THE ACCURACY OF SAMPLING METHODS IN ECOLOGY

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1. Introduction. For a number of years journals on ecology have contained articles on sampling techniques for estimating the distribution of common species of plants in various regions. Although much experimental work has been done on this problem and although the problem is essentially statistical in nature, no theoretical work of any consequence seems to have been attempted. This paper considers the question of the relative accuracy of common sampling methods from a theoretical point of view by means of geometrical probability and statistical distribution theory.

There are three common methods of sampling used by ecologists. They are designated by the names of coverage, abundance, and frequency. For each of these methods of sampling, there are two common choices of sampling unit, namely, the quadrat and the transect. By the coverage of a species in a region is meant the total area covered by the projection on the ground of the crowns of the plants of this species. By abundance is meant the total number of plants of this species in the region. By frequency is meant the number of sampling units in the region in which at least one plant of the species occurs. A quadrat is a sampling unit in the form of a square, usually several yards on a side. A transect is a sampling unit in the form of a straight line, coverage in this case being the length of line covered by the projection of the crowns.

In this paper it will be assumed that plants possess circular crowns. Further, it will be assumed that plant species distribute themselves at random in the region to be sampled. This is not necessarily the case, since there is often a tendency for plants of a given species to distribute themselves at random or otherwise in groups rather than as single plants. However, if sampling units are somewhat comparable in size, the relative accuracy of these methods of sampling based on a random distribution would be expected to hold fairly well for distributions somewhat removed from this ideal situation. Further, by the proper choice of sampling unit size, some non-random distributions behave very much as though they were random.

The accuracy of a sampling method may be measured by the variance of the estimate of the quantity which is of interest. Here interest will be centered on the total coverage of a given species in the region being sampled. Thus, two sampling methods will be said to be equally accurate for coverage if they produce equal variances for the estimate of total coverage.

The quadrat unit of sampling will be considered first for the three methods of sampling, after which the transect unit will be considered.

2. Quadrat coverage. Let the region to be sampled be a square B units on a side. Let there be n quadrats, each a square A units on a side, distributed at random in the region. Finally, let the total number of plants of the species in question in the region be N, with the distribution of the radius of their crowns given by a frequency function $f(r)$ whose explicit form will be specified later.

First, consider a single plant of radius r and a single quadrat. The problem is to determine the variance of a, the area of that part of the plant lying in the quadrat. Now the probability that this plant will be found in any particular part of the region is obtained by treating the plant as a circle of radius r which is thrown at random in the region and then applying geometrical probability to the position of the center of the circle. Thus, considering only those situations when the center of the circle lies in the region, the probability that the circle will cover an area of at least $a > \frac{1}{2}\pi r^2$ units of the quadrat is given by the ratio of the area of the subregion inside the quadrat whose boundary is the locus of centers of circles of radius r which have precisely a units of their area inside the quadrat, to the area of the region. Probabilities of this type may be treated as functions of a. The expressions below for such probabilities follow directly from Fig. 1, which displays one corner of the quadrat.

$$\begin{aligned} &P_1[a < \text{area} < \pi r^2] = 4S_1/B^2, && a \geq \tfrac{1}{2}\pi r^2 \\ &P_2[0 < \text{area} < a] = 4S_2/B^2, && a < \tfrac{1}{2}\pi r_2 \\ &P_3[a = \pi r^2] = (A - 2r)^2/B^2, \\ &P_4[a = 0] = P_4. \end{aligned} \tag{1}$$

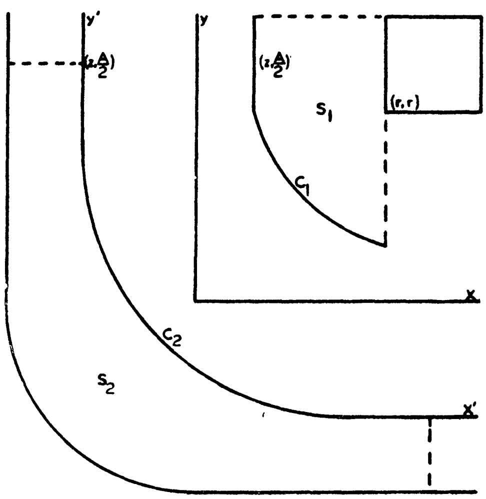

FIG. 1

Now

$$S_1 = (A - r)(r - z) - \int_z^r y\,dx, \tag{2}$$

where y is the ordinate of curve C_1. Likewise

$$S_2 = A(r + z) - z^2 + \tfrac{1}{4}\pi r^2 + \int_0^{-z+\sqrt{r^2-z^2}} y'\,dx', \tag{3}$$

