ON THE ANALYSIS OF A CERTAIN SIX-BY-SIX FOUR-GROUP LATTICE DESIGN[1]

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1. Introduction. The lattice consists of groups of randomized incomplete blocks with certain restrictions being imposed on the randomization within each group, and the number of varieties is a perfect square. For example, if the number of varieties is $k^2 = 36$, then the orthogonal groups for a triple lattice, not considering randomizing within the blocks or between blocks, are as follows: (the numbers signify varieties).

GROUP X

Blocks						
(1)	1	2	3	4	5	6
(2)	7	8	9	10	11	12
(3)	13	14	15	16	17	18
(4)	19	20	21	22	23	24
(5)	25	26	27	28	29	30
(6)	31	32	33	34	35	36

GROUP Y

Blocks						
(1)	1	7	13	19	25	31
(2)	2	8	14	20	26	32
(3)	3	9	15	21	27	33
(4)	4	10	16	22	28	34
(5)	5	11	17	23	29	35
(6)	6	12	18	24	30	36

GROUP Z

Blocks						
(1)	1	8	15	22	29	36
(2)	2	9	16	23	30	31
(3)	3	10	17	24	25	32
(4)	4	11	18	19	26	33
(5)	5	12	13	20	27	34
(6)	6	7	14	21	28	35

This design is constructed so that no variety appears with another variety more than once in the same block. This important characteristic makes the analysis simple, as it enables the results to be treated as a factorial design. The analysis is well described by Yates [3, 4, 5] and Cochran [1].

Suppose another group, U, is now formed from a six by six lattice, for example, the following group:

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[1] Certain of the ideas presented here are embodied in the author's unpublished doctoral thesis by the same title, Library, George Washington University, Washington, D. C., 1943.

[2] The author wishes to express his appreciation to W. G. Cochran of Iowa State College, who advised freely in the preparation of the original thesis, and to Frank M. Weida of George Washington University.

GROUP U

Blocks						
(1)	31	26	21	16	11	6
(2)	1	32	27	22	17	12
(3)	7	2	33	28	23	18
(4)	13	8	3	34	29	24
(5)	19	14	9	4	35	30
(6)	25	20	15	10	5	36

The important characteristic, that no variety appears with another variety in the same block more than once, does not hold. For example, varieties 1 and 22 appear together in both groups Z and U.

It is the purpose of this paper to develop the statistical method for the analysis of such a design, where each group is duplicated, and to apply the results to an actual problem. The least square solution, as developed here, uses only the intra-block information to correct the varieties for block effects. In a second article the solution using both intra- and inter-block information will be given.

2. Estimation of the block and varietal effects. It is reasonable to assume in varietal trials that the general mean, and any particular block and variety effects, operate additively to produce the true mean of y associated with this block and variety. In particular, if y_{eij} is the yield of the plot for the jth variety in the ith block of the eth replicate, the following hypothesis may be set up, namely:

$$y_{eij} = \mu + \rho_e + \beta_{ei} + \nu_j + \epsilon_{eij}. \tag{1}$$

Where μ is the true or population mean yield, ρ_e is the population replicate effect of the eth replicate, β_{ei} is the population block effect of the ith block in the eth replicate, ν_j is the population variety effect of the jth variety, and ϵ_{eij} is the experimental error of the eij plot. Since the design has eight replicates, that is, each group is duplicated, the block effects may be estimated from unpaired and paired replicates or partners.

It is assumed that the ϵ_{eij} are independently and normally distributed with common variance $\frac{1}{W}$. Without loss of generality, it also may be assumed that the sum of the replicate effects, the sum of the block effects within any replication, and the sum of the variety effects are respectively equal to zero.

