a system of infinitely many hard balls with the same diameter r moving discontinuously in $\mathbf{R}^d$. We denote the configuration space of hard balls by $\mathscr{X}$:

$$\mathscr{X} = \left\{\xi = \{x_i\} \subset \mathbf{R}^d \colon |x_i - x_j| \geq r,\ i \neq j\right\},$$

the position of a ball being represented by its center. The space $\mathscr{X}$ is a compact subset of $\mathfrak{M}$ with the vague topology.

Let $\mathbf{C}(\mathscr{X})$ be the space of all real-valued continuous functions on $\mathscr{X}$, and let $\mathbf{C}_0(\mathscr{X})$ be the set of functions of $\mathbf{C}(\mathscr{X})$ which depend only on the configurations in some compact set K. The system is described by the $\mathscr{X}$-valued Markov process ξ_t whose generator is the smallest closed extension of the operator $\overline{\mathbf{K}}$ on $\mathbf{C}_0(\mathscr{X})$ given by

$$\mathbf{K} f(\xi) = \sum_{x \in \xi} \int_{\mathbf{R}^d} dy \{ f(\xi^{x,y}) - f(\xi)\} p(|x-y|)\chi(y|\xi \setminus \{x\}), \qquad f \in \mathbf{C}_0(\mathscr{X}),$$

where χ is the function defined by (1.1), p is a nonnegative function satisfying $(p.1)$, $(p.2)$ and $(p.4)$, and

$$\xi^{x,y} = \begin{cases} (\xi \setminus \{x\}) \cup \{y\}, & \text{if } x \in \xi,\ y \notin \xi, \\ \xi, & \text{otherwise}. \end{cases}$$

The measure $p(|x-y|)\chi(y|\xi \setminus \{x\})\,dy$ gives the rate of the movement of a ball at the position x to the position y when the entire configuration is ξ. From the property $(\Psi.1)$, the ball of the system moves by random jumps under the hard core condition.

In this paper we study the behavior of a tagged particle in the process. In order to follow the motion of the tagged particle it is convenient to regard the process ξ_t as a Markov process $(y(t), \zeta_t)$ on the locally compact space $\mathbf{R}^d \times \mathscr{X}_0$, where

$$\mathscr{X}_0 = \{\zeta \in \mathscr{X} \colon \zeta \cap U_r(0) = \varnothing\}.$$

$y(t)$ is the position of the tagged particle and ζ_t is the entire configuration seen from the tagged particle. We can see that ζ_t is a Markov process whose generator $\overline{\mathscr{K}}$ is the smallest closed extension of the operator on $\mathbf{C}_0(\mathscr{X}_0)$ given by

$$\mathscr{K} = \mathscr{K}_1 + \mathscr{K}_2,$$

$$\mathscr{K}_1 f(\zeta) = \int_{\mathbf{R}^d} du \{ f(\tau_{-u}\zeta) - f(\zeta)\} p(|u|)\chi(u|\zeta),$$

$$\mathscr{K}_2 f(\zeta) = \sum_{x \in \zeta} \int_{U_r(0)^c} dy \{ f(\zeta^{x,y}) - f(\zeta)\} p(|x-y|)\chi(y|\zeta \setminus \{x\}),$$

$$f \in \mathbf{C}_0(\mathscr{X}_0),$$

where $\mathbf{C}(\mathfrak{X}_0)$ and $\mathbf{C}_0(\mathfrak{X}_0)$ are defined in the same way as $\mathbf{C}(\mathfrak{X})$ and $\mathbf{C}_0(\mathfrak{X})$, respectively. We denote by S_t the semigroup associated with generator $\overline{\mathscr{K}}$ and by $(\Omega, \mathscr{F}, P_\nu^0, \zeta_t)$ the associated process with initial distribution ν.

For any $\mu \in \mathscr{G}(z, \Psi)$ we define

$$\mu_0(d\eta) = \frac{\chi(0|\eta)}{c_3}\mu(d\eta),$$

where $c_3 = \int_{\mathfrak{M}} \chi(0|\eta)\mu(d\eta)$. In [10] we proved that there exists $z_3 \in (0, \infty)$ such that if $z \in (0, z_3)$ and if $\#\mathscr{G}(z, \Psi) = 1$ and $\mu \in \mathscr{G}(z, \Psi)$, then $(P_{\mu_0}^0, \zeta_t)$ is an ergodic reversible Markov process.

The process $y(t)$ is driven by the process ζ_t in the following way. Let $A \in \mathscr{B}(\mathbf{R}^d)$ and let $\Xi(A)$ be the measurable subset of $\mathfrak{X}_0 \times \mathfrak{X}_0$ defined by

$$\Xi(A) = \{(\eta, \zeta) \in (\mathfrak{X}_0 \times \mathfrak{X}_0) \setminus \Delta \colon \zeta = \tau_{-u}\eta \text{ for some } u \in A\},$$

where

$$\Delta = \{(\zeta, \zeta) \colon \zeta \in \mathfrak{X}_0\} \cup \left(\{\zeta \in \mathfrak{X}_0 \colon \zeta = \tau_{-u}\zeta \text{ for some } u \in \mathbf{R}^d \setminus \{0\}\}^2\right).$$

Define a σ-finite random measure N by

$$N((0,t] \times A) = \sum_{s \in (0,t]} \mathbb{1}_{\Xi(A)}(\eta_{s-}, \eta_s), \qquad t > 0.$$

Then,

$$y(t) = y(0) + \int_0^t \int_{\mathbf{R}^d} N(ds\,du)u.$$

Our second main result is the following theorem.

THEOREM 1.2. *If $z \in (0, z_2 \wedge z_3)$ and if $\#\mathscr{G}(z, \Psi) = 1$ and $\mu \in \mathscr{G}(z, \Psi)$, then the process $\varepsilon y(t/\varepsilon^2)$ on $(\Omega, \mathscr{F}, P_{\mu_0}^0)$ converges to $\sigma_0 B(t)$ as $\varepsilon \to 0$ in distribution with respect to the J_1-topology on Skorohod's function space $\mathbf{D}[0, \infty)$, where σ_0 is a positive constant.*

2. Uniqueness of the unbounded cluster. Let $C(x, \eta)$ be the cluster containing x for η, which is defined in (1.2). We denote the collection of all unbounded clusters for η by $\mathscr{C}(\eta)$:

$$\mathscr{C}(\eta) = \{C(x, \eta) \colon C(x, \eta) \text{ is unbounded, } x \in \mathbf{R}^d\}.$$

In this section we study the number of elements of $\mathscr{C}(\eta)$. Burton and Keane [1] proved the uniqueness of the infinite cluster of a site percolation model on $\mathbf{Z}^d$ under a translation invariant finite energy probability measure. Using their technique, we show the uniqueness of the unbounded cluster under a translation invariant Gibbs state, which is a key part of proving the ergodicity of the environment process.

PROPOSITION 2.1. *If $\mu \in \operatorname{ex} \mathscr{G}_\Theta(z, \Phi)$ and $\mu(\mathfrak{M}^*) > 0$, then $\#\mathscr{C}(\eta) = 1$ for μ-almost all η.*

PROOF. Put

$$\mathscr{A}(\eta) = \{A_{x,\eta}: A_{x,\eta} \text{ is unbounded}, x \in \mathbf{R}^d\}.$$

Since $\#\mathscr{A}(\eta) \geq \#\mathscr{C}(\eta)$ and $\#\mathscr{C}(\eta)$ is constant μ-a.s. from the ergodicity of μ, it is sufficient to show that $\#\mathscr{A}(\eta) \leq 1$ μ-a.s. for the proof of this proposition. This is trivial when $h = \infty$. So we assume that $h < \infty$. First, note that $\#\mathscr{A}(\eta)$ is constant, μ-a.s., from the ergodicity of μ.