where y' is the ordinate of curve C_2 with respect to the primed axes and z is negative. Using the formula for the area of a segment of a circle, the equation of C_1 is easily found to be

$$x\sqrt{r^2 - x^2} + r^2 \sin^{-1}\frac{x}{r} + y\sqrt{r^2 - y^2} + r^2 \sin^{-1}\frac{y}{r} = a, \qquad x^2 + y^2 \geq r^2 \tag{4}$$

$$x\sqrt{r^2 - x^2} + r^2 \sin^{-1}\frac{x}{r} + y\sqrt{r^2 - y^2} + r^2 \sin^{-1}\frac{y}{r} + 2xy + \tfrac{1}{2}\pi r^2 = 2a, \qquad x^2 + y^2 < r^2 \tag{5}$$

where the value of a is given in terms of z by

$$z\sqrt{r^2 - z^2} + r^2 \sin^{-1}\frac{z}{r} + \frac{\pi r^2}{2} = a. \tag{6}$$

The equation of C_2 is given by (5) with z negative. These equations do not permit the solution of y in terms of x; however, they can be thrown into the following parametric form with t as parameter:

$$\begin{aligned} x &= r \sin\left\{\frac{t}{2} + \frac{1}{2}\cos^{-1}\left[\frac{a/r^2 - t}{\sin t}\right]\right\}, \\ y &= r \sin\left\{\frac{t}{2} - \frac{1}{2}\cos^{-1}\left[\frac{a/r^2 - t}{\sin t}\right]\right\}, \end{aligned} \tag{4'}$$

$$\begin{aligned} x &= r \sin\left\{\frac{t}{2} + \frac{1}{2}\cos^{-1}\left[\frac{2a/r^2 - \pi/2 + \cos t - t}{1 + \sin t}\right]\right\}, \\ y &= r \sin\left\{\frac{t}{2} - \frac{1}{2}\cos^{-1}\left[\frac{2a/r^2 - \pi/2 + \cos t - t}{1 + \sin t}\right]\right\}. \end{aligned} \tag{5'}$$

Since a may be treated as a parameter, equations (4) and (5), and hence (4′) and (5′), represent a system of curves C_1 and C_2. Unfortunately, equations (4′) and (5′) are not convenient for integration purposes either, but they are convenient for numerical work. This system of curves can be approximated satisfactorily by means of simpler curves. One set of such approximating curves is the following system of circles:

$$(x - r)^2 + (y - r)^2 = (r - z)^2, \qquad z \geq 0 \tag{7}$$

$$(x - \sqrt{r^2 - z^2})^2 + (y - \sqrt{r^2 - z^2})^2 = (-z + \sqrt{r^2 - z^2})^2, \qquad z < 0. \tag{8}$$

Although inequalities may be obtained between the approximating and true curves, these are of little value for determining the accuracy of essential moments obtained by using these approximating curves; therefore the accuracy of these approximating curves will be judged empirically by means of Fig. 2 in which the true curves are plotted by means of (4′) and (5′) for $z = .6, .3, 0, -.3, -.6, -.9$, of r with solid lines and the approximating circles (7) and (8) with broken lines. Although the circles appear to fit poorly for relatively large positive values of z, this is not serious because these values occur increasingly less often than other values of z for a random circle and because the use of these circles is confined to the rate of change of area bounded by these curves and the lines $x = r$ and $y = r$. Since the true curves are approaching the circles with decreasing positive z, their rate of change of area would not differ much from that for the circles even though the circles include larger areas for a given z. In the paragraph following (11), further evidence will be presented to show that for the computation of the first two moments of a, these curves give a good approximation.

