

seem sufficiently stressed; in particular, the Wiener-Khinchine relation between the periodogram and correlogram is noted (section 30.68) as "an interesting relation", whereas it is a fundamental relation in the modern method of approach to time-series, giving much deeper insight into the correct interpretation of classical periodogram analysis.

These criticisms, which could be extended to cover minor errors and misprints, are not intended to detract seriously from what is a remarkable achievement. An excellent sense of proportion has been maintained throughout between mathematical theory and illustrative discussion and examples. This makes this treatise, if both the breadth and level of the subject matter are taken into account, at present unique. It will be an indispensable reference book to every teacher and advanced student of the theory of statistics.

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**Sequential Analysis of Statistical Data: Applications.** Prepared by the Statistical Research Group, Columbia University for the Applied Mathematics Panel, National Defense Research Committee, Office of Scientific Research and Development. SRG Report 255, Revised; AMP Report 30.2R, Revised. New York: Columbia University Press, September 1945. pp. vii, 17; iv, 80; v, 57; iii, 25; iii, 18; iii, 39; ii, 41. \$6.25. (London: Oxford University Press, 1946.)

REVIEWED BY JOHN W. TUKEY

*Princeton University*

Many of the features of this compendium are familiar to most of the readers of this review, but for the benefit of the others I shall enumerate them briefly. It consists of a heavy looseleaf binder containing 7 booklets of distinctive colors—each saddle stitched and usable separately. It is the last word (to date) in presenting sequential analysis to the statistician who may wish to use it in practice. It covers five elementary cases (each in a booklet, the two others being used for introduction and appendices):

- Acceptance or rejection by percent defective (Sec. 2)
- Comparative percent satisfactory (Double dichotomy) (Sec. 3)
- Acceptance or rejection by the adequacy of the mean (with known variability) (Sec. 4)
- Acceptance or rejection by the exact value of the mean (with known variability) (Sec. 5)
- Acceptance or rejection by the smallness of the variability (Sec. 6)

These cases are covered in complete detail, with illustrative examples, tables and charts. A copy should be accessible to every teacher of statistics and to every statistician in industry or experimental work who can propose new techniques of testing.

With this general introduction let us go on and explain what the reader will not find and what further work in this line the reviewer awaits with keen interest. The classical testing procedure was to test a sample of predetermined size and then decide to accept or reject. Long ago curtailed sampling and double sampling were developed to cut corners legitimately and reduce inspection costs. There are two situations, each more frequent in war than in peace, where it is clearly desirable to reduce the *average* number of items tested to a minimum:

- (I) Where essentially all lots are accepted and the test is destructive so that the items tested are the main loss of production, or
- (II) Where the cost of testing an item is large in comparison with the cost of production.

Subject to a practically unimportant allowance for the finite size of the lots, and to an allowance of unknown importance for the quality of lots presented, the methods of sequential analysis minimize this average number among all methods so far considered. When situation (I) or (II) holds without modifying complications, then, the best known method is sequential analysis, the natural descendant of double sampling. Otherwise, the situation is far from clear, and much judgement is involved in setting up a practically efficient scheme. The reader will get no help on this problem of judgement, nor in the problem of setting risks from the book under review—he will get every needed help with the mathematical problem of setting up a sequential plan to meet chosen risks, including complete tables of all necessary functions, including natural logarithms.

There is no reason to suppose that sequential analysis is the last word in testing procedures for the general problem of efficient testing, but what should be the next step ahead is not a step for the mathematical statistician. What is needed now is a careful analysis, by the operational research techniques so useful during the war, of a half-dozen industrial testing situations to determine what properties of the testing procedure are involved in cost and to what extent. Do we want the minimum average sample size, the minimum average square of the sample size—or what? With this there should go a corresponding operational study of the advantages of different OC curves, including those of what now seems to be a peculiar shape. Given these studies, we could put the problem in mathematical statistics to the mathematical statistician which he would then solve. But with the present lack of operational research groups in industry, it is probable that we will proceed in an unnatural way, and that the mathematical statistician will take the next step forward. For reasons of mathematical simplicity it is not unlikely that the sample plan with the minimum average squared sample size will come next.

The credit for the book is clearly assigned on the inside cover of each pamphlet in the following words: "So many members of the Statistical Research Group (Columbia) have participated in the preparation of this report, a previous edition of which was prepared by H. A. Freeman, that its authorship is attributed to the group as a whole. The responsibility for planning and preparing this

edition has been shared by H. A. Freeman, M. A. Girshick, and W. Allen Wallis, with the cooperation of Kenneth J. Arnold, Milton Friedman, Edward Paulson, and others. The theory of sequential analysis is mainly the work of A. Wald."

It may be of interest to notice a few minor points for the record. On page 1.01 it is indicated that 100% inspection is 100% effective—this seems far from industrial experience. Another badly needed set of operational studies would be on the influence of the sampling plan on inspector's inspection. On page 2.27, the footnote suggests that when a tabular procedure is used instead of a graphic one, that more decimal places should be kept—the logic of this is not clear. On page 4.14 it is stated that "similarly, if all patches had tested 400 minutes, the experiment would have terminated at 9.4 . . .". Clearly no such experiment can terminate after a fractional number of tests. On page A.09 it is stated that "Finally it should be mentioned that truncation of any kind ought generally to be avoided". This seems to the reviewer to be a rash statement, for when not only average sample size but all other properties entering into the practical efficiency of a sampling plan are considered, this decision will almost certainly be reversed. The relatively small number of these detailed points is an evidence of careful and competent workmanship.

A footnote to the Appendix (B) on some principles of sequential analysis states: "Any mathematician who may stray into this Appendix should be assured that the validity of the conclusions in no case depends upon the type of reasoning presented here; indeed, even for intuitive or heuristic arguments mathematicians may prefer those given in SRG 75". This warning and caveat seems unduly strong—the appendix is recommended to all mathematically minded newcomers to sequential analysis.

The same appendix warns the reader in a few places that the theory set forth does not allow for the fact that samples come in units. If the reader tries to apply the theory to cases far from normal inspection practice, for example with risks of 0.25 and average sample sizes of 12, he will then find out that this does occasionally make a difference. In conventional circumstances the approximation will not bother him.