The parameters are estimated by the method of least squares, subject to the restrictions stated in the preceding paragraph. This involves choosing the parameters so that

$$\begin{aligned} &S\left(y_{eij} - mx_1 - r_e x_{2e} - \frac{b_{ei} - b'_{ei}}{2} x_{3ei} - \frac{b_{ei} + b'_{ei}}{2} x_{4ei} - v_j x_{5j}\right)^2 \\ &\quad + \lambda_1 \sum_1^8 r_e + \sum_{e=1}^8 \lambda_{2e} \sum_{i=1}^6 \frac{b_{ei} - b'_{ei}}{2} + \sum_{e=1}^8 \lambda_{3e} \sum_{i=1}^6 \frac{b_{ei} + b'_{ei}}{2} + \lambda_4 \sum_{j=1}^{36} V_j \end{aligned} \tag{2}$$

is a minimum.[3] Here y_{eij} is the dependent variate, and $x_1, \cdots, x_{5j}$ are the independent variates. In ordinary regression problems, the values of the x variates, as well as the y variate, constitute a part of the original data. However, in this case the y variate only is given, and the x variates must be constructed. Thus, for the design, one takes $x_1 = 1$ for all values; $x_{2e} = 1$ for all values in replicate e, but zero elsewhere; $x_{3ei} = 1$ for all values in the ith block of the eth replicate and -1 for all values in its partner, but zero elsewhere; $x_{4ei} = 1$ for all values in the ith block of the eth replicate and also 1 in its partner, but zero elsewhere; and $x_{5j} = 1$ where variety j occurs, but zero elsewhere.

One now takes the partial derivatives of the above equation with respect to the parameters and forms the normal equations. It can be shown that $(\lambda_1, \cdots, \lambda_4)$ are each zero. The normal equations not involving λ's are:

Leading term

$$m \qquad Nm + k^2 \sum_{e=1}^{8} r_e + 2k \sum_{e=1}^{8} \sum_{i=1}^{6} \frac{b_{ei} - b'_{ei}}{2} + 2k \sum_{e=1}^{8} \sum_{i=1}^{6} \frac{b_{ei} + b'_{ei}}{2} + 8 \sum_{j=1}^{36} V_j = G.$$

$$(3) \qquad r_e \qquad k_2 m + k^2 r_e + k \sum_{i=1}^{6} b_{ei} + \sum_{j=1}^{36} v_j = R_e .$$

$$\frac{b_{ei} - b'_{ei}}{2} \qquad k(r_e - r'_e) + 2k \frac{(b_{ei} - b'_{ei})}{2} = B_{ei} - B'_{ei} .$$

Equations having $\frac{b_{ei} + b'_{ei}}{2}$ as leading terms.

Equations having v_j as leading terms.

In the above, N is the total number of values, k is the number of plots in a block, r_e is the eth replicate effect, b_{ei} is the ith block effect in the eth replicate, v_j is the jth variety effect, G is the total sum of all values, R_e is the sum of the values in the eth replicate, B_{ei} is the sum of values in the ith block of the eth replicate and v_j is the sum of the yields of the jth variety. The primes denote similar values of the partner terms.

Using the restrictions

$$(4) \qquad \sum_{e=1}^{8} r_e = \sum_{e=1}^{8} \sum_{i=1}^{6} \frac{b_{ei} + b'_{ei}}{2} = \sum_{e=1}^{8} \sum_{i=1}^{6} \frac{b_{ei} - b'_{ei}}{2} = \sum_{j=1}^{36} v_j = 0,$$

the values of the following parameters are directly obtainable:

$$(5) \qquad m = \frac{G}{N}, \qquad r_e = \frac{R_e}{k^2} - \frac{G}{N} \quad \text{and} \quad \frac{b_{ei} - b'_{ei}}{2} = \frac{B_{ei} - B'_{ei}}{2k} - \frac{r_e - r'_e}{2}.$$

[3] **S, for summation, will be used to represent summation over all values. Σ will be used in a more restricted sense.**

The values of the $\frac{b_{ei} + b'_{ei}}{2}$ and v_i effects cannot be obtained directly. In order to simplify the solution of these equations, only the mean, confounded blocks, and the varietal effects will be used. The results later will be corrected for replicate effects. If each of the yields is added to the corresponding yield in its partner, one gets an equation of the form

$$y_{ik} = \mu + \beta_i + \nu_k + \epsilon_{ik} \tag{6}$$

where μ, β_i, and ν_k are now twice their original values. These parameters are estimated by the method of least squares subject to the restrictions previously given. To distinguish them from the estimates derived in equation (3) they are designated with double primes (″). If B_{ix} is the total for block i in group X, that is from both pairs, and similarly for the other groups, the normal equations are:

Leading term

$$\begin{array}{lll} m'' & 144m'' & = G. \\ b''_{x1} & 6m'' + 6b''_{x1} + (v''_1 + v''_2 + v''_3 + v''_4 + v''_5 + v''_6) & = B_{1x}. \\ & \cdots & \\ b''_{u6} & 6m'' + 6b''_{u6} + (v''_5 + v''_{10} + v''_{15} + v''_{20} + v''_{25} + v''_{36}) & = B_{u6}. \\ v''_1 & 4m'' + b''_{x1} + b''_{y1} + b''_{z1} + b''_{u2} + 4v''_1 & = V_1. \\ & \cdots & \\ v''_{36} & 4m'' + b''_{x6} + b''_{y6} + b''_{z1} + b''_{u3} + 4v''_{36} & = V_{36}. \end{array} \tag{7}$$

Let T_{xi} be the total of all the varieties appearing in block 1 of group X from all the replicates; T_{yi} the total of all values appearing in block i of group Y from all the replicates, etc. Also

$$C_{xi} = 4B_{xi} - T_{xj} \qquad C_{yi} = 4B_{yi} - T_{yj} \quad \text{etc.} \tag{8}$$

Solving the equations:

$$b''_{xi} = \frac{C_{xi}}{18} \qquad b''_{yi} = \frac{C_{yi}}{18}$$

and

$$\begin{aligned} b''_{z1} &= \tfrac{1}{432}[(25C_{z1} + C_{z3} + C_{z5}) + 3(C_{u2} + C_{u4} + C_{u6})] \\ b''_{z2} &= \tfrac{1}{432}[(25C_{z2} + C_{z4} + C_{z6}) + 3(C_{u1} + C_{u4} + C_{u5})] \\ &\cdots \\ b''_{u6} &= \tfrac{1}{432}[(25C_{u6} + C_{u2} + C_{u4}) + 3(C_{z1} + C_{z3} + C_{z5})]. \end{aligned} \tag{9}$$

The values of b'''s calculated as above contain the replicate effects. To correct for this, adjust the values so that the sum of the block effects for each replicate is zero. From the corrected b'' values and the normal equations with the v'''s as leading terms, the corrected varietal sums are calculated. These are:

$$4v''_j + 4m'' = V_j - \Sigma b''_{ei} \text{ (sum over blocks in which } v_j \text{ appears)} \tag{10}$$

where $4v''_j + 4m''$ is the corrected varietal total for the jth variety.

3. Test of significance and the analysis of variance. If the x's have the previously defined values, the following identity occurs:

$$Sy^2 = mG + \sum_{e=1}^{8} r_e R_e + \sum_{e=1}^{8}\sum_{i=1}^{6} \frac{(b_{ei} - b'_{ei})}{2}(B_{ei} - B'_{ei})$$

$$(11) \qquad + \sum_{e=1}^{8}\sum_{i=1}^{6} \frac{(b_{ei} + b'_{ei})}{2}(B_{ei} + B'_{ei}) + \sum_{j=1}^{36} v_j V_j$$

$$+ S\left(y - mx_1 - r_e x_{2e} - \frac{b_{ei} - b'_{ei}}{2} x_{3i} - \frac{b_{ei} + b'_{ei}}{2} x_{4i} - v_j x_{5j}\right)^2.$$

In the equation (11)

$$(12) \qquad mG + \sum_{e=1}^{8} r_e R_e + \sum_{e=1}^{8}\sum_{i=1}^{6} \frac{(b_{ei} - b'_{ei})}{2}(B_{ei} - B'_{ei}) + \sum_{e=1}^{8}\sum_{i=1}^{6} \frac{(b_{ei} + b'_{ei})}{2}(B_{ei} + B'_{ei}) + \sum_{j=1}^{36} v_j V_j,$$

is the reduction in the sum of squares due to regression and

$$(13) \qquad S\left(y - mx_1 - r_e x_{2e} - \frac{b_{ei} - b'_{ei}}{2} x_{3i} - \frac{(b_{ei} + b'_{ei})}{2} x_{4i} - v_j x_5\right)^2,$$

is the residual sum of squares. The reductions attributable to

$$\sum_{e=1}^{8}\sum_{i=1}^{6} \frac{(b_{ei} - b'_{ei})}{2}(B_{ei} - B'_{ei})$$

and

$$(14) \qquad \sum_{e=1}^{8}\sum_{i=1}^{6} \frac{(b_{ei} + b'_{ei})}{2}(B_{ei} + B'_{ei}),$$

will be designated as component (a) and component (b) respectively. The residual mean square will be denoted by s^2.