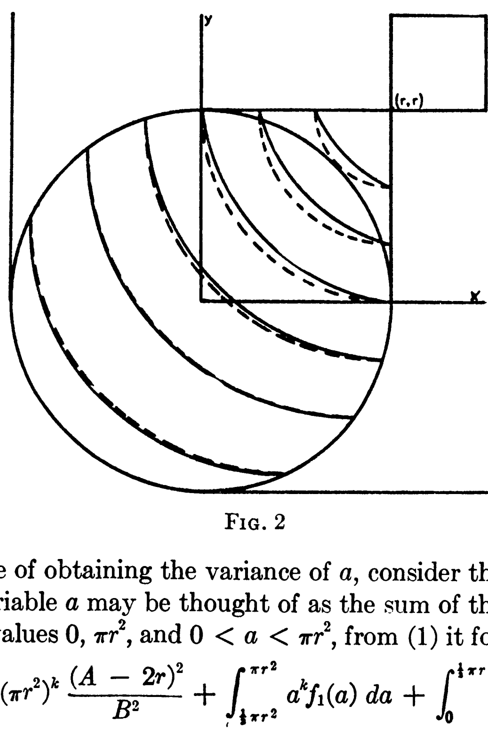

FIG. 2

For the purpose of obtaining the variance of a, consider the expected value of a^k. Since the variable a may be thought of as the sum of three variables which assume only the values 0, πr^2, and $0 < a < \pi r^2$, from (1) it follows that

$$E(a^k) = (\pi r^2)^k \frac{(A - 2r)^2}{B^2} + \int_{\frac{1}{2}\pi r^2}^{\pi r^2} a^k f_1(a)\, da + \int_0^{\frac{1}{2}\pi r^2} a^k f_2(a)\, da,$$

where $f_1(a)$ and $f_2(a)$ are the frequency functions for $z \geq 0$ and $z < 0$ respectively. Now since

$$P_1[a < \text{area} < \pi r^2] = \int_a^{\pi r^2} f_1(a)\, da,$$

and

$$P_2[0 < \text{area} < a] = \int_0^a f_2(a)\, da,$$

it follows from (1), (2), and (3) that

$$f_1(a)\,da = -dP_1 = -4\frac{dS_1}{B^2} = \frac{4}{B^2}\left[A - r + \frac{d}{dz}\int_z^r y\,dx\right]dz,$$

and

$$f_2(a)\,da = dP_2 = 4\frac{dS_2}{B^2} = \frac{4}{B^2}\left[A - 2z + \frac{d}{dz}\int_0^{-z+\sqrt{r^2-z^2}} y'\,dx'\right]dz.$$

Using the approximating curves (7) and (8), these integrals become:

$$\int_z^r y\,dx = r(r-z) - \frac{\pi}{4}(r-z)^2,$$

$$\int_0^{-z+\sqrt{r^2-z^2}} y'\,dx' = \left(1 - \frac{\pi}{4}\right)(-z + \sqrt{r^2 - z^2})^2.$$

Hence,

$$f_1(a)\,da = \frac{4}{B^2}\left[A - 2r\left(1 - \frac{\pi}{4}\right) - \frac{\pi}{2}z\right]dz,$$

and

$$f_2(a)\,da = \frac{4}{B^2}\left[A - 2z - 2\left(1 - \frac{\pi}{4}\right)\left(2\sqrt{r^2 - z^2} - \frac{r^2}{\sqrt{r^2 - z^2}}\right)\right]dz.$$

Hence,

$$E(a^k) = (\pi r^2)^k\,\frac{(A - 2r)^2}{B^2} + \frac{4}{B^2}\int_0^r a^k\left[A - 2r\left(1 - \frac{\pi}{4}\right) - \frac{\pi}{2}z\right]dz$$
$$+ \frac{4}{B^2}\int_{-r}^0 a^k\left[A - 2z - 2\left(1 - \frac{\pi}{4}\right)\left(2\sqrt{r^2 - z^2} - \frac{r^2}{\sqrt{r^2 - z^2}}\right)\right]dz.$$

Substituting the value of a from (6), standard integrals give the following values for $k = 1$ and $k = 2$:

(9) $$E(a) = \frac{\pi r^4}{B^2}\left[\left(\frac{A}{r}\right)^2 + .13\right],$$

(10) $$E(a^2) = \frac{\pi^2 r^6}{B^2}\left[\left(\frac{A}{r}\right)^2 + 1.15\left(\frac{A}{r}\right) + .46\right],$$

where certain constants involving π have been evaluated to two decimals.

