It is easily seen that an "outer" estimate of p is still given by U_{k+1}. However, an "inner" estimate is now given by U_{k-1}, leading to a lower end point of the confidence interval which is unnecessarily small.

The method of obtaining a confidence interval for p discussed in this note is in a certain sense the reverse of the method discussed in an earlier paper of the author [2]. There it was shown how confidence intervals for p can be used to obtain confidence intervals for quantiles, which then can be used to obtain tolerance intervals.

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ABSTRACTS OF PAPERS

(*Abstracts of papers presented at the Minneapolis meeting of the Institute, September 4–7, 1951*)

1. **On Stieltjes Integral Equations of Stochastic Processes.** MARIA CASTELLANI, University of Kansas City.

This paper considers two methods of solving certain S-integral equations.

a. *A Fredholm-Stieltjes integral equation of generating functions.* We give the F-S integral equation $\int_E A(s, x)\, dg(x) = f(s)$, where $A(s, x) = \Sigma_{k=0}^{\infty} \alpha_k(x)s^{-k}$ and $f(s) = \Sigma a_k s^k$ for $s \to \varphi(s)$ and $a_0 = 0$ if $k = 0$. Let us assume that $u(x)$ and $v(x)$ are respectively solutions of $\int_E A(s, x)\cdot A(-s_1, x)\, du(x) = 1/(S - S_1)$ and $\int_E A(s, x)\, dv(x) = 0$. If we consider

$$\int_E A(s, x)A(-s_1, x)f(s_1)\, du(x) = f(s_1)/(S - S_1)$$

and if $\gamma(x)$ is the coefficient of $-1/S_1$ in the serial expansion of $A(-s, x)f(s_1)$, then under fairly general conditions the required solutions are given, almost everywhere, by $g(x) = \text{const.} \int^x dv(x) + \int^x \gamma(x)\, du(x)$. The proof is based on a Murphy D'Arcais linear operator and on the ρ operation of S-integrals.

b. *A Volterra-Stieltjes integral of recurrent random functions.* Let us have over a time interval (τ, t) an unknown rf$\delta(t - \tau)$ satisfying the following recursive equation: $\delta(t - \tau) = \delta(\tau) - \int_\tau^t \delta(x - \tau)\rho(x)\, dF(x)$ where $F(x)$ is a df and $\rho(x)$ is bounded. We assume the interval divided into n parts and also that the set of the n discrete values of δ satisfy the following relation: $\delta(t - \tau)/\delta(\tau) = \Pi_{s=\tau}^{s=t}(1 - \rho(s)\Delta F(s))$. If $F = F_1 + F_2$, where the F_1 is a continuous function and F_2 is a jump function over a set S of points, then by a generalized method of Cantelli, taking finer and finer partitions, we obtain as a limit $\delta(t - \tau)/\delta(\tau) = \left[\exp\left(-\int_\tau^t \rho(x)\, dF_1(x)\right)\right] \Pi_{s\epsilon S}(1 - \rho(s)\, dF_2(s))$. This gives almost everywhere the required solutions.

2. An Unfavorable Aspect of the Liklihood Ratio Test. L. M. COURT, Rutgers University.

The likelihood ratio test has many desirable properties. For example, it is not only consistent, but as Wald has shown, uniformly consistent. Still, it can at times be a poor test, e.g., under certain circumstances when the size of the test region is properly selected, the probability of rejecting the hypothesis to be tested when it is true *exceeds* the probability of rejecting it when any alternative is true. Both Stein and Rubin have given examples of this. Stein's example, quoted by Lehmann in his notes on "Testing Hypotheses" (pp. 1–5) consists of a family of *discrete* distributions (five-valued, to be precise) in which a simple hypothesis is tested against a composite alternative. The writer, using a geometrical construction, gives an example (actually, a broad class of examples, much broader than Stein's which is a 2-parameter class) in which the distributions are *continuous*. The hypothesis to be tested is first simple, then composite; the alternative, always composite.