The common test required is that of the null hypothesis that the variety effects $v_1, v_2, \cdots, v_{36}$ are all zero. This test is made by calculating the reduction (R_t) to the sum of squares on all variates, and the reduction (R_v) due to the regression on all variates, except the variety effects. $R_t - R_v$ is called the additional reduction to the sum of squares due to the v's after fitting the remaining variates.

The ratio $(R_t - R_v)/(82 - 47)s^2$ is distributed as F, as shown by Yates (6), with 35 and 205 degrees of freedom. The 35, 205, 82, and 47 degrees of freedom pertain respectively to varieties, error, all constants fitted, and the total constants less the constants for varieties.

Referring to formula (11) and the parameter effects, the sum of squares in the "Analysis of Variance Table" follow directly for replicate and component (a). Nair[2] in his recent article gives in detail the method for getting out the reduction to the sum of squares for the entangled components. He shows that the reductions for component (b) and the varieties may be written as:

$$\frac{1}{8}\sum_{e=x}^{u}\sum_{i=1}^{6} b''_{ei}C_{ei} \quad \text{and} \quad \sum_{j=1}^{36}\frac{V_j^2}{8} - \frac{G^2}{N} \tag{15}$$

where the b'' have been corrected for replicate effects. It is well to note that $\frac{1}{8}\sum_{e=x}^{u}\sum_{i=1}^{6} b''_{ei}C_{ei}$ is the reduction due to intra-block effects, freed of varietal effects.

The reduction due to varieties corrected for block effects is given by

$$\frac{1}{8}\sum_{e=x}^{u}\sum_{i=1}^{6} b''_{ei}C_{ei} + \left(\sum_{j=1}^{36}\frac{V_j^2}{8} - \frac{G^2}{N}\right) - \left(\sum_{e=x}^{u}\sum_{i=1}^{6}\frac{B_{ei}}{6} - \frac{G^2}{N}\right). \tag{16}$$

This reduction can be used for testing the variety effects.

ANALYSIS OF VARIANCE TABLE

	D/F	
Replicate	(8–1)	$\sum_{e=1}^{8}\frac{R_e^2}{36} - \frac{G^2}{N}$
Component (a)	4(6–1)	$\frac{1}{8}\sum_{e=1}^{8}\sum_{i=1}^{6}\frac{(B_{ei} - B'_{ei})^2}{12} - \sum_{e=1}^{8}\frac{(R_e - R'_e)^2}{72}$
Component (b)	4(6–1)	$\frac{1}{8}\sum_{e=x}^{u}\sum_{i=1}^{6} b''_{ei}C_{ei}$
Varieties *(ignoring blocks)*	(36–1)	$\sum_{j=1}^{36}\frac{V_j^2}{8} - \frac{G^2}{N}$
Error	205	obtained by subtraction
Total	8(36)–1	$Sy^2 - \frac{G^2}{N}$

4. Standard error of adjusted varietal means. For obtaining the standard error of the difference between two varieties adjusted by the intra-block information, this difference between two varieties can be expressed as a linear function of the plot yields. The standard error of the difference then can be obtained from the standard error of a linear function. To obtain the coefficients, it is well to draw a sketch of the plots, and put the coefficient of each plot on the diagram. In this way the proper multipliers can be found in a convenient manner.