If circles with centers outside the region but overlapping the region were also measured, then geometrical probability would give the following value for $E(a)$:

(11) $$E(a) = \frac{\pi r^4}{B^2}\left(\frac{A}{r}\right)^2.$$

Since in (1) only circles with centers inside the region are assumed measured, $E(a)$ will be only very slightly larger than this last value; consequently the approximate result in (9) is only slightly in error. For a quadrat ten yards on a side and plants averaging two yards in diameter, the error is in the neighborhood of one tenth of one percent; consequently formula (10) may be expected to be quite accurate as well. Another approximating system of curves lying largely on the opposite side of the true curves from the circles gave formula (10) with .46 replaced by .26, both of which have a negligible effect on $E(a^2)$ for ordinary applications.

Formula (10) was derived on the assumption that the same circle was thrown repeatedly at random in the region. Consider now the situation when the circle varies in size according to the frequency function $f(r)$. Treating a and r as two statistical variables, their joint frequency function may be expressed as:

$$f(a, r) = f(r)f(a \mid r),$$

where $f(a \mid r)$ is the frequency function of a when r has the fixed value r. Letting $\mathcal{E}(a^k)$ represent the expected value of a^k when r is permitted to vary according to $f(r)$,

$$\begin{aligned} \mathcal{E}(a^k) &= \int \int a^k f(a, r)\, da\, dr \\ &= \int f(r) \int a^k f(a \mid r)\, da\, dr \\ &= \int f(r) E(a^k)\, dr, \end{aligned}$$

where all integrals are taken over the regions for which a and r are defined. Consequently, from (10) and (11)

$$\mathcal{E}(a^2) = \frac{\pi^2}{B^2}\left[A^2\nu_4 - 1.15A\nu_5 + .46\nu_6\right],$$

and

$$\mathcal{E}(a) = \frac{\pi}{B^2} A^2\nu_2, \tag{12}$$

where the ν's represent moments of r. Hence the variance of a is given by:

$$\sigma_a^2 = \frac{\pi^2}{B^2}[A^2\nu_4 - 1.15A\nu_5 + .46\nu_6 - A^4\nu_2^2/B^2]. \tag{13}$$

Finally, let there be n quadrats, N circles whose radii vary according to $f(r)$, and let the total area of quadrat covered by the N circles be denoted by s. Then

$$\mathcal{E}(s) = nN\mathcal{E}(a), \tag{14}$$

and

(15) $$\sigma_s^2 = nN\sigma_a^2$$

The purpose of measuring s is to use it to obtain an estimate of T, the total area of the N circles. But

(16) $$T = N\mathcal{E}(\pi r^2) = N\pi\nu_2 .$$

Substituting the value of ν_2 from (12) and using (14),

$$T = B^2\mathcal{E}(s)/nA^2.$$

Hence an estimate of T will be given by

(17) $$T_1 = B^2 s/nA^2.$$

Using (15) and (13), the variance of this estimate will be given by

(18) $$\sigma_{T_1}^2 = \frac{\pi^2 B^2 N}{nA^2}\left[\nu_4 - 1.14\frac{\nu_5}{A} + .46\frac{\nu_6}{A^2} - \frac{A^2}{B^2}\nu_2^2\right].$$

3. Quadrat abundance. In this method the sampler merely counts the number of plants of the given species in each quadrat. Although this method was designed to estimate the total number of plants, it may be adapted to estimate total coverage as well. Since it is the practice to count a plant as lying in the quadrat only if its stem is in the quadrat, the probability that this event will occur is given by:

(19) $$P_q = A^2/B^2.$$

Since there are n quadrats and N circles, the number of circles with centers lying in quadrats, which will be denoted by s, will follow the binomial distribution; hence