Let $2 \leq n < \infty$. Suppose that $\mathscr{A}(\eta) = \{A_1(\eta), A_2(\eta), \dots, A_n(\eta)\}$, μ-a.s. η. Then there exists $l > 0$ such that

$$\mu(A_i \cap U_l(0) \neq \varnothing, i = 1,2) > 0.$$

From a property of a Gibbs state we have

$$(2.1)\quad \mu(\{\eta \in \mathfrak{M}: \eta = \zeta \setminus U_l(0), A_i(\zeta) \cap U_l(0) \neq \varnothing, i = 1,2\}) > 0,$$

so $\mu(\#\mathscr{A} < n) > 0$. This is a contradiction. Hence,

$$(2.2)\qquad \mu(2 \leq \#\mathscr{A} < \infty) = 0.$$

Suppose that $\#\mathscr{A}(\eta) = \infty$ and $\mathscr{A}(\eta) = \{A_1(\eta), A_2(\eta), \dots\}$, μ-a.s. η. We introduce the following notion. Let $l_0 > 0$. A point $x \in l_0\mathbf{Z}^d$ is called an l_0-encounter site for $\eta \in \mathfrak{M}$, if $A_{x,\eta}$ is unbounded and $A_{x,\eta} \setminus \overline{U_{l_0/2}(x)}$ has exactly three unbounded connected components. Denote by $\mathscr{N}_\eta(l)$ the number of unbounded connected components of the open set $A_{0,\eta} \setminus \overline{U_l(0)}$. By an argument used to show (2.1) we obtain that there exists $l_1 > 0$ such that

$$\mu(\mathscr{N}(l_1) \geq 3) > 0.$$

Since $\mathscr{N}_\eta(\cdot)$ is a right-continuous increasing function and satisfies, for μ-almost all η, $\mathscr{N}_\eta(l) - \mathscr{N}_\eta(l-) \leq 1$ for all $l > 0$, there exists $l_0 > 0$ such that

$$\mu\left(\mathscr{N}\left(\frac{l_0}{2}\right) = 3\right) > 0.$$

Then, we have

$$\mu(\{\eta: 0 \text{ is an } l_0\text{-encounter site for } \eta\}) = \varepsilon > 0.$$

It follows from the ergodic theorem that for almost all η,

$$(2.3)\quad \text{the number of } l_0\text{-encounter sites for } \eta \text{ in } V_k = O(k^d), \qquad k \to \infty,$$

where $V_k = [-(k + \frac{1}{2})l_0, (k + \frac{1}{2})l_0]^d$, $k \in \mathbf{N}$.

Put

$$\mathscr{Y}_i(\eta) = \{Y: Y \text{ is an unbounded connected component of } A_i(\eta) \setminus V_k\}.$$

Any l_0-encounter site x for η with $x \in A_i(\eta) \cap V_k$ determines a partition $\mathbf{P} = \{\mathbf{P}_1, \mathbf{P}_2, \mathbf{P}_3\}$ of $\mathscr{Y}_i(\eta)$. If $\mathbf{P} = \{\mathbf{P}_1, \mathbf{P}_2, \mathbf{P}_3\}$ and $\mathbf{P}' = \{\mathbf{P}'_1, \mathbf{P}'_2, \mathbf{P}'_3\}$ are different partitions of $\mathscr{Y}_i(\eta)$ determined by l_0-encounter sites for η, then there is an ordering of each such that $\mathbf{P}_1' \supset \mathbf{P}_1 \cup \mathbf{P}_2$. Denote by $\mathscr{P}_i(\eta)$ the collection of all partitions of $\mathscr{Y}_i(\eta)$ determined by l_0-encounter sites for η in $A_i(\eta) \cap V_k$. Using Lemma 2 in [1], if $\#\mathscr{P}_i \neq 0$, then

$$\#\mathscr{P}_i(\eta) \leq \#\mathscr{Y}_i(\eta) - 2.$$

Since $|Y \cap (V_{k+h/l_0} \setminus V_k)| \geq |U_h(0)|$ for any $Y \in \mathscr{Y}_i(\eta)$, we have

$$\sum_{i=1}^{\infty} \#\mathscr{Y}_i(\eta) = O(k^{d-1}), \qquad k \to \infty;$$

so

(2.4) the number of l_0-encounter sites for η in $V_k = O(k^{d-1})$, $\qquad k \to \infty$.

This contradicts (2.3). Hence, we have

$$\mu(\#\mathscr{A} = \infty) = 0. \tag{2.5}$$

This completes the proof of Proposition 2.1. □

3. Random electrical networks. In this section we study effective conductivity in a random electrical network associated with the process $x(t)$ and show a property which is a key part of proving the nondegeneracy of the diffusion matrix D^*. We begin with the definition of an electrical network and effective conductivity in the context of an electrical network, which is a generalization of that in [4]. Let A be a bounded measurable subset of $\mathbf{R}^d$, and let (q, m) be a pair of a measurable kernel q on $A \times \mathscr{B}(A)$ and a finite measure m on A satisfying

$$q(x, dy)m(dx) = q(y, dx)m(dy).$$

We call the pair (q, m) an *electrical network*. Throughout this section we assume that $q(\cdot, A)$ is bounded. Let A_0 and A_1 be disjoint subsets of A. The *effective conductivity* between A_0 and A_1 in (q, m), denoted by $\mathscr{I}_{A_0, A_1}(q, m)$, is defined by

$$\mathscr{I}_{A_0, A_1}(q, m) = \tfrac{1}{2}\int_A m(dx) \int_A q(x, dy)(\varphi(y) - \varphi(x))^2,$$

where φ is a function on A satisfying the following conditions (3.1):

$$\begin{aligned} \int_A q(x, dy)(\varphi(y) - \varphi(x)) &= 0, \qquad x \in A \setminus (A_0 \cup A_1), \\ \varphi(x) &= 0, \qquad x \in A_0, \\ \varphi(x) &= 1, \qquad x \in A_1. \end{aligned} \tag{3.1}$$

Although the solution of (3.1) is not always unique, all solutions give the same value $\mathscr{I}_{A_0, A_1}(q, m)$. Indeed, if we denote two solutions of (3.1) by φ and φ' and put $\psi = \varphi - \varphi'$ and $\psi' = \varphi + \varphi'$, then

$$\int_A m(dx)\int_A q(x,dy)\{(\varphi(y) - \varphi(x))^2 - (\varphi'(y) - \varphi'(x))^2\}$$

$$= \int_A m(dx)\int_A q(x,dy)(\psi(y) - \psi(x))(\psi'(y) - \psi'(x))$$

$$= -2\int_A m(dx)\psi(x)\int_A q(x,dy)(\psi'(y) - \psi'(x))$$

$$= 0.$$

Denote by $Pot(q, A_0, A_1)$ the set of all functions satisfying (3.1).

LEMMA 3.1. (i) *Let* $\varphi \in Pot(q, A_0, A_1)$. *Then*,

$$\mathscr{I}_{A_0, A_1}(q, m) = \int_{A_0} m(dx)\int_{A\setminus A_0} q(x,dy)\varphi(y)$$

$$= \int_{A_1} m(dx)\int_{A\setminus A_1} q(x,dy)(1 - \varphi(y)).$$

(ii) $\mathscr{I}_{A_0, A_1}(q, m) = \min_{\varphi'}\left\{\frac{1}{2}\int_A m(dx)\int_A q(x,dy)(\varphi'(y) - \varphi'(x))^2\right\}$,

where the minimum extends over the measurable functions φ' *on* A *satisfying* $\varphi' = 0$ *on* A_0 *and* $\varphi' = 1$ *on* A_1.

PROOF. Applying first the symmetry of $q(x,dy)m(dx)$ and the condition $\varphi \in Pot(q, A_0, A_1)$, we have

$$\mathscr{I}_{A_0, A_1}(q, m) = \int_A m(dx)\varphi(x)\int_A q(x,dy)(\varphi(x) - \varphi(y))$$
$$= \int_{A_1} m(dx)\int_{A\setminus A_1} q(x,dy)(1 - \varphi(y)). \tag{3.2}$$

Similarly,

$$\int_{A_0} m(dx)\int_{A\setminus A_0} q(x,dy)\varphi(y) - \int_{A_1} m(dx)\int_{A\setminus A_1} q(x,dy)(1 - \varphi(y))$$
$$= \int_A m(dx)\int_A q(x,dy)(\varphi(y) - \varphi(x)) = 0. \tag{3.3}$$

Thus, we obtain (i). We next turn to the second assertion. Let $\varphi \in Pot(q, A_0, A_1)$, and let φ' be a function on A such that $\varphi' = 0$ on A_0 and $\varphi' = 1$ on A_1. Put $\psi = \varphi' - \varphi$. Since $\psi(x) = 0$, for $x \in A_0 \cup A_1$ and

$q(x, dy)m(dx) = q(y, dx)m(dy)$, we have

$$
\begin{aligned}
&\int_A m(dx) \int_A q(x, dy)\{(\varphi'(y) - \varphi'(x))^2 - (\varphi(y) - \varphi(x))^2\} \\
&\quad = \int_A m(dx) \int_A q(x, dy)(\psi(y) - \psi(x))^2 \\
&\qquad + 2\int_A m(dx) \int_A q(x, dy)(\varphi(y) - \varphi(x))(\psi(y) - \psi(x)) \\
&\quad \geq -4\int_A m(dx)\psi(x) \int_A q(x, dy)(\varphi(y) - \varphi(x)) \\
&\quad = 0.
\end{aligned}
\tag{3.4}
$$

