that is, of classifying a student as one who will do unsatisfactory work when he actually does satisfactory work.

3. Conclusions. In using the above classification equation to classify the 305 trainees used in this study, 21 errors of Type I were made or 22.9 percent, while 50 errors of Type II were made or 23.9 percent. These percentages seem reasonably close to the expected 20.6 percent.

NOTE ON AN IDENTITY IN THE INCOMPLETE BETA FUNCTION

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Since the incomplete beta function has proved of some importance in statistics, it would appear that any additional information concerning its properties might at some time prove useful. In a paper by the author, [1], two identities in the incomplete beta function were incidentally obtained. They are as follows:

$$(p + q)I_x(p, q) = pI_x(p + 1, q) + qI_x(p, q + 1) \tag{1}$$

and

$$(p + q + 1)^{[2]}I_x(p, q) = (p + 1)^{[2]}I_x(p + 2, q) + 2pqI_x(p + 1, q + 1) + (p + 1)^{[2]}I_x(p, q + 2), \tag{2}$$

where the incomplete beta function $I_x(p, q) = \dfrac{B_x(p, q)}{B(p, q)}$, etc., and $(p + 1)^{[2]}$, etc. refer to the standard factorial notation.

Written in the above form these two identities suggest a possible general identity to which they belong as special cases. The third special case suggested is:

$$\begin{aligned}(p + q + 2)^{[3]}I_x(p, q) &= (p + 2)^{[3]}I_x(p + 3, q) \\ &+ 3(p + 1)^{[2]}qI_x(p + 2, q + 1) + 3p(q + 1)^{[2]}I_x(p + 1, q + 2) \\ &+ (q + 2)^{[3]}I_x(p, q + 3).\end{aligned} \tag{3}$$

The general formula suggested is

$$(p + q + n - 1)^{[n]} I_x(p, q) = \sum_{r=0}^{n} \binom{n}{r} (p + n - r - 1)^{[n-r]} (q + r - 1)^{[r]} I_x(p + n - r, q + r). \tag{4}$$

To prove the general formula we write (4) as

$$(p + q + n - 1)^{[n]} I_x(p, q) = \sum_{r=0}^{n} \binom{n}{r} (p + n - r - 1)^{[n-r]} \cdot (q + r - 1)^{[r]} \frac{B_x(p + n - r, q + r)}{B(p + n - r, q + r)}. \tag{5}$$

By expanding and simplifying it is easy to show that

$$\frac{(p+n-r-1)^{[n-r]}\,(q+r-1)^{[r]}}{\mathrm{B}(p+n-r,\,q+r)} = \frac{(p+q+n-1)^{[n]}}{\mathrm{B}(p,\,q)}. \tag{6}$$

Using (6) the right hand side of (5) reduces to

$$\frac{(p+q+n-1)^{[n]}}{\mathrm{B}(p,\,q)} \sum_{r=0}^{n} \binom{n}{r} \mathrm{B}_x\,(p+n-r,\,q+r). \tag{7}$$

The summed function in (7) reduces to

$$\int_0^x x^{p-1}\,(1-x)^{q-1}\,[x+(1-x)]^n\,dx = \mathrm{B}_x\,(p,\,q), \tag{8}$$

which proves the identity.

Although the general identity is quite simple to prove, it does not seem to have appeared in the literature.

REFERENCE

[1] BANCROFT, T. A. "On biases in estimation due to the use of preliminary tests of significance," *Annals of Math. Stat.*, Vol. 15 (1944), No. 2.