First consider the case where the two varieties appear together in the same block in both groups Z and U. One such pair consists of varieties (1) and (22), for which the varietal effects are designated by v_1'' and v_2''. From equation (10) we have:

$$\begin{aligned} 4v_1'' &= V_1 - 4m'' - b''_{x1} - b''_{y1} - b''_{z1} - b''_{u2} \\ 4v_{22}'' &= V_{22} - 4m'' - b''_{x4} - b''_{y4} - b''_{z1} - b''_{u2}\,. \end{aligned} \tag{17}$$

The linear function of the difference between the varietal effects is:

$$4(v''_{22} - v''_{1}) = V_{22} - V_1 + b''_{x1} - b''_{x4} + b''_{y1} - b''_{y4} \tag{18}$$

where

$$b''_{x1} = \frac{1}{3k}(4B_{x1} - T_{x1}) \qquad b''_{x4} = \frac{1}{3k}(4B_{x4} - T_{x4})$$

$$b''_{y1} = \frac{1}{3k}(4B_{y1} - T_{y1}) \qquad b''_{y4} = \frac{1}{3k}(4B_{y4} - T_{y4}).$$

The multipliers [except for the common factor 4 shown on the left of equation (18)] are:

Number of Plots	*Multipliers*	*Contribution to variance*
4	$\pm\frac{3k+2}{3k}$	$\frac{36k^2+48k+16}{W(9k^2)}$
4	$\pm\frac{3k-2}{3k}$	$\frac{36k^2-48k+16}{W(9k^2)}$
4	$\pm\frac{4}{3k}$	$\frac{64}{W(9k^2)}$
$4(k+2)$	$\pm\frac{3}{3k}$	$\frac{36(k-2)}{W(9k^2)}$
$12(k+2)$	$\pm\frac{1}{3k}$	$\frac{12(k-2)}{W(9k^2)}$
Total		$\frac{72k^2+48k}{W(9k^2)}$

The variance per plot of the difference between two varietal means for varieties which occur together in the same block in groups Z and U is:

$$\frac{72k^2+48k}{2W(16)(9k)} = \frac{3k+2}{12Wk} \tag{19}$$

and for $k = 6$ is $\frac{5}{18W}$.

Similarly the variance per plot of the difference between varietal means for other combinations are as follows:

Combinations	*formula for $k = 6$*
both in same block in groups Z *or* U	$\frac{7}{24W}$
both in same block in groups X *or* Y	$\frac{8}{27W}$
not together in the same block	$\frac{67}{216W}$.

5. Numerical Analysis. (a) The data. In order to illustrate the application of the method developed, an experiment used to test the yields of 36 hybrid corn varieties is presented. This experiment was carried out on the Arlington Experimental Farm, and the results are used through the courtesy of A. E. Brandt[4] and M. H. Jenkins.[5]

Except for randomization, the plot yields are as shown in tables I to IV.

(b). *Calculation for analysis of variance table.* From page 9, the total sum of squares, the sum of squares for replicates, and the sum of squares for varieties ignoring blocks are obtained by substitution. They are:

$$(90.9)^2 + (81.4)^2 + \cdots + (101.0)^2 - c = 33{,}546.92$$

$$\frac{(3291.8)^2 + (3300.3)^2 + \cdots + (2978.2)^2}{36} - c = 2{,}289.68$$

and

$$\frac{(741.2)^2 + (695.6)^2 + \cdots + (743.1)^2}{8} - c = 15{,}825.09$$

where

$$c = \frac{(25{,}935.9)^2}{288} = 2{,}335{,}662.80$$

The block sum of squares, eliminating varieties, is made up of two parts, component (a) and component (b). From the formula on page 9 the reduction for component (a) is:

$$\frac{(559.2 - 540.2)^2 + (547.0 - 522.4)^2 + \cdots + (515.8 - 507.7)^2}{12}$$

$$- \frac{(3291.8 - 3300.2)^2 + (3256.5 - 3304.7)^2 + \cdots + (3284.6 - 2978.2)^2}{72} = 1{,}415.27.$$

Component (b) consists of differences giving an estimate of block yield freed of varietal effects. The C's are first calculated by using formula (8) and the results are as follows:

$$C_{x1} = (4B_{x1} - T_{x1}) = 4(1{,}099.4) - 4203.2 = 194.4 \text{ etc.}$$

The b's are calculated by using formulas given by (9), and then correcting for replicate effects by imposing the conditions that

$$\sum_{i=1}^{6} b''_{ei} = 0 \qquad (e = x \cdots u)$$

The corrected b''_{ei} are:

$$b''_1 = 6.79556, \quad b''_2 = -3.83777, \cdots \quad b''_{24} = -0.34306.$$

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TABLE I

GROUP X

Plot Yield Replicate 1

							Σ
	1 90.9	2 81.4	3 95.4	4 96.9	5 96.6	6 98.0	559.2
	7 96.0	8 92.9	9 86.3	10 96.4	11 84.5	12 90.9	547.0
	13 79.3	14 97.5	15 95.9	16 80.2	17 101.8	18 112.1	566.8
	19 83.4	20 92.6	21 91.2	22 88.3	23 71.1	24 111.8	538.4
	25 93.7	26 86.1	27 68.0	28 109.2	29 94.9	30 93.3	545.2
	31 85.3	32 81.2	33 99.5	34 86.9	35 93.4	36 88.9	535.2
Σ	528.6	531.7	536.3	557.9	542.3	595.0	3291.8

Plot Yield Replicate 1′

							Σ
	1 105.8	2 83.5	3 68.6	4 99.0	5 91.8	6 91.5	540.2
	7 98.6	8 71.9	9 81.0	10 98.6	11 90.3	12 82.0	522.4
	13 70.5	14 90.4	15 86.2	16 88.8	17 99.7	18 113.5	549.1
	19 94.6	20 89.6	21 106.8	22 99.2	23 73.2	24 90.9	554.3
	25 91.6	26 86.9	27 74.6	28 104.1	29 102.3	30 96.0	555.5
	31 91.1	32 95.1	33 98.8	34 95.5	35 88.1	36 110.2	578.8
Σ	552.2	516.4	516.0	585.2	545.4	584.1	3300.3

TABLE II

GROUP Y

Plot Yield Replicate 2

						Σ
1 85.9	7 96.6	13 85.4	19 86.4	25 88.8	31 89.7	532.8
2 87.1	8 83.5	14 86.9	20 88.1	26 89.1	32 102.7	537.4
3 88.4	9 85.5	15 90.4	21 99.7	27 80.5	33 100.2	544.7
4 88.5	10 72.3	16 89.5	22 95.6	28 105.5	34 82.6	534.0
5 87.0	11 88.1	17 101.8	23 82.5	29 86.8	35 100.1	546.3
6 90.8	12 83.4	18 101.2	24 120.2	30 72.6	36 93.1	561.3
						3256.5

Plot Yield Replicate 2′

						Σ
1 95.1	7 96.0	13 85.4	19 74.3	25 93.0	31 86.7	530.5
2 96.5	8 84.2	14 79.0	20 88.1	26 95.6	32 101.3	544.7
3 95.4	9 83.3	15 95.9	21 99.0	27 66.0	33 105.3	544.9
4 81.4	10 95.0	16 91.6	22 101.4	28 108.5	34 90.5	568.4
5 91.8	11 107.6	17 84.2	23 66.1	29 87.6	35 101.6	538.9
6 92.2	12 89.6	18 104.1	24 116.0	30 81.5	36 93.9	577.3
						3304.7

TABLE III

GROUP Z

Plot Yield Replicate 3

						Σ
1 87.3	8 79.9	15 86.9	22 105.0	29 99.4	36 90.3	548.8
2 90.0	9 93.0	16 93.1	23 90.3	30 73.9	31 94.1	534.4
3 91.2	10 92.9	17 90.6	24 92.3	25 76.0	32 97.2	540.2
4 87.8	11 98.2	18 125.1	19 102.1	26 88.3	33 98.1	599.6
5 84.3	12 93.7	13 73.9	20 84.4	27 66.7	34 89.8	492.8
6 90.1	7 91.3	14 85.4	21 95.4	28 93.7	35 101.6	557.5
						3273.3

Plot Yield Replicate 3′

						Σ
1 99.4	8 83.5	15 70.9	22 99.9	29 103.0	36 69.0	525.7
2 92.2	9 96.0	16 98.1	23 74.7	30 89.8	31 103.6	554.4
3 91.9	10 106.4	17 92.7	24 108.3	25 88.8	32 103.4	591.5
4 99.0	11 105.4	18 101.2	19 78.1	26 90.5	33 98.1	572.3
5 63.7	12 94.4	13 80.7	20 78.4	27 71.3	34 86.9	475.4
6 81.5	7 88.6	14 90.4	21 84.1	28 93.7	35 88.9	527.2
						3246.5