This completes the proof of (ii). □

We introduce a random electrical network $(q_{\eta,\Lambda}, m_\Lambda)$ associated with the process $x(t)$. Let $b = (h/\sqrt{d+2}) \wedge r_0$, $\beta = 2[r_0/b] + 3$ and $a = \beta b$, where $[c]$ stands for the integer part of $c \geq 0$. Then, $a > 2r_0 + b$ and $\operatorname{ess\,inf}\{p(\alpha), \alpha \in [0, b\sqrt{d+1})\} > 0$. For any l with $l \gtrsim 2a$ put

$$
\begin{aligned}
\Lambda &= \Lambda(l) = [-l - a, l + a] \times [-l, l]^{d-1}, \\
\Lambda_0 &= \Lambda_0(l) = [-l - a, -l + a] \times [-l, l]^{d-1}, \\
\Lambda_1 &= \Lambda_1(l) = [l - a, l + a] \times [-l, l]^{d-1}.
\end{aligned}
$$

We define a measurable kernel $q_{\eta,\Lambda}$ on $\Lambda \times \mathscr{B}(\Lambda)$ and a measure m_Λ on Λ by

$$q_{\eta,\Lambda}(x, dy) = \mathbb{I}_\Lambda(x)\mathbb{I}_\Lambda(y)q_\eta(x, dy), \tag{3.5}$$

$$m_\Lambda(dx) = \mathbb{I}_\Lambda(x)\,dx. \tag{3.6}$$

It is obvious that

$$q_{\eta,\Lambda}(x, dy)m_\Lambda(dx) = q_{\eta,\Lambda}(y, dx)m_\Lambda(dy), \qquad \eta \in \mathfrak{M}. \tag{3.7}$$

The main result of this section is the following proposition.

PROPOSITION 3.1. *There exist $z_2 \in (0, z_1]$ and a positive function $c_2(z)$ on $(0, z_2)$ such that if $z \in (0, z_2)$ and $\mu \in \mathscr{G}(z, \Phi)$, then*

$$\liminf_{l\to\infty} (2l)^{2-d} \mathscr{I}_{\Lambda_0(l), \Lambda_1(l)}(q_{\eta,\Lambda(l)}, m_{\Lambda(l)}) \geq c_2(z), \qquad \mu\text{-a.e.} \tag{3.8}$$

In case $h = \infty$ we can take $z_1 = \infty$.

Lower bounds similar to (3.8) were shown by Grimmett and Kesten [7], Chayes and Chayes [2] and Grimmett and Marstrand [6] in the case of the electrical network associated with the simple random walk on the Bernoulli percolation cluster. Then the proof of Proposition 3.1 is completed if we show a suitable relation between this network and the electrical network $(q_{\eta,\Lambda}, m_\Lambda)$. To this end we first introduce an electrical network $(Q^\Gamma_{\mathbf{a}}, M_\Gamma)$ associated with a simple random walk on a cluster. For any $k \in \mathbf{N}$ we put

$$\Gamma = \Gamma(k) = a\mathbf{Z}^d \cap \left([-(k+1)a, (k+1)a] \times [-ka, ka]^{d-1}\right),$$
$$\Gamma_0 = \Gamma_0(k) = a\mathbf{Z}^d \cap \left(\{-(k+1)a\} \times [-ka, ka]^{d-1}\right),$$
$$\Gamma_1 = \Gamma_1(k) = a\mathbf{Z}^d \cap \left(\{(k+1)a\} \times [-ka, ka]^{d-1}\right),$$

where a is the same constant as introduced above. Let $\mathbf{a} \in \{0,1\}^{a\mathbf{Z}^d}$. Define a measurable kernel $Q_{\mathbf{a}}^{\Gamma}$ on $\Gamma \times \mathscr{B}(\Gamma)$ and a measure $M_\Gamma(dx)$ on Γ by

$$Q_{\mathbf{a}}^{\Gamma}(x, dy) = \sum_{\substack{u \in \Gamma \\ |u-x|=a}} \mathbf{a}(x)\mathbf{a}(u)\delta_u(dy), \tag{3.9}$$

$$M_\Gamma(dx) = \sum_{u \in \Gamma} \delta_u(dx). \tag{3.10}$$

Then,

$$Q_{\mathbf{a}}^{\Gamma}(x, dy) M_\Gamma(dx) = Q_{\mathbf{a}}^{\Gamma}(y, dx) M_\Gamma(dy), \qquad \mathbf{a} \in \{0,1\}^{a\mathbf{Z}^d}.$$

Given $u \in a\mathbf{Z}^d$, we define a function $\chi_\eta(u)$ of $\eta \in \mathfrak{M}$ by

$$\chi_\eta(u) = \begin{cases} 1, & \text{if } I_a(u) \cap \eta = \varnothing, \\ 0, & \text{if } I_a(u) \cap \eta \neq \varnothing, \end{cases}$$

where (and also in the sequel) for $c > 0$ and $v \in \mathbf{R}^d$, $I_c(v)$ denotes the cube

$$\prod_{i=1}^{d} \left(v_i - \frac{c}{2}, v_i + \frac{c}{2}\right).$$

LEMMA 3.2. *Suppose that* $(k + \frac{1}{2})a \le l < (k + \frac{3}{2})a$ *for some* $k \in \mathbf{N}$. *Then*

$$\mathscr{I}_{\Lambda_0, \Lambda_1}(q_{\eta,\Lambda}, m_\Lambda) \ge c_1 \mathscr{I}_{\Gamma_0, \Gamma_1}\left(Q_{\chi_\eta}^{\Gamma}, M_\Gamma\right), \qquad \eta \in \mathfrak{M},$$

where $c_1 = (b^{2d}/\beta)\ \operatorname{ess\,inf}\{p(\alpha): \alpha \in [0, b\sqrt{d+1})\}$.

PROOF. Put

$$\Gamma_{u,v}^b = \left\{\frac{j}{\beta}u + \left(1 - \frac{j}{\beta}\right)v : j = 1, 2, \dots, \beta\right\}, \qquad u, v \in \Gamma, |u-v| = a,$$

$$\Gamma^b = \bigcup_{\substack{u,v \in \Gamma \\ |u-v|=a}} \Gamma_{u,v}^b, \qquad \Gamma_0^b = \bigcup_{\substack{u,v \in \Gamma_0 \\ |u-v|=a}} \Gamma_{u,v}^b, \qquad \Gamma_1^b = \bigcup_{\substack{u,v \in \Gamma_1 \\ |u-v|=a}} \Gamma_{u,v}^b.$$

First, we define a measurable kernel q_1 on $\Lambda \times \mathscr{B}(\Lambda)$ by

$$q_1(x, dy) = \sum_{u,v \in \Gamma^b} Q(u,v) \mathbb{1}_{I_b(u)}(x) \mathbb{1}_{I_b(v)}(y)\, dy,$$

where

$$Q(u,v) = \begin{cases} \chi_\eta(u')\chi_\eta(v'), & \text{if } u,v \in \Gamma^b_{u',v'},\ u',v' \in \Gamma, \\ & |u-v| = b,\ |u'-v'| = a, \\ 0, & \text{otherwise}. \end{cases}$$

It is obvious that $q_1(x, dy)m_\Lambda(dx) = q_1(y, dx)m_\Lambda(dy)$. Since

$$\bigcup_{\substack{v \in \Gamma^b \\ |u-v|=b}} I_b(v) \subset U_h(x), \qquad \text{if } u \in \Gamma^b,\ x \in I_b(u),$$

and

$$\chi(x|\eta) \geq \chi_\eta(u)\chi_\eta(v) \qquad \text{if } x \in \bigcup_{u' \in \Gamma_{u,v}} I_b(u'),\ |u-v| = a,$$

we see that $q_{\eta,\Lambda}(x, dy) \geq \operatorname{ess\,inf}\{p(\alpha)\colon \alpha \in [0, b\sqrt{d+1})\} q_1(x, dy)$. Then, by Lemma 3.1(ii), we have

$$\mathscr{I}_{\Lambda_0,\Lambda_1}(q_{\eta,\Lambda}, m_\Lambda) \geq \operatorname{ess\,inf}\left\{p(\alpha) : \alpha \in \left[0, b\sqrt{d+1}\right)\right\} \mathscr{I}_{J_0,J_1}(q_1, m_\Lambda), \tag{3.11}$$

where $J_0 = \bigcup_{u \in \Gamma^b_0} I_b(u)$, $J_1 = \bigcup_{u \in \Gamma^b_1} I_b(u)$.