(20) $$\mathcal{E}(s) = nNP_q ,$$

and

(21) $$\sigma_s^2 = nNP_q(1 - P_q).$$

From (16) and (20) it follows that

$$T = \pi\nu_2\mathcal{E}(s)/nP_q .$$

Therefore an estimate of T will be given by

(22) $$T_2 = \pi B^2 m_2 s/nA^2,$$

where m_2 is a sample estimate of ν_2 obtained by measuring the diameters of k plants and calculating their mean area. Since m_2 and s are independent, a standard formula for the variance of a product of two independent variables may be applied to give

$$\sigma_{T_2}^2 = \left[\frac{\pi B^2}{nA^2}\right]^2 [\mathcal{E}(m_2^2)\sigma_s^2 + \mathcal{E}^2(s)\sigma_{m_2}^2].$$

But

$$\sigma^2_{m_2} = \frac{\nu_4 - \nu_2^2}{k},$$

and

$$\mathcal{E}(m_2^2) = \frac{\nu_4 - \nu_2^2}{k} + \nu_2^2.$$

Consequently, with the aid of (19), (20), and (21)

$$\sigma^2_{T_2} = \pi^2 \left\{ \frac{N}{n} \frac{B^2 - A^2}{A^2} \left[\nu_2^2 + \frac{\nu_4 - \nu_2^2}{k} \right] + \frac{N^2}{k} [\nu_4 - \nu_2^2] \right\}. \tag{23}$$

4. Quadrat frequency. In this method the sampler records the number of quadrats observed and the number of those quadrats which contained at least one plant of the given species. Given N plants, the probability p that at least one of them will be found in a given quadrat is given by

$$p = 1 - (1 - P_q)^N,$$

where P_q is given in (19). For n quadrats the expected number of quadrats in which at least one plant will be found is therefore np. Letting w represent the number of such quadrats,

$$\mathcal{E}(w) = n[1 - (1 - P_q)^N].$$

Solving for N,

$$N = \log\left[1 - \frac{\mathcal{E}(w)}{n}\right] \Big/ \log\,[1 - P_q].$$

Consequently, from (16) an estimate of T will be given by

$$T_3 = \pi m_2 \log\left[1 - \frac{w}{n}\right] \Big/ \log\,[1 - P_q].$$

Neither the mean nor the variance of T_3 will exist because T_3 is a discrete variable which becomes infinite for $w = n$. Unless the density of the species is very low, values of w near n will occur quite often and hence cause T_3 to vary widely. Consequently the frequency method will be inferior to the abundance method except when the mean density is low, in which case the abundance method is practically as easy to apply. Because the frequency method is obviously inferior to the abundance method, it will not be considered further here.

5. Transect coverage. In this type of sampling a line is laid down and the length of line covered by a plant of the species in question is recorded. Let there be n such lines, each L units in length.

If a circle of radius r is thrown at random in the region, it will cross a line only if its center lies within the subregion, indicated in Fig. 3, composed of a rectangle of width $2r$ and length L with semi-circular ends. From this figure it is clear that the probability of the circle intersecting some positive length less than z of the line is given by four times the shaded area s_3, divided by the area of the region. From this same diagram the following equations of the indicated curves result:

$$C_1:\quad \left(x - \frac{L}{2}\right)^2 + y^2 = r^2, \qquad \frac{L}{2} < x < \frac{L}{2} + r$$

$$C_2:\quad y = \sqrt{r^2 - z^2/4}, \qquad 0 \leq x \leq \frac{L}{2} - \frac{z}{2}$$

$$C_3:\quad \left(x - \frac{L}{2} + z\right)^2 + y^2 = r^2, \qquad \frac{L}{2} - \frac{z}{2} < x < \frac{L}{2} - z + r$$