3. Impartial Decision Rules and Sufficient Statistics. RAGHU RAJ BAHADUR AND LEO A. GOODMAN, University of Chicago.

In the following, (1) refers to the paper "On a problem in the theory of k populations," by R. R. Bahadur (*Annals of Math. Stat.* Vol. 21 (1950), pp. 362–375). The present paper provides certain improvements of the main result contained in (1). The authors define the class of impartial decision rules in terms of permutations of the k samples (rather than in terms of the k ordered values of an arbitrarily chosen real valued statistic (cf. (1)). This definition is intuitively more appealing than the one adopted in (1), and permits a unified treatment of discrete and absolutely continuous populations. The authors show that if the same function is a sufficient statistic for each of k independent samples of equal size, then the conditional expectation given the sufficient statistics of an impartial decision rule is also an impartial decision rule. They also give a characterization of impartiality which relates the present definition to that of (1). These results, together with Theorem 1 of (1), yield the desired improvements. An illustration of the argument indicated here is given.

4. Contributions to the Statistical Theory of Counter Data. G. E. ALBERT, University of Tennessee and Oak Ridge National Laboratory, AND M. L. NELSON, Oak Ridge National Laboratory.

Let a sequence f of events be such that the number occurring in time T is a chance variable having a Poisson distribution with mean aT, $a > 0$. A counting device generates a new sequence g since, due to a resolving time $u > 0$, it fails to record all of f. An event in f (i) *can* be recorded only if none has been recorded during a time u preceding it, (ii) *will* be recorded if it follows its predecessor in f by more than time u, (iii) either will be recorded with probability p or not recorded with probability $1 - p$ if it follows its predecessor in f by time $\leq u$. The choices $p = 0$ and $p = 1$ give the so called Type I and Type II counters respectively. The distribution theory of g is obtained as a generalization of the Type II theory given by Feller in "On probability problems in the theory of counters," *Courant Anniversary Volume*, 1948. The Cornish-Fisher normalization is applied to obtain confidence intervals for the constant a from observations on g of either the time to a preassigned count or the count to a preassigned time. These intervals turn out essentially independent of p whenever the product au is small; thus the Type I theory reported at an earlier meeting covers most of the cases of practical importance.

5. On the Use of Wald's Classification Statistic. HARMAN LEON HARTER, Michigan State College.

In 1944 Wald published a paper introducing the statistic V and giving a general outline of its use in problems of classification. Recently the author published a paper giving the distribution of V in various cases. The present paper takes the form of an effort to relate the two earlier papers and apply the results of the latter. The technique of classifying an individual into one of two groups is studied in detail. Let the individual under consideration belong to the population π, which is known to be identical with one or the other of two known populations π_1 and π_2. Then one may wish to test the hypothesis $H_1: \pi = \pi_1$, against the alternative hypothesis $H_2: \pi = \pi_2$. The values of the statistic V under these two hypotheses are given, and a method is outlined for testing H_1 against H_2, where an error of the second kind is k times as costly as an error of the first kind. A numerical example is given for the univariate case, for which the distribution of V is given in the author's earlier paper. The same procedure can be applied in the multivariate case when the distribution is known.

6. Polynomial Determination in a Field of Integers Modulo P. EDWARD C. VARNUM, Barber-Colman Company.

From a study of integers mod 2 applied to on-off relay circuits, a generalization to any prime modulus, p, has been made to construct a $p^n \times p^n$ matrix by which a polynomial in n variables may be determined when the p^n values of the polynomial are given for all the combinations of the p values of each of the n variables.

7. About Some Symmetrical Distributions from the Perks' Family of Functions. JOSEPH TALACKO, Marquette University.

The Perks' system of functions includes a family of symmetrical nonnormal distributions, from which two probability densities are of growing interest in theoretical statistics: the Verhulst's distribution (logistic distribution) and the hyperbolic cosine distribution. In the first part of this paper properties of this family of probability functions are discussed and the characteristic functions for Verhulst's and the hyperbolic secant distributions introduced. The Verhulst's probability function $f(t) = \delta e^{-\delta t}/(1 + e^{-\delta t})^2$ has $C(\nu) = (\pi\nu/\delta)$ cosech $(\pi\nu/\delta)$, and the hyperbolic secant probability function $\varphi(t) = (2\delta/\pi)(1/(e^{\delta t} + e^{-\delta t}))$ has $C(\nu) = \text{sech}\,(\pi\nu/(2\delta))$. The second part is concerned with some previously uninvestigated distributions of certain statistics for samples from Verhulst's population. In particular, distributions of sample means and sum of squares are discussed. In an appendix a table of numerical values of Verhulst's functions is given.