Second, we define a measurable kernel Q_2 on $\Lambda \times \mathscr{B}(\Lambda)$ and a measure M_2 on Λ by

$$Q_2(x, dy) = \sum_{u \in \Gamma^b} Q(x,u)\delta_u(dy),$$
$$M_2(dx) = \sum_{u \in \Gamma^b} \delta_u(dx).$$

It is obvious that $Q_2(x, dy)M_2(dx) = Q_2(y, dx)M_2(dy)$. Let $\varphi_2 \in Pot(Q_2, \Gamma^b_0, \Gamma^b_1)$. We define a function φ_1 on Λ by

$$\varphi_1(x) = \begin{cases} \varphi_2(u), & \text{if } x \in I_b(u),\ u \in \Gamma^b, \\ 0, & \text{if } x \in \Lambda \setminus \left(\bigcup_{u \in \Gamma^b} I_b(u) \right). \end{cases}$$

Then $\varphi_1 \in Pot(q_1, \Lambda_0, \Lambda_1)$. Hence, by Lemma 3.1(i),

$$\begin{aligned} \mathscr{I}_{J_0,J_1}(q_1, m_\Lambda) &= \int_{J_0} m_\Lambda(dx) \int_{\Lambda \setminus J_0} q_1(x, dy)\varphi_1(y) \\ &= b^{2d} \int_{\Gamma^b_0} M_2(dx) \int_{\Gamma^b \setminus \Gamma^b_0} Q_2(x, dy)\varphi_2(y) \\ &= b^{2d} \mathscr{I}_{\Gamma^b_0,\Gamma^b_1}(Q_2, M_2). \end{aligned} \tag{3.12}$$

Let $\varphi_3 \in Pot(Q^\Gamma_{\chi_\eta}, \Gamma_0, \Gamma_1)$. Then we define a function φ_2' on Γ^b by

$$\varphi_2'(x) = \frac{j}{\beta}\varphi_3(u) + \left(1 - \frac{j}{\beta}\right)\varphi_3(v),$$

for $x = (j/\beta)u + (1 - j/\beta)v$, $u, v \in \Gamma$, $|u - v| = a$, $j = 1, 2, \dots, \beta$. Then $\varphi_2' \in Pot(Q_2, \Gamma_0^b, \Gamma_1^b)$, and so

$$
\begin{aligned}
\mathscr{I}_{\Gamma_0^b, \Gamma_1^b}(Q_2, M_2) &= \int_{\Gamma_0^b} M_3(dx) \int_{\Gamma^b \setminus \Gamma_0^b} Q_2(x, dy) \varphi_2'(y) \\
&= \frac{1}{\beta} \int_{\Gamma_0} M_\Gamma(dx) \int_{\Gamma \setminus \Gamma_0} Q^\Gamma_{\chi_\eta}(x, dy) \varphi_3(y) \qquad (3.13)\\
&= \frac{1}{\beta} \mathscr{I}_{\Gamma_0, \Gamma_1}\left(Q^\Gamma_{\chi_\eta}, M_\Gamma\right).
\end{aligned}
$$

From (3.11)–(3.13) we complete the proof of Lemma 3.2. □

PROOF OF PROPOSITION 3.1. From the conditions (Φ.1) and (Φ.2) [or (Φ.2′)] of Φ, for any $\theta \in (0, 1)$ we can choose $z(\theta) \in (0, z_1)$ such that

$$Z_{I_a(u), \eta, z(\theta)} \le \frac{1}{\theta} \qquad (3.14)$$

for any $u \in \mathbf{R}^d$ and $\eta \in \mathfrak{M}$. Then, if $\mu \in \mathscr{G}(z(\theta), \Phi)$, we can construct $\{0, 1\}^{a\mathbf{Z}^d}$-valued random variables $\mathbf{a}_1$ and $\mathbf{a}_2$ on a probability space $(\hat{\Omega}, \hat{\mathscr{F}}, \hat{P})$ satisfying

$$
\begin{aligned}
&\hat{P}\left(\mathbf{a}_1(u) \ge \mathbf{a}_2(u), u \in a\mathbf{Z}^d\right) = 1, \\
&\hat{P}\left(\mathbf{a}_1(u_i) = 1, i = 1, 2, \dots, n\right) = \mu\left(\chi_\eta(u_i) = 1, i = 1, 2, \dots, n\right), \\
&\hat{P}\left(\mathbf{a}_2(u_i) = 1, i = 1, 2, \dots, n\right) = \theta^n,
\end{aligned}
$$

for any sequence $u_1, u_2, \dots, u_n$ of $a\mathbf{Z}^d$ such that $u_i \ne u_j$, $1 \le i < j \le n$. It is obvious that

$$\mathscr{I}_{\Gamma_0, \Gamma_1}\left(Q^\Gamma_{\chi_\eta}, M_\Gamma\right) = \mathscr{I}_{\Gamma_0, \Gamma_1}\left(Q^\Gamma_{\mathbf{a}_1}, M_\Gamma\right), \qquad (3.15)$$

in the sense of distribution. From Lemma 3.1(ii) we have

$$\mathscr{I}_{\Gamma_0, \Gamma_1}\left(Q^\Gamma_{\mathbf{a}_1}, M_\Gamma\right) \ge \mathscr{I}_{\Gamma_0, \Gamma_1}\left(Q^\Gamma_{\mathbf{a}_2}, M_\Gamma\right). \qquad (3.16)$$

Since $(Q^\Gamma_{\mathbf{a}_2}, M_\Gamma)$ is a random electrical network in which the sites of Γ are taken to be occupied independently by unit conductors with probability θ and vacant with probability $1 - \theta$, it has been proven that there exists $\theta_c \in (0, 1)$ such that if $\theta > \theta_c$,

$$\liminf_{k \to \infty} (2k - 1)^{2-d} \mathscr{I}_{\Gamma_0, \Gamma_1}\left(Q^\Gamma_{\mathbf{a}_2}, M_\Gamma\right) > 0, \quad \text{a.s.} \qquad (3.17)$$

(See [3], [6] and [7].) Therefore, by (3.15)–(3.17) and Lemma 3.2, we obtain (3.8) for any $\mu \in \mathscr{G}_\Theta(z, \Phi)$ with $z \in (0, z(\theta))$, $\theta > \theta_c$.

The only assertion left to be proved is that if $h = \infty$, then (3.8) holds for any $\mu \in \mathscr{G}_\Theta(z, \Phi)$ with $z \in (0, \infty)$. For this purpose, let $\theta > \theta_c$ and $z \in (0, \infty)$. We can choose $j \in \mathbf{N}$ so that

$$Z_{I_a(u),\eta,z} \le \left\{1 - (1-\theta)^{1/j}\right\}^{-1}$$

for any $u \in \mathbf{R}^d$ and $\eta \in \mathfrak{M}$. Set $\tilde{a} = aj$ and for $k \ge 3l$ put

$$\tilde{\Gamma} = \tilde{\Gamma}(k) = \tilde{a}\mathbf{Z}^d \cap \left(\left[-(\tilde{k}+1)\tilde{a}, (\tilde{k}+1)\tilde{a}\right] \times \left[-\tilde{k}\tilde{a}, \tilde{k}\tilde{a}\right]^{d-1}\right),$$

$$\tilde{\Gamma}_0 = \tilde{\Gamma}_0(k) = \tilde{a}\mathbf{Z}^d \cap \left(\left\{-(\tilde{k}+1)\tilde{a}\right\} \times \left[-\tilde{k}\tilde{a}, \tilde{k}\tilde{a}\right]^{d-1}\right),$$

$$\tilde{\Gamma}_1 = \tilde{\Gamma}_1(k) = \tilde{a}\mathbf{Z}^d \cap \left(\left\{(\tilde{k}+1)\tilde{a}\right\} \times \left[-\tilde{k}\tilde{a}, \tilde{k}\tilde{a}\right]^{d-1}\right)$$

and

$$\Gamma_u^a = \Gamma_u^a(k) = \begin{cases} \Gamma(k) \cap I_{\tilde{a}}(u), & \text{if } u \in \tilde{\Gamma}(k) \setminus \left(\tilde{\Gamma}_0(k) \cup \tilde{\Gamma}_1(k)\right), \\ \Gamma_0(k) \cap I_{\tilde{a}}(u), & \text{if } u \in \tilde{\Gamma}_0(k), \\ \Gamma_1(k) \cap I_{\tilde{a}}(u), & \text{if } u \in \tilde{\Gamma}_1(k), \end{cases}$$

where $\tilde{k}$ is the positive integer satisfying $(\tilde{k} + \frac{3}{2})j \ge k - \frac{3}{2} > (\tilde{k} + \frac{1}{2})j$. We define

$$\tilde{\chi}_\eta(u) = 1 - \prod_{u' \in \Gamma_u^a} \left(1 - \chi_\eta(u')\right).$$

Then, one can proceed in the same way as in the proof of the case of $h < \infty$, and one obtains the desired result. □

4. Proofs of theorems. In this section we give the proofs of Theorems 1.1 and 1.2. First we show Theorem 1.1. To study the random walk with random obstacles $x(t)$, we introduce an $\mathfrak{M}$-valued process η_t on $(\Omega, \mathscr{F}, P_\eta)$ defined by

$$\eta_t = \tau_{-x(t)}\eta, \tag{4.1}$$

where $\tau_u A = \{x + u\colon x \in A\}$, $A \subset \mathbf{R}^d$, $u \in \mathbf{R}$. The process η_t describes the environment seen from $x(t)$. For any probability measure ν on $\mathfrak{M}$, (η_t, P_ν) is a Markov process with initial distribution ν. We denote the semigroup and the generator on $L^2(\mathfrak{M}, \mu)$ associated with the process by $\{T_t\}_{t\ge 0}$ and $\mathscr{L}$, respectively. By simple calculation we see that

$$\mathscr{L}f(\eta) = \int_{\mathbf{R}^d} q_\eta(0, du)\{f(\tau_{-u}\eta) - f(\eta)\}, \qquad f \in L^2(\mathfrak{M}, \mu). \tag{4.2}$$

To begin with, we show the reversibility and the ergodicity of the Markov process (η_t, P_{μ^*}) in the following lemmas.

LEMMA 4.1. *Suppose that $z \in (0, z_1)$ and $\mu \in \mathscr{G}_\Theta(z, \Phi)$. Then (η_t, P_{μ^*}) is a reversible Markov process, that is,*

$$\langle T_t f, g \rangle_{\mu^*} = \langle f, T_t g \rangle_{\mu^*}, \qquad \mathscr{F}, g \in L^2(\mathfrak{M}, \mu),\ t \geq 0,$$

where $\langle \cdot , \cdot \rangle_\nu$ is the L^2-inner product with respect to a measure ν.

PROOF. First note that

$$(4.3) \qquad \tau_{-u}\eta \in \mathfrak{M}^* \quad \text{if } u \in U_h(0) \text{ and } \eta \in \mathfrak{M}^* \text{ satisfies } \chi(u|\eta) \neq 0.$$

Then by the translation invariance of μ and (4.2),

$$\begin{aligned}
&\mu(\mathfrak{M}^*)\{\langle \mathscr{L}f, g\rangle_{\mu^*} - \langle f, \mathscr{L}g\rangle_{\mu^*}\} \\
&\quad = \int_{\mathfrak{M}} \mu(d\eta) \int_{\mathbf{R}^d} du\, f(\tau_{-u}\eta) g(\eta) p(|u|) \chi(u|\eta) \chi(0|\eta) 1_{\mathfrak{M}^*}(\eta) 1_{\mathfrak{M}^*}(\tau_{-u}\eta) \\
&\qquad - \int_{\mathfrak{M}} \mu(d\eta) \int_{\mathbf{R}^d} du\, g(\tau_{-u}\eta) f(\eta) p(|u|) \chi(u|\eta) \chi(0|\eta) 1_{\mathfrak{M}^*}(\eta) 1_{\mathfrak{M}^*}(\tau_{-u}\eta) \\
&\quad = \int_{\mathfrak{M}} \mu(d\eta) \int_{\mathbf{R}^d} du\, f(\tau_{-u}\eta) g(\eta) p(|u|) \chi(0|\tau_{-u}\eta) \chi(0|\eta) 1_{\mathfrak{M}^*}(\eta) 1_{\mathfrak{M}^*}(\tau_{-u}\eta) \\
&\qquad - \int_{\mathfrak{M}} \mu(d\eta) \int_{\mathbf{R}^d} du\, g(\tau_{u}\eta) f(\eta) p(|u|) \chi(0|\tau_{u}\eta) \chi(0|\eta) 1_{\mathfrak{M}^*}(\eta) 1_{\mathfrak{M}^*}(\tau_{u}\eta) \\
&\quad = 0.
\end{aligned}$$

Since $\mathscr{L}$ is the generator for T_t, Lemma 4.1 is proved. □

LEMMA 4.2. *Suppose that $z \in (0, z_1)$ and $\mu \in \operatorname{ex} \mathscr{G}_\Theta(z, \Phi)$. Then (η_t, P_{μ^*}) is ergodic.*

PROOF. Let f be a bounded measurable function satisfying $T_t f = f$ for any $t \geq 0$. The ergodicity of (η_t, P_{μ^*}) follows from showing that f is constant μ^*-a.s. From (4.2) and Lemma 4.1 we have

$$-2\langle \mathscr{L}f, f\rangle_{\mu^*} = \int_{\mathfrak{M}^*} \mu^*(d\eta) \int_{\mathbf{R}^d} du \{f(\tau_{-u}\eta) - f(\eta)\}^2 p(|u|) \chi(0|\eta) \chi(u|\eta).$$

Since f is T_t-invariant for any $t \geq 0$, $\mathscr{L}f = 0$; so, when $h < \infty$,

$$\int_{\mathfrak{M}} \mu(d\eta) \int_{U_h(0)} du \{f(\tau_{-u}\eta) - f(\eta)\}^2 \chi(0|\eta) \chi(u|\eta) 1_{\mathfrak{M}^*}(\eta) = 0.$$

From (4.3) and the translation invariance of μ, for any $x \in \mathbf{R}^d$,

$$\int_{U_h(x)} du\, |f(\tau_{-u}\eta) - f(\tau_{-x}\eta)| 1_{\mathfrak{M}^*}(\tau_{-x}\eta) 1_{\mathfrak{M}^*}(\tau_{-u}\eta) = 0, \qquad \mu\text{-a.s.}$$

Using this equality repeatedly, we have for almost all η,

$$|f(\tau_{-x}\eta) - f(\eta)| \prod_{k=0}^{m} 1_{\mathfrak{M}^*}(\tau_{-y_k}\eta) \prod_{k=0}^{m-1} |U_h(y_k) \cap U_h(y_{k+1}) \cap C(0, \eta)| = 0,$$

for any sequence $y_1, y_2, \dots, y_m$ with $y_1 = 0$, $y_m = x$ and $y_2, \dots, y_{m-1} \in \mathbf{Q}^d$. On the other hand, by the definition of $C(0, \eta)$, if $\eta \in \mathfrak{M}^*$, then for any $x \in C(0, \eta)$ there exists a sequence $y_1, y_2, \dots, y_m$ such that $y_1 = 0$, $y_m = x$ and $y_2, \dots, y_{m-1} \in \mathbf{Q}^d \cap C(0, \eta)$ and

$$\prod_{k=0}^{m-1} |U_h(y_k) \cap U_h(y_{k+1}) \cap C(0, \eta)| > 0.$$

Hence, for any $x \in C(0, \eta)$,

$$|f(\tau_{-x}\eta) - f(\eta)| \mathbb{I}_{\mathfrak{M}^*}(\tau_{-x}\eta) \mathbb{I}_{\mathfrak{M}^*}(\eta) = 0, \qquad \mu\text{-a.s.} \tag{4.4}$$

By Proposition 2.1, for almost all η, unboundedness of the clusters $C(0, \eta)$ and $C(x, \eta)$ implies $x \in C(0, \eta)$; so (4.4) holds for any $x \in \mathbf{R}^d$. Put

$$\hat{\mathfrak{M}} = \{\eta \in \mathfrak{M} : \tau_u \eta \in \mathfrak{M}^*, \text{ for some } u \in \mathbf{R}^d\}.$$

Since $\hat{\mathfrak{M}}$ is translation invariant, $\mu(\hat{\mathfrak{M}}) = 1$ by the ergodicity of μ. Define a measurable function $\hat{f}$ by

$$\hat{f}(\eta) = \begin{cases} f(\tau_u \eta), & \text{if } \tau_u \eta \in \mathfrak{M}^*, u \in \mathbf{R}^d, \\ 0, & \text{if } \eta \notin \hat{\mathfrak{M}}. \end{cases}$$

From (4.4), $\hat{f}$ is translation invariant, so $\hat{f}$ is constant μ-a.s. Since $\hat{f} = f$ on $\mathfrak{M}^*$, f is constant μ^*-a.s. A similar argument applies when $h = \infty$. □

By the construction of η_t, $x(t)$ is an antisymmetric additive functional of η_t, that is,

$$x(t)(\eta_{t/2-\cdot}) = -x(t)(\eta_\cdot), \quad t \geq 0, \tag{4.5}$$

$$x(t+s)(\eta_\cdot) = x(t)(\eta_\cdot) + x(s)(\eta_{t+\cdot}), \qquad t, s \geq 0. \tag{4.6}$$

De Masi, Ferrari, Goldstein and Wick [3] proved an invariance principle for antisymmetric additive functionals of ergodic reversible Markov processes, which is a generalization of the theorem of Kipnis and Varadhan [8]. Applying their invariance principle, we obtain that if $z \in (0, z_1)$ and $\mu \in \text{ex}\, \mathscr{G}(z, \Phi)$, $\varepsilon x(t/\varepsilon^2)$ converges to $D^*(\mu)B(t)$ as $\varepsilon \to 0$, where $D^*(\mu)$ is a nonnegative definite $d \times d$ matrix. Then, the only assertion left to be proved is the nondegeneracy of $D^*(\mu)$.

The reversibility of the Markov process (η_t, P_μ) is derived from the translation invariance of μ. From this and the fact that $x(t)$ is an antisymmetric additive functional of η_t, we can show that if $\mu \in \mathscr{G}_\Theta(z, \Phi)$, then

$$\lim_{t \to \infty} \frac{1}{t} E_\mu\left[x_1(t)^2\right] = \sigma(\mu)^2, \tag{4.7}$$

where $\sigma(\mu)$ is a nonnegative constant determined by

$$\sigma(\mu)^2 = \int_{\mathfrak{M}} \mu(d\eta) \int_{\mathbf{R}^d} q_\eta(0, du) u_1^2 - 2\int_0^\infty dt \langle T_t, F, F\rangle_\mu,$$

$$F(\eta) = \int_{\mathbf{R}^d} q_\eta(0, du) u_1.$$

(See, for instance, [11].) On the other hand, if $\mu \in \mathscr{G}_\Theta(z, \Phi)$ with $z \in (0, z_1)$, then

$$\lim_{t\to\infty} \frac{1}{t} \int_{\mathfrak{M}\setminus\mathfrak{M}^*} \mu(d\eta) E_\eta\big[x_1(t)^2\big] = 0,$$

so

$$e_1\big(D^*(\mu)^2\big)\,{}^t e_1 = \lim_{t\to\infty} \frac{1}{t} E_{\mu^*}\big[x_1(t)^2\big] = \frac{\sigma(\mu)^2}{\mu(\mathfrak{M}^*)},$$

where $e_1 = (1, 0, \dots, 0)$ and ${}^t e_1$ denotes the transpose of e_1. For any rotation R on $\mathbf{R}^d$ with $R(0) = 0$,

$$\big(x(t), P_\eta\big) = \big(R(x(t)), P_{R^{-1}(\eta)}\big),$$

in the sense of distribution. Then, for any $\mu \in \mathscr{G}_\Theta(z, \Phi)$, there exists $\mu' \in \mathscr{G}_\Theta(z, \Phi)$ such that

$$\big(x(t), P_{\mu'}\big) = \big(R(x(t)), P_\mu\big),$$

in the sense of distribution. Hence, it is enough to show the following lemma for the proof of the nondegeneracy of the matrix $D^*(\mu)$ for any $\mu \in \mathscr{G}_\Theta(z, \Phi)$ with $z \in (0, z_2)$.

Lemma 4.3. *Let $c_2(z)$ be the function on $(0, z_2)$ in Proposition 3.1. If $z \in (0, z_2)$ and $\mu \in \mathscr{G}_\Theta(z, \Phi)$, then $\sigma(\mu) \geq 2c_2(z)$.*

The proof of this lemma is given in Section 5.

Next we give the proof of Theorem 1.2. In [10] we showed the theorem except for the strict positivity of σ_0, which is the nonnegative constant determined by

$$(\sigma_0)^2 = \int_{\mathbf{R}^d} du \int_{\mathfrak{X}_0} \mu_0(d\eta) p(|u|) \chi(u|\eta) u_1^2 - 2\int_0^\infty dt \,\langle S_t \hat{F}, \hat{F}\rangle_{\mu_0},$$

$$\hat{F}(\eta) = \int_{\mathbf{R}^d} du\, p(|u|) \chi(u|\eta) u_1.$$

Then it remains only to prove the following lemma.

Lemma 4.4. *If $z \in (0, z_2)$ and $\mu \in \mathscr{G}_\Theta(z, \Psi)$, then*

$$\int_{\mathbf{R}^d} du \int_{\mathfrak{X}_0} \mu_0(d\eta) p(|u|) \chi(u|\eta) u_1^2 - 2\int_0^\infty dt \langle S_t \hat{F}, \hat{F}\rangle_{\mu_0} > 0.$$

PROOF. First, note that

$$(4.8)\qquad \begin{aligned} c_3\langle \mathscr{K}_1 f, f\rangle_{\mu_0} &= \langle \mathscr{L} f, f\rangle_{\mu}, \qquad f\in \mathbf{C}_0(\mathfrak{X}_0),\\ c_3\langle \hat{F}, f\rangle_{\mu_0} &= \langle F, f\rangle_{\mu}, \qquad f\in \mathbf{C}_0(\mathfrak{X}_0). \end{aligned}$$

Since $\mathscr{K}_2$ is nonpositive, we obtain

$$(4.9)\qquad c_3\langle -\mathscr{K} f, f\rangle_{\mu_0} \ge \langle -\mathscr{L} f, f\rangle_{\mu}, \qquad f\in \mathbf{C}_0(\mathfrak{X}_0).$$

By the self-adjointness of the operator $\mathscr{L}$, if

$$\int_0^\infty dt\langle T_t F, F\rangle_\mu \le c$$

for some $c > 0$, then

$$\langle F, f\rangle_\mu^2 \le c\langle -\mathscr{L} f, f\rangle_\mu, \qquad f\in \mathbf{C}_0(\mathfrak{X}_0);$$

so, from (4.8) and (4.9),

$$\langle \hat{F}, f\rangle_{\mu_0}^2 \le \frac{c}{c_3}\langle -\mathscr{K} f, f\rangle_{\mu_0}, \qquad f\in \mathbf{C}_0(\mathfrak{X}_0).$$

By the self-adjointness of the operator $\mathscr{K}$, this implies that

$$\int_0^\infty dt\langle S_t\hat{F}, \hat{F}\rangle_{\mu_0} \le \frac{c}{c_3}.$$

Then, we have

$$(4.10)\qquad \frac{1}{c_3}\int_0^\infty dt\langle T_t F, F\rangle_\mu \ge \int_0^\infty dt\langle S_t\hat{F}, \hat{F}\rangle_{\mu_0}.$$

Hence,

$$\begin{aligned} &\int_{\mathbf{R}^d} du \int_{\mathfrak{X}_0}\mu_0(d\eta)p(|u|)\chi(u|\eta)u_1^2 - 2\int_0^\infty dt\langle S_t\hat{F}, \hat{F}\rangle_{\mu_0}\\ &\quad\ge \frac{1}{c_3}\left\{\int_{\mathbf{R}^d} du\int_{\mathfrak{M}}\mu(d\eta)p(|u|)\chi(u|\eta)\chi(0|\eta)u_1^2 - 2\int_0^\infty dt\langle T_t F, F\rangle_\mu\right\}\\ &\quad= \frac{\sigma(\mu)^2}{c_3}. \end{aligned}$$

Therefore, Lemma 4.4 follows from Lemma 4.3. □

5. Proof of Lemma 4.3. To prove Lemma 4.3 we first construct Markov processes. For l and a with $0 < a \le \frac{2}{3}l$ put

$$W = \mathbf{R}\times[-l, l]^{d-1} = \bigcup_{k\in 2l\mathbf{Z}} \tau_{ke_1}\Lambda.$$

Let $\tilde{q}_{\eta,\Lambda}$ be the measurable kernel on $\Lambda \times \mathscr{B}(\Lambda)$ defined by

$$\begin{aligned}\tilde{q}_{\eta,\Lambda}(x, dy) &= \mathbb{I}_{\Lambda \setminus (\Lambda_0 \cup \Lambda_1)}(x)\mathbb{I}_{\Lambda \setminus (\Lambda_0 \cup \Lambda_1)}(y) q_\eta(x, dy)\\ &\quad + \mathbb{I}_{\Lambda \setminus \Lambda_0}(x)\mathbb{I}_{\Lambda_0}(y) q_\eta(x, \Lambda_0)\, dy\\ &\quad + \mathbb{I}_{\Lambda \setminus \Lambda_1}(x)\mathbb{I}_{\Lambda_1}(y) q_\eta(x, \Lambda_1)\, dy\\ &\quad + \mathbb{I}_{\Lambda_0}(x)\mathbb{I}_{\Lambda \setminus \Lambda_0}(y) q_\eta(y, \Lambda_0)\, dy\\ &\quad + \mathbb{I}_{\Lambda_1}(x)\mathbb{I}_{\Lambda \setminus \Lambda_1}(y) q_\eta(y, \Lambda_1)\, dy,\end{aligned}$$

and define a measurable kernel $q_\eta^l(x, dy)$ on $\mathbf{R}^d \times \mathscr{B}(\mathbf{R}^d)$ by

$$(5.1)\quad q_\eta^l(x, A) = \sum_{k \in 2l\mathbf{Z}} \tilde{q}_{\eta,\Lambda}(x + ke_1, \tau_{ke_1} A), \qquad x \in \mathbf{R}^d,\ A \in \mathscr{B}(\mathbf{R}^d).$$

For any $\eta \in \mathfrak{M}$ we define a linear operator on $\mathbf{C}_\infty(\mathbf{R}^d)$ by

$$L_\eta^l \varphi(x) = \int_{\mathbf{R}^d} q_\eta^l(x, dy)\{\varphi(y) - \varphi(x)\}, \qquad \varphi \in \mathbf{C}_\infty(\mathbf{R}^d).$$

For any $x \in W$ and $\eta \in \mathfrak{M}$ we denote by $(X^l(t), P_{\eta,x}^l)$ the W-valued right-continuous Markov process starting from $x \in W$ with generator $(L_\eta^l, \mathbf{C}_\infty(\mathbf{R}^d))$. Denote by $Y^l(t)$ the Markov process $X^l(t)/\sim$, where $x \sim y \Leftrightarrow x - y = 2kle_1$ for some $k \in \mathbf{Z}$. $Y^l(t)$ can be regarded as a Markov process with state space $V = V(l) = [-l, l)^d$. Let $\tilde{m}^l$ be a probability measure on V defined by

$$(5.2)\qquad \tilde{m}^l(dx) = (2l)^{-d}\mathbb{I}_V(x)\, dx.$$

Put $P_\eta^l = \int_V \tilde{m}^l(dx) P_{\eta,x}^l$. *Let* $E_{\eta,x}^l$ and E_η^l denote the expectation with respect to $P_{\eta,x}^l$ and P_η^l, respectively. Since $q_\eta^l(x, dy)\mathbb{I}_W(x)\, dx = q_\eta^l(y, dx)\mathbb{I}_W(y)\, dy$, we have the following lemma.

LEMMA 5.1. *For any $\eta \in \mathfrak{M}$, $(Y^l(t), P_\eta^l)$ is a reversible Markov process, that is,*

$$\left\langle \tilde{T}_\eta^l(t)\varphi, \psi\right\rangle_{\tilde{m}^l} = \left\langle \varphi, \tilde{T}_\eta^l(t)\psi\right\rangle_{\tilde{m}^l}, \qquad \varphi, \psi \in \mathbf{C}(V),\ t \ge 0,$$

where $\{\tilde{T}_\eta^l(t)\}_{t \ge 0}$ is the semigroup for the process $Y_\eta^l(t)$.

Let F_η^l and G_η^l be functions on W defined by

$$(5.3)\qquad F_\eta^l(x) = \int_W q_\eta^l(x, dy)(y_1 - x_1), \qquad x \in W,$$

$$(5.4)\qquad G_\eta^l(x) = \int_W q_\eta^l(x, dy)(y_1 - x_1)^2, \qquad x \in W.$$

Since $F_\eta^l(x) = F_\eta^l(y)$ and $G_\eta^l(x) = G_\eta^l(y)$ if $x \sim y$, we can regard F_η^l and G_η^l as functions on V. Since $X_1^l(t)$ is an additive antisymmetric functional of the reversible Markov process $Y^l(t)$, we have the following lemma.

LEMMA 5.2. *For any* $\eta \in \mathfrak{M}$,

$$\lim_{t\to\infty} \frac{1}{t} E_\eta^l\left(X_1^l(t)^2\right) = \sigma^l(\eta)^2,$$

where $\sigma^l(\eta)$ *is the nonnegative constant determined by*

$$\sigma^l(\eta)^2 = \int_V \tilde{m}^l(dx) G_\eta^l(x) - 2\int_0^\infty dt \left\langle T_\eta^l(t) F_\eta^l, F_\eta^l \right\rangle_{\tilde{m}^l}.$$

By Fatou's lemma, a lower bound of σ is given by means of σ^l.

LEMMA 5.3. *Let* $\mu \in \mathscr{G}(z, \Phi)$. *Then*,

$$\sigma(\mu)^2 \geq \limsup_{t\to\infty} \int_{\mathfrak{M}} \mu(d\eta) \sigma^l(\eta)^2.$$

PROOF. We shall prove this lemma only in the case $h < \infty$. In the case $h = \infty$ we can proceed similarly. First we show

$$\lim_{l\to\infty} \int_{\mathfrak{M}} \mu(d\eta) \left\langle \tilde{T}_\eta^l(t) F_\eta^l, F_\eta^l \right\rangle_{\tilde{m}^l} = \langle T_t F, F \rangle_\mu, \qquad t > 0. \tag{5.5}$$

Let $\alpha \in (0,1)$ and put $\kappa = \inf\{t > 0: Y^l(t) \notin V(l - a - h)\}$. We decompose the integral as follows:

$$\begin{aligned}
&\int_{\mathfrak{M}} \mu(d\eta) \left\langle \tilde{T}_\eta^l(t) F_\eta^l, F_\eta^l \right\rangle_{\tilde{m}^l} \\
&\quad = \int_{\mathfrak{M}} \mu(d\eta) \int_{V(l - l^\alpha)} \tilde{m}^l(dx) F_\eta^l(x) E_{\eta,x}^l\left[F_\eta^l(Y^l(t)) : t < \kappa\right] \\
&\qquad + \int_{\mathfrak{M}} \mu(d\eta) \int_{V(l - l^\alpha)} \tilde{m}^l(dx) F_\eta^l(x) E_{\eta,x}^l\left[F_\eta^l(Y^l(t)) : t \geq \kappa\right] \\
&\qquad + \int_{\mathfrak{M}} \mu(d\eta) \int_{V(l) \setminus V(l - l^\alpha)} \tilde{m}^l(dx) F_\eta^l(x) E_{\eta,x}^l\left[F_\eta^l(Y^l(t))\right].
\end{aligned} \tag{5.6}$$

By simple calculation we can show that the second and the third terms on the right-hand side vanish as l tends to infinity. We put $\kappa_x = \inf\{t > 0: x + x(t) \notin V(l - a - h)\}$. Then it is easy to see that

$$\left\{Y^l(t), t \in [0, \kappa), P_{\eta,x}^l\right\} = \left\{x + x(t), t \in [0, \kappa_x), P_{\tau_{-x}\eta}\right\},$$

in the sense of distribution. Noting that

$$F_\eta^l(x) = \int_{\mathbf{R}^d} q_\eta(x, dy)(y_1 - x_1) = F(\tau_{-x}\eta), \qquad x \in V(l - a - h),$$

by the translation invariance of μ,

$$\int_{\mathfrak{M}} \mu(d\eta) \int_{V(l-l^\alpha)} \tilde{m}^l(dx) F_\eta^l(x) E_{\eta,x}^l\left[F_\eta^l(Y^l(t)) : t < \kappa\right]$$

$$= \int_{V(l-l^\alpha)} \tilde{m}^l(dx) \int_{\mathfrak{M}} \mu(d\eta) F(\tau_{-x}\eta) E_{\tau_{-x}\eta}\left[F(\eta_t) : t < \kappa_x\right]$$

$$= \int_{V(l-l^\alpha)} \tilde{m}^l(dx) \int_{\mathfrak{M}} \mu(d\eta) F(\eta) E_\eta\left[F(\eta_t) : t < \kappa_x\right],$$

and the rightmost member converges to $\int_{\mathfrak{M}} \mu(d\eta) F(\eta) E_\eta[F(\eta_t)]$. Thus, we obtain (5.5). On the other hand, since $q_\eta^l(x, dy) = q_\eta(x, dy)$ for $x \in V(l - a - h)$, by the translation invariance of μ,

$$\lim_{l\to\infty} \int_{\mathfrak{M}} \mu(d\eta) \int_V \tilde{m}^l(dx) G_\eta^l(x)$$

$$(5.7) \qquad = \lim_{l\to\infty} \int_{\mathfrak{M}} \mu(d\eta) \int_V \tilde{m}^l(dx) \int_{\mathbf{R}^d} q_\eta(x, dy)(y_1 - x_1)^2$$

$$= \int_{\mathfrak{M}} \mu(d\eta) \int_{\mathbf{R}^d} q_\eta(0, dy) u_1^2.$$

Using Fatou's lemma, from (5.5) and (5.7),

$$\limsup_{l\to\infty} \int_{\mathfrak{M}} \mu(d\eta) \sigma^l(\eta)^2$$

$$= \lim_{l\to\infty} \int_{\mathfrak{M}} \mu(d\eta) \int_V \tilde{m}^l(dx) G_\eta^l(x) - 2 \liminf_{l\to\infty} \int_{\mathfrak{M}} \mu(d\eta) \int_0^\infty dt \left\langle \tilde{T}_\eta^l(t) F_\eta^l, F_\eta^l \right\rangle_{\tilde{m}^l}$$

$$\leq \int_{\mathfrak{M}} \mu(d\eta) \int_{\mathbf{R}^d} q_\eta(0, du) u_1^2 - 2 \int_0^\infty dt \langle T_t F, F \rangle_\mu$$

$$= \sigma(\mu)^2.$$

This completes the proof of Lemma 5.3. □

By Lemma 5.3, the proof of Lemma 4.3 is complete if we show the proof of the following lemma.

LEMMA 5.4. *If $z \in (0, z_2)$ and $\mu \in \mathscr{G}_\Theta(z, \Phi)$, then*

$$\liminf_{l\to\infty} \sigma^l(\eta)^2 \geq 2c_2(z), \qquad \mu\text{-a.s.}$$

PROOF. Let $\varphi_\eta^l \in Pot(\tilde{q}_{\eta,\Lambda}, \Lambda_0, \Lambda_1)$ and set $\psi_\eta^l(x) = x_1 - 2l\varphi_\eta^l(x) + l$. Since $\psi_\eta^l(x) = \psi_\eta^l(x + 2le_1)$, $x \in \Lambda_0$, ψ_η^l can be regarded as a periodic function on W. We first show that

$$L_\eta^l \psi_\eta^l(x) = F_\eta^l(x), \qquad x \in W. \tag{5.8}$$

This equation is obvious for $x \in \Lambda \setminus (\Lambda_0 \cup \Lambda_1)$. On the other hand, for $x \in \Lambda_0$,

$$\begin{aligned}
&L_\eta^l \psi_\eta^l(x) - F_\eta^l(x)\\
&\quad = \int_W q_\eta^l(x, dy)\big(\psi_\eta^l(y) - \psi_\eta^l(x) - y_1 + x_1\big)\\
&\quad = \int_{\Lambda\setminus\Lambda_0} q_\eta^l(x, dy)\big(\psi_\eta^l(y) - \psi_\eta^l(x) - y_1 + x_1\big)\\
&\qquad + \int_{\Lambda\setminus\Lambda_1} q_\eta^l(x + 2le_1, dy)\big(\psi_\eta^l(y) - \psi_\eta^l(x) - y_1 + x_1\big)\\
&\quad = -2l\left\{\int_{\Lambda\setminus\Lambda_0} \tilde{q}_{\eta,\Lambda}(x, dy)\varphi_\eta^l(y) + \int_{\Lambda\setminus\Lambda_1} \tilde{q}_{\eta,\Lambda}(x + 2le_1, dy)\big(\varphi_\eta^l(y) - 1\big)\right\}.
\end{aligned}$$

Since $\tilde{q}_{\eta,\Lambda}(x, dy) = \tilde{q}_{\eta,\Lambda}(x', dy)$ for $x, x' \in \Lambda_0$ or $x, x' \in \Lambda_1$, by Lemma 3.1 we have

$$\int_{\Lambda\setminus\Lambda_0} \tilde{q}_{\eta,\Lambda}(x, dy)\varphi_\eta^l(y) = \frac{1}{4al}\mathscr{I}_{\Lambda_0,\Lambda_1}\big(\tilde{q}_{\eta,\Lambda}, m_\Lambda\big), \tag{5.9}$$

$$\int_{\Lambda\setminus\Lambda_1} \tilde{q}_{\eta,\Lambda}(x, dy)\big(1 - \varphi_\eta^l(y)\big) = \frac{1}{4al}\mathscr{I}_{\Lambda_0,\Lambda_1}\big(\tilde{q}_{\eta,\Lambda}, m_\Lambda\big). \tag{5.10}$$

Thus, we obtain $L_\eta^l\psi_\eta^l(x) = F_\eta^l(x)$, $x \in \Lambda \setminus \Lambda_1$. Noting that the functions $L_\eta^l\psi_\eta^l$ and F_η^l are periodic, we conclude (5.8).

By (5.8) we obtain

$$\int_0^\infty dt\big\langle \tilde{T}_\eta^l(t)F_\eta^l, F_\eta^l\big\rangle_{\tilde{m}^l} = -\int_V \tilde{m}^l(dx)F_\eta^l(x)\psi_\eta^l(x). \tag{5.11}$$

Using the symmetry of $\tilde{q}_{\eta,\Lambda}(x, dy)m_\Lambda(dx)$ and the equations

$$F_\eta^l(x) = \int_\Lambda \tilde{q}_{\eta,\Lambda}(x, dy)(y_1 - x_1) + \int_\Lambda \tilde{q}_{\eta,\Lambda}(x + 2le_1, dy)(y_1 - x_1),$$

$$G_\eta^l(x) = \int_\Lambda \tilde{q}_{\eta,\Lambda}(x, dy)(y_1 - x_1)^2 + \int_\Lambda \tilde{q}_{\eta,\Lambda}(x + 2le_1, dy)(y_1 - x_1)^2, \quad x \in \Lambda_0,$$

we have

$$\sigma^l(\eta)^2 = \int_V \tilde{m}^l(dx)\left\{G_\eta^l(x) + 2F_\eta^l(x)\psi_\eta^l(x)\right\}$$

$$= (2l)^{-d}\int_\Lambda m_\Lambda(dx)\int_\Lambda \tilde{q}_{\eta,\Lambda}(x,dy)\Big\{(y_1 - x_1)^2$$

$$+2\big(x_1 - 2l\varphi_\eta^l(x) + l\big)(y_1 - x_1)\Big\}$$

$$= (2l)^{-d}\int_\Lambda m_\Lambda(dx)\int_\Lambda \tilde{q}_{\eta,\Lambda}(x,dy)(y_1 - x_1)\big(y_1 + x_1 + 2l - 4l\varphi_\eta^l(x)\big)$$

$$= 2(2l)^{1-d}\int_\Lambda m_\Lambda(dx)x_1\int_\Lambda \tilde{q}_{\eta,\Lambda}(x,dy)\big(\varphi_\eta^l(x) - \varphi_\eta^l(y)\big)$$

$$= 2(2l)^{1-d}\Bigg\{\int_{\Lambda_0} m_\Lambda(dx)x_1\int_{\Lambda\setminus\Lambda_0}\tilde{q}_{\eta,\Lambda}(x,dy)\varphi_\eta^l(y)$$

$$+\int_{\Lambda_1} m_\Lambda(dx)x_1\int_{\Lambda\setminus\Lambda_1}\tilde{q}_{\eta,\Lambda}(x,dy)\big(1 - \varphi_\eta^l(y)\big)\Bigg\}$$

$$= 2(2l)^{2-d}\mathscr{I}_{\Lambda_0,\Lambda_1}\big(\tilde{q}_{\eta,\Lambda}, m_\Lambda\big).$$

In the preceding we have used (5.9) and (5.10) in the last step. Since $\mathscr{I}_{\Lambda_0,\Lambda_1}(\tilde{q}_{\eta,\Lambda}, m_\Lambda) = \mathscr{I}_{\Lambda_0,\Lambda_1}(q_{\eta,\Lambda}, m_\Lambda)$, by Proposition 3.1 we obtain Lemma 5.4. □

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DEPARTMENT OF MATHEMATICS
FACULTY OF SCIENCE
CHIBA UNIVERSITY
CHIBA, 263
JAPAN