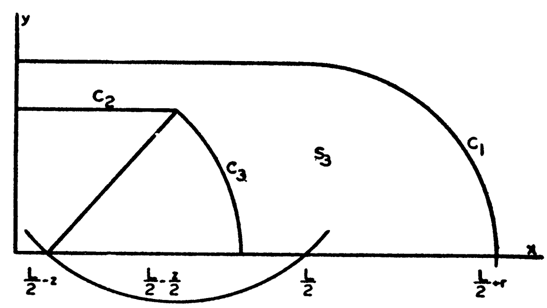

FIG. 3

Applying geometrical probability,

$$P_5\,[0 < \text{intercepted length} < z] = 4\,\frac{S_3}{B^2} = \int_0^z f(z)\,dz,$$

where $f(z)$ is the frequency function for z. But

$$S_3 = \frac{L}{2}\,[r - \sqrt{r^2 - z^2/4}] + \frac{z}{2}\sqrt{r^2 - z^2/4} + \frac{\pi r^2}{4} - \int_{\frac{1}{2}L-\frac{1}{2}z}^{\frac{1}{2}L-z+r} \sqrt{r^2 - \left(x - \frac{L}{2} + z\right)^2}\,dx.$$

Standard integrals give

$$P_5 = \frac{4}{B^2}\left\{\frac{L}{2}\,[r - \sqrt{r^2 - z^2/4}] + \tfrac{3}{4}z\sqrt{r^2 - z^2/4} + \frac{r^2}{2}\sin^{-1}\frac{z}{2r}\right\}.$$

Consequently,

$$f(z)\,dz = dP_5 = \frac{4}{B^2}\left\{\frac{8r^2 + Lz - 3z^2}{8\sqrt{r^2 - z^2/4}}\right\}dz.$$

From this relation the following moments are readily obtained:

$$E(z) = \pi r^2 L/B^2$$

$$E(z^2) = [\tfrac{16}{3} L r^3 - \pi r^4]/B^2.$$

For variable r these formulas become:

$$\mathcal{E}(z) = \pi \nu_2 L/B^2,$$

$$\mathcal{E}(z^2) = [\tfrac{16}{3} L \nu_3 - \pi \nu_4]/B^2,$$

$$\sigma_z^2 = [\tfrac{16}{3} L \nu_3 - \pi \nu_4]/B^2 - \pi^2 \nu_2^2 L^2/B^4.$$

Let ξ denote total z for N circles and n quadrats, then

$$\mathcal{E}(\xi) = nN\pi \nu_2 L/B^2, \tag{24}$$

and

$$\sigma_\xi^2 = nN\{[\tfrac{16}{3} L \nu_3 - \pi \nu_4]/B^2 - \pi^2 \nu_2^2 L^2/B^4\}.$$

From (16) and (24)

$$T = B^2 \mathcal{E}(\xi)/nL.$$

Hence an estimate of T will be given by

$$T_4 = B^2 \xi/nL, \tag{25}$$

and its variance will be given by

$$\sigma_{T_4}^2 = \frac{N}{n}\left\{\frac{B^2}{L^2}[\tfrac{16}{3} L \nu_3 - \pi \nu_4] - \pi^2 \nu_2^2\right\}. \tag{26}$$

6. Transect abundance. Since the probability, P_t, of a circle of radius r intersecting a line of length L is the area of the band with semi-circular ends indicated in Fig. 3, divided by the area of the region,

$$P_t = [2rL + \pi r^2]/B^2.$$

Hence, letting s represent the total number of intersections, as in the case of quadrat abundance,

$$\begin{aligned} E(s) &= nNP_t, \\ E(s^2) &= nNP_t(1 - P_t) + n^2 N^2 P_t^2, \\ \mathcal{E}(s) &= nN[2L\nu_1 + \pi \nu_2]/B^2, \\ \mathcal{E}(s^2) &= \frac{nN}{B^4}\{B^2[2L\nu_1 + \pi\nu_2] + [nN - 1][4L^2\nu_2 + 4\pi L \nu_3 + \pi^2 \nu_4]\}. \end{aligned} \tag{27}$$

For simplicity of formula if $nN - 1$ is replaced by nN, the variance of s becomes

$$\begin{aligned} \sigma_s^2 = \frac{nN}{B^4}\{B^2[2L\nu_1 + \pi\nu_2] \\ + nN[4L^2(\nu_2 - \nu_1^2) + 4\pi L(\nu_3 - \nu_1\nu_2) + \pi^2(\nu_4 - \nu_2^2)]\}. \end{aligned} \tag{28}$$