8. A Large-Sample Test for the Variation of Sample Covariance Matrices. DAYLE D. RIPPE, University of Michigan.

A test criterion is developed to determine whether a given sample covariance matrix could be obtained as a result of taking a random sample of size N from a k-variate normal population with a given covariance matrix. The test is based upon the fact that the maximum likelihood estimate of the population covariance u_{ij} is $\hat{u}_{ij} = m_{ij} = (N - 1)^{-1} \sum_{\alpha=1}^{N} (x_{i\alpha} - \bar{x}_i)(x_{j\alpha} - \bar{x}_j)$. The test criterion for large samples is $\lambda = (N - 1)(\ln \mid u_{ij} \mid - \ln \mid m_{ij} \mid + \sum_i \sum_j u^{ij} m_{ij} - k)$, where λ is distributed as a chi-square with $\frac{1}{2}k(k + 1)$ degrees of freedom minus the number of independent linear restrictions among the variables $(m_{ij} - u_{ij})$. The results of the application of this criterion to the sampling problems in correlation theory compare favorably with exact sampling results, and the range of application is extended considerably. The criterion is sufficiently general in application to furnish a large-sample test for completeness of factorization in matrix factorization (or factor analysis) for the case of complete initial estimates of the communalities. It is applicable to any of the common orthogonal forms of solution. The degrees of freedom of the chi-square involved is $\frac{1}{2}(k - s)(k - s + 1)$ after s of the total of k possible common components have been removed. The test may also be applied to determine the significance of component loadings in the common factor solution.

9. Probability Models for Analyzing Time Changes in Attitudes. T. W. ANDERSON, Columbia University.

Statistical inference in Markov chains is studied with particular application to data in which the finite number of states are states of attitudes of individuals in "panel surveys." Each individual's sequence of states over a finite number of time points is considered as an observation from a Markov chain with the same stationary transition probability matrix $P = (p_{ij})$. $n_i(0)$ individuals hold state i at the time origin. The maximum likelihood estimate of P is obtained. Tests are obtained for the hypothesis that P is a given matrix (or that certain elements are given numbers) and for the hypothesis that the transition matrix is stationary against alternatives that it varies over time. Extension of results to higher order cases is straightforward. A test of the hypothesis that the process is first order against the alternative that it is second order is given. When the state is defined in terms of two attitudes, a test is given for the hypothesis that the two attitudes change independently of each other. Asymptotic distributions of the estimates and of the test criteria are obtained under the assumption that $n_i(0) \to \infty$.

10. The Variance of a Weighted Average Using Estimated Weights. PAUL MEIER, Princeton University.

In various experimental designs (e.g., lattices) the problem of estimation involves averaging of two or more means with different variances. The proper weights (invariances) must be estimated from the experimental data. This problem has been treated by Cochran for the case of a large number of samples of equal size. We consider the case of two or more samples and find adjustments of order $O(\Sigma 1/n_i)$, both for the increase in variance and the bias of estimating the variance. Bounds on the increase in variance due to the use of estimated weights are given. Exact computations are made for several special cases. For the case of two means with weights based on ten degrees of freedom the adjustments reduce the maximum bias from approximately 15 per cent to less than 2 per cent.