TABLE IV

GROUP U

Plot Yield Replicate 4

						Σ
31 98.5	26 99.3	21 97.6	16 90.9	11 102.5	6 91.5	580.3
1 95.9	32 92.3	27 70.0	22 99.2	17 85.6	12 100.6	543.6
7 77.8	2 79.9	33 98.1	28 104.8	23 88.9	18 115.7	565.2
13 78.6	8 81.3	3 84.1	34 86.9	29 91.3	24 111.8	534.0
19 82.6	14 89.0	9 75.0	4 94.1	35 108.3	30 96.7	545.7
25 87.3	20 76.2	15 89.7	10 83.7	5 82.2	36 96.7	515.8
						3284.6

Plot Yield Replicate 4′

						Σ
31 83.1	26 88.3	21 84.8	16 94.5	11 73.6	6 84.4	508.7
1 80.9	32 82.6	27 73.9	22 94.8	17 75.8	12 83.4	491.4
7 93.3	2 85.0	33 100.2	28 87.8	23 83.2	18 94.7	544.2
13 74.6	8 68.2	3 45.2	34 79.0	29 87.6	24 102.1	456.7
19 69.1	14 81.2	9 80.3	4 68.8	35 83.7	30 86.4	469.5
25 89.5	20 71.7	15 75.1	10 97.1	5 73.3	36 101.0	507.7
						2978.2

To get the reduction due to component (b) the above results are substituted in

$$\frac{1}{8}\sum_{e=x}^{u}\sum_{i=1}^{6} b''_{ei} C_{ei} = 1{,}389.96.$$

The necessary results are now available for the "Analysis of Variance Table."

THE ANALYSIS OF VARIANCE TABLE

Source of Variation	*Degrees of Freedom*		*Sum of Squares*		*Mean Square*
Replications		7		2,289.68	327.097
Component (a)	20		1,415.27		
Component (b)	20		1,389.96		
Blocks (eliminating varieties)		40		2,805.23	70.131
Varieties (ignoring blocks)		35		15,825.09	
Error		205		12,626.92	61.595
Total		287		33,546.92	

(c). *Test of significance.* There frequently will be large differences between varieties so that a test of significance may not be needed. If a test is needed, one involving only intra-block information may be used. For this purpose, it is necessary to calculate the sum of squares for varieties eliminating block effects as shown by formula (15): 13,946.28. The mean square will be 399.893, and $F = \frac{399.893}{61.595} = 6.49$ which is highly significant.

(d). *Corrected varietal totals and means.* The right-hand side of equation (10) gives the corrected variety totals, and when divided by eight gives the varietal means. These corrected varietal means can then be compared to determine the best variety. The corrected varietal totals and means are:

Corrected Varietal Totals

1 743.22	2 669.05	3 652.00	4 705.80	5 672.04	6 720.32
7 747.59	8 658.58	9 664.57	10 751.39	11 739.54	12 735.95
13 642.31	14 700.44	15 686.41	16 713.21	17 730.79	18 857.26
19 665.95	20 675.40	21 756.25	22 801.34	23 619.46	24 868.84
25 704.17	26 699.83	27 567.71	28 814.04	29 763.51	30 679.44
31 721.79	32 757.48	33 783.42	34 726.05	35 780.48	36 760.37

Corrected Varietal Means

1 92.902	2 83.631	3 81.500	4 88.225	5 84.005	6 90.040
7 93.449	8 82.322	9 83.071	10 93.924	11 92.442	12 91.994
13 80.289	14 87.555	15 85.801	16 89.151	17 91.349	18 107.158
19 83.244	20 84.425	21 94.531	22 100.168	23 77.432	24 108.605
25 88.021	26 87.479	27 70.964	28 101.755	29 95.439	30 84.930
31 90.224	32 94.685	33 97.927	34 90.756	35 97.560	36 95.046

When comparing one variety with another it is necessary to know the standard error of the mean difference, in order to judge whether this difference is significant. The formulas for the standard error of the difference between mean yields differ for those sets of varieties which occur together in the same block in groups Z and U, in groups Z or U, in groups X or Y, or do not occur together in the same block. The formulas for these calculations are, respectively:

(19), (20), (21), and (22). For example, the standard error of the difference between two variety means, such as variety 1 and 2, is

$$\frac{8}{27}(61.595) = 4.27.$$

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