From (16) and (27)

$$T = \frac{\pi \nu_2 B^2 \mathfrak{E}(s)}{n[2L\nu_1 + \pi\nu_2]}.$$

Hence an estimate of T will be given by

$$T_5 = \frac{\pi B^2 s}{n[\pi + 2L\alpha]},$$

where α is an estimate of ν_1/ν_2. In order to obtain a satisfactory estimate of ν_1/ν_2, data for the distribution of diameters of common California shrubs were analyzed. It was found that Pearson's type three curve gave an excellent fit. Since the moments of this type distribution are given by

$$\nu_m = \rho^m \prod_{j=1}^{m-1} [1 + jV^2], \tag{29}$$

where ρ is the mean and V is the coefficient of variation, σ/ρ, then $\nu_1/\nu_2 = 1/\rho\theta$, where $\theta = 1 + V^2$, and the above estimate becomes

$$T_5 = \frac{B^2 s}{n}\left[1 - \frac{2L}{\pi\rho\theta + 2L}\cdot\frac{1}{1 + \varphi}\right], \tag{30}$$

where $\varphi = \pi\theta[\bar{r} - \rho]/[\pi\rho\theta + 2L]$ and where $1/\bar{r}$ is chosen as an estimate of $1/\rho$. Since $\bar{r}$ will be approximately normally distributed for samples considerably smaller than those usually taken to find $\bar{r}$, assume that it is normally distributed with mean zero and variance $\sigma^2\pi^2\theta^2/k[\pi\rho\theta + 2L]^2$. Since L is large relative to σ and since k will usually exceed twenty-five, this variance is very small, and hence the probability of φ exceeding one numerically is extremely small. Although the value $\varphi = -1$ is theoretically possible on the normality assumption, such a value would not permit the existence of either the mean or variance of $1/[1 + \varphi]$. However, if φ is restricted to a range of, say, ten standard deviations about zero, then $|\varphi| < 1$ for ordinary conditions and the variance will exist. Further, because φ assumes such small values, with this finite range the variance of $1/[1 + \varphi]$ is the same as the variance of φ itself if higher powers in this variance are neglected. Since s and φ are independent, the same product formula that was used for quadrat abundance may be employed here, together with the various approximations indicated above, to yield

$$\begin{aligned}\sigma^2_{T_5} = \left(\frac{N\pi\rho\theta}{2L + \pi\rho\theta}\right)^2 \Bigg\{ \Bigg[& \frac{B^2}{nN}(2L\nu_1 + \pi\nu_2) \\ & + 4L^2(\nu_2 - \nu_1^2) + 4\pi L(\nu_3 - \nu_1\nu_2) + \pi^2(\nu_4 - \nu_2^2)\Bigg] \\ & \cdot\left[1 + \frac{4L^2V^2}{k(2L + \pi\rho\theta)^2}\right] + \frac{4L^2V^2}{k(2L + \pi\rho\theta)^2}[2L\nu_1 + \pi\nu_2]^2\Bigg\}.\end{aligned} \tag{31}$$

7. Comparison of methods. Formulas (18) and (23) may be compared for relative accuracy of these two quadrat methods of measuring coverage. Formulas (26) and (31) may be compared for relative accuracy of these two transect methods of measuring coverage. Finally, formulas (18) and (26), and formulas (23) and (31), may be compared to determine what length transect will give the same accuracy as a quadrat of given size. All such comparisons will necessarily have to be done numerically by considering typical values for the parameters involved. The moments occurring in these formulas are expressible by means of (29) in terms of ρ and V if the form of $f(r)$ is that assumed here. For the data analyzed to determine $f(r)$ it was found that V was approximately 1/3. These numerical comparisons will not be made here.

The question of which type of sampling method should be employed now becomes a question of balancing relative ease or cost of sampling against size samples needed to produce equivalent accuracy as determined by means of these formulas. If total frequency is desired rather than total coverage, these formulas may be altered to handle this situation as well.