11. Distribution of Ratios of Quadratic Forms. JOHN GURLAND, University of Chicago.

The problem of finding the distribution of a quadratic form and of a ratio of quadratic forms in normally distributed random variables is considered. By transforming the problem of ratios into one of linear combinations of independent variables each having a χ^2 distribution, a solution is given in terms of Laguerre polynomials which is more general than that of Pitman and Robbins ("Application of the method of mixtures to quadratic forms in normal variates," *Annals of Math. Stat.*, Vol. 20, 1949, pp. 552–560). The convergence of the expansion is established, and a new system of polynomials is suggested which would afford a solution for all distributions of quadratic forms and ratios of quadratic forms in normally distributed random variables. Once the convergence of expansions in terms of the new system polynomials is established, the system will be applicable in a much wider class of distributions than the Gram-Charlier series.

12. The Large-Sample Power of Tests Based on Permutations of Observations. (Preliminary Report.) WASSILY HOEFFDING, University of North Carolina.

The results of this paper can be illustrated by the following example. Let $t(x) = t(x_1, \cdots, x_N)$ be the usual t-statistic for testing whether two samples $x_1, \cdots, x_m$ and

$x_{m+1}, \cdots, x_N$ came from the same (normal) population. The critical region of the standard test of size α is of the form $| t(x) | \geq \lambda_N$. As $N \to \infty$, λ_N approaches a value $\lambda = \lambda(\alpha)$. A nonparametric test proposed by E. J. G. Pitman can be described as follows. Let $x^{(1)}, \cdots, x^{(N!)}$ be the $N!$ permutations of the sample values $x = (x_1, \cdots, x_N)$, so numbered that $|t(x^{(1)})| \geq \cdots \geq | t(x^{(N!)}) |$. Let k be the largest integer $\leq N! \ \alpha + 1$. Then the hypothesis that the two samples came from the same (arbitrary) population is rejected if and only if $| t(x) | > | t(x^{(k)}) |$, and the size of the test is $\leq \alpha$. The critical value $| t(x^{(k)}) | = \lambda_N'$, say, is a random variable. It is shown that as $N \to \infty$, λ_N' tends in probability to λ under general conditions which cover the case of two samples from two normal populations. It follows that in large samples the power of the nonparametric test approaches that of the standard parametric test. Similar results hold for tests of certain linear hypotheses, the correlation coefficient test, etc. (Work sponsored by the Office of Naval Research.)

13. A Complete Class of Decision Procedures for Distributions with Monotone Likelihood Ratio. Herman Rubin, Stanford University.

Let $P(X \leq x \mid \tau) = \int_{-\infty}^{x} f(x, \tau)\, d\mu(x)$, where τ lies in some interval of the reals, and if $x_1 > x_2$, $\tau_1 > \tau_2$, then $f(x_1, \tau_1)f(x_2, \tau_2) - f(x_2, \tau_1)f(x_1, \tau_2) \geq 0$. This is a generalization of the exponential family, where $f(x, \tau) = \omega(\tau) \exp (x, \tau)$. Suppose the terminal decision d ranges over a closed subset D of the reals. Then if the loss function $W(d, \tau)$ satisfies certain monotonicity restrictions (which are usually met in multiple decision and estimation problems), a complete class of decision procedures based on a single observation are those which are unrandomized, except possibly at jumps of μ, and are monotone.

14. Some Nonparametric Results for Experimental Designs. John E. Walsh, Bureau of the Census.

In experimental designs, the quantities investigated are often grouped into blocks as a method of obtaining a higher precision for the experiment. This grouping may result in high correlation among observations within the same block. Also there may be substantial variance differences between blocks. Then the t-statistic is not necessarily applicable for comparing the effects of the treatments under investigation. This paper presents some nonparametric results which are usually valid for a well known type of experimental design if there is statistical independence among blocks (number of blocks ≥ 4). These nonparametric results are reasonably efficient, compared to those based on the t-statistic, for the case where the totality of observations are independent, normally distributed, and have the same variance. High precision can sometimes be obtained by designing the experiment to yield large positive correlation within blocks and then using the nonparametric results.

