ABSTRACTS OF PAPERS

(Presented on December 26, 1940, at the Chicago meeting of the Institute)

On the Calculation of the Probability Integral on Non-Central t and an Application. C. C. CRAIG, University of Michigan.

It seems not to have been noted that the probability integral for non-central t can be calculated by means of an infinite series in incomplete β-functions which converges rapidly for small samples. The application here considered is to a test based on the randomization principle which is the subject of E. J. G. Pitman's paper: *Significance tests which may be applied to samples from any populations* (*Roy. Stat. Soc. Jour.*, Vol. 4 (1937), pp. 119–130). In case the samples come from normal populations with equal variance but with unequal means, the chance that the hypothesis of equal population means will be accepted on this test is given by this probability integral which is evaluated in some illustrative numerical examples.

On Some New Results in the Sampling of Discrete Random Variables. WILLIAM G. MADOW, Bureau of the Census.

Many statistical tables may be regarded as the result of subsampling finite populations classified into $r \times s \times \cdots$ tables. The main aim of this paper is to derive the associated statistical theory including both the finite and limiting distributions. After evaluating the fundamental distributions and the moments it is shown that under certain conditions, the limiting distribution is multinomial, while under other conditions the limiting distribution is multivariate normal. These results are then applied to determine the adequate size of sample, and the sampling proportions from various strata.

On the Use of Inverse Probability in Sample Inspection. W. EDWARDS DEMING and WILLIAM G. MADOW, Bureau of the Census.

The theory of inspection by sampling is abstractly equivalent to one part of the theory of subsampling. The theory of subsampling finite populations is considered in this paper in order to investigate the differences that occur when the methods of fiducial inference and inverse probability are used, particularly in regard to determining the adequate size of sample. In sample inspection, the prior distribution of failures is almost always known, at least approximately. In using any system of sample inspection, a number of failures will pass undetected. On the basis of certain prior distributions of failures, distributions are derived for the number and percent of failures remaining after each of several different possible systems of sample inspection has been applied. Formulas giving the cost of partial inspection are used together with these distributions in order to determine methods of sample inspection having various desired properties.

On a Convergent Iterative Procedure for Adjusting a Sample Frequency Table When Some of the Marginal Totals are Known. FREDERICK F. STEPHAN, Cornell University and W. EDWARDS DEMING, Bureau of the Census.

The 5 per cent sample taken with the 1940 Population Census presents an interesting problem of estimation in which the estimates are connected by equations of condition.

These equations arise from the fact that certain sums of estimates derived from the sample should equal the corresponding frequencies derived from the tabulations of the census enumeration, i.e. the distribution of each of several variables may be known but their joint distribution may only be estimated from a cross tabulation of the data furnished by the sample. The adjustment of the sample estimates is accomplished by the principle of least squares and an outline of the various types of conditions for two and three variables is presented. The solution of the normal and condition equations is tedious when hundreds of sets of estimates must be adjusted but a simple iterative procedure is available (see *Annals of Math. Stat.*, Vol. 11 (1940), pp. 427–444).

The Return Period of Flood Flows. E. J. GUMBEL, New School for Social Research (N. Y.)

For any statistical variable the return period is defined as the mean number of trials necessary in order that a certain value of the variable or a greater one returns. The return period is a theoretical statistical function such as the distribution or the probability. In hydraulics the corresponding observed values are the recurrence and exceedance intervals.

The main thesis is that *the flood flows are the largest values of flows* which have to be considered as *unlimited* variables. The method of return periods applied to the largest values leads without further assumptions to a formula which gives the return period $f(x)$ of a flood superior to x, and at the same time the most probable flood to be reached not at a certain time, but within a certain period. This formula contains only two constants, which are linear functions of the mean annual flood and the standard deviation. Fuller's formula turns out to be an asymptotic expression of my formula.

This method applied to the Connecticut, Columbia, Merrimack, Cumberland, Tennessee and Mississippi rivers shows a very good fit between theory and observation, superior to the methods applied heretofore.

A Note on the Power of the Sign Test. W. M. STEWART, University of Wisconsin.

Let us consider a set of N non-zero differences, of which x are positive and $N - x$ are negative; and suppose that the hypothesis tested, H_0, implies in independent sampling that x will be distributed about an expected value of $N/2$ in accordance with the binomial $(\frac{1}{2} + \frac{1}{2})^N$. As a quick test of H_0, we may choose to test the hypothesis h_0 that x has the above probability distribution. Defining r to be the smaller of x and $N - x$, the test consists in rejecting h_0 and therefore H_0 whenever $r \leq r(\epsilon, N)$, where $r(\epsilon, N)$ is determined by N and the significance level ϵ.

In applying such a test it is of interest to know how frequently it will lead to a rejection of H_0 when H_0 is false and the actual situation H implies that the probability law of x is $(q + p)^N$, with $p \neq \frac{1}{2}$, thereby indicating an expectation of an unequal number of + and − differences. The probability of rejecting H_0 when H_1 implying $p = p_1$ is true, is termed the *power* of the test of H_0 relative to the alternative H_1.

A table is given for the 5% significance level ($\epsilon = .05$) showing the minimum value of N for which the power of the test relative to $p = p_1$ exceeds β for values of β from .05 to .95 at intervals of .05; and for p_1 from .60 to .95 (and thereby for p_1 from .40 to .05) at intervals of .05. The case of $\beta > .99$ is also considered for these values of p_1.

A New Explanation of Non-Normal Dispersion. HILDA GEIRINGER, Bryn Mawr College.

The starting point of the *Lexis theory* consists in this fact: It is to be expected, on the average, that two expressions Σ and Σ' which can be computed from the results of $m \cdot n$ observations are equal, provided that the corresponding $m \cdot n$ chance variables $x_{\mu\nu}$ are equally and independently distributed. Let a_μ be the average $a_\mu = 1/n \sum_{\nu=1}^{n} x_{\mu\nu}$ and a the average of the a_μ ($\mu = 1, \cdots, m$). Then

$$\sum{}' = \frac{m}{mn - 1} s'^2 = \frac{m}{mn - 1} \frac{\sum_\mu \sum_\nu (x_{\mu\nu} - a)^2}{mn}$$

$$\sum = \frac{m}{m - 1} s^2 = \frac{m}{m - 1} \frac{\sum_\mu (a_\mu - a)^2}{m}.$$

We see, however, that rows and columns do not play the same role here because Σ depends only on the a_μ, the average values of the rows. If the observed value of Σ happens to be larger (smaller) than the value of Σ', we speak of supernormal (subnormal) dispersion. It is well known that supernormal dispersion can be explained by assuming that the $m \cdot n$ theoretical populations are only equal "by rows" but not by columns (there are m different distributions); in the same way one can explain the case of subnormal dispersion by admitting that the distributions are equal "by columns," but not by rows.

Another explanation which may sometimes seem more plausible is the following: All the $m \cdot n$ distributions are supposed to be equal, but we *omit the assumption of mutual independence*. Then one can prove that the supernormal or subnormal dispersion corresponds respectively to an appropriately defined "positive" or "negative correlation." The fact that normal dispersion occurs rather rarely in social questions is then reflected by the idea that social phenomena are in fact not independent of each other but are usually only assumed so for the purpose of simplicity. In that way the more frequent occurrence of supernormal dispersion likewise finds an adequate explanation.