15. Efficient Tests and Confidence Intervals for Mortality Rates. John E. Walsh, Bureau of the Census.

This paper presents large-sample tests and confidence intervals for the "true value" of a mortality rate based on insurance data. These results have efficiencies of nearly 100%; i.e., they utilize nearly all the "information" contained in the data. The procedure used in obtaining the tests and confidence intervals consists in constructing a suitable t-statistic. The construction requires that the data be divided into between 300 and 400 statistically independent subgroups of approximately the same size. One possible way of accomplishing this is by subdividing the data according to the first three letters of the last name of the person insured and then appropriately combining the resulting groups. The amount of work required in applying the results of this paper is not appreciably greater than that required in obtaining the usual point estimate of the mortality rate; in fact, a procedure which yields the point estimate as a byproduct is followed.

16. Sufficient Statistics when the Carrier of the Distribution Depends on the Parameter. D. A. S. Fraser, University of Toronto.

A "statistic of selection" is defined by a mapping from the space of the distribution to the space of Borel sets over that space. This statistic is sufficient if the parameter is a "parameter of selection," that is, if the parameter θ determines only the carrier of the distribution, the relative density being independent of the parameter. For more general distributions a theorem in this paper facilitates obtaining sufficient statistics, subject to continuity conditions.

17. Bayes Solutions and Likelihood Ratio Tests of Some Simple and Composite Hypotheses. (Preliminary Report.) Allan Birnbaum, Columbia University.

Let H_0 be a hypothesis concerning the density function $p_\theta(e)$, to be tested against the composite alternative H_1, by means of the acceptance region A in the space of the minimal sufficient statistic $t(e)$. For various distributions in which $t(e)$ is not real-valued, necessary and sufficient conditions are given for A to be a Bayes solution or the limit of a sequence of Bayes solutions. The likelihood ratio test, for a wide class of simple and composite hypotheses, is proved to be the limit of a sequence of Bayes solutions. A condition which is necessary and sufficient for the admissibility of the likelihood ratio test is derived. The distributions considered include: (1) $p_\theta(e)$ of general Koopman-Darmois form; the result here is applied to various examples. (2) $p_\theta(e)$ rectangular; generalizations of this result are indicated. Methods of approximating these tests are discussed. Applications to problems of "combining" independent significance tests are made; a definition of admissibility of methods of combination is proposed, according to which some current methods are inadmissible; a minimax multidecision procedure is proposed and developed, to replace certain current methods of combining tests.

18. The Impossibility of Certain Affine Resolvable Balanced Incomplete Block Designs. S. S. Shrikhande, Nagpur College of Science, India.

Three theorems on the impossibility of an Affine Resolvable Design (R. C. Bose, "A note on the resolvability of balanced incomplete designs," *Sankyhā*, Vol. 6 (1942), pp. 105–110) with parameters $v = nk = n^2(n-1)t + n^2$, $b = nr = n(n^2t + n + 1)$, $\lambda = nt + 1$, with $n \geq 2, t \geq 0$ (n and t integral) are proved. Theorem 1. *An Affine Resolvable Design with the above parameters does not exist when n and t are odd and* (i) $n((n-1)t + 1)$ *is not a perfect square, or* (ii) $n((n-1)t+1)$ *is a perfect square and* $nt + 1 \equiv 2 \pmod 4$, *and the square-free part of n contains a prime of the form* $4i + 3$. Theorem 2. *An Affine Resolvable Design with the above parameters does not exist when n is odd and t is even and* (i) $(n-1)t + 1$ *is not a perfect square, or* (ii) $(n-1)t + 1$ *is a perfect square and* $n + t \equiv 1 \pmod 4$, *and the square-free part of n contains a prime* $4i + 3$. Theorem 3. *An Affine Resolvable Design with the above parameters does not exist for any value of t if* $n \equiv 2 \pmod 4$ *and the square-free part of n contains a prime* $4i + 3$. The proofs depend on showing the impossibility of a Group Design obtained from the Affine Resolvable Design by making use of results due to Bose and Connor (Abstracts No. 4 and No. 6, *Annals of Math. Stat.*, Vol. 22 (1951), pp. 311–312). The theorem on the impossibility of finite projective planes (R. H. Bruck and H. J. Ryser, "The nonexistence of certain finite projective planes," *Canadian Jour. Math.*, Vol. 1 (1949), pp. 88–93) is contained here as a particular case.

19. On Sufficiency and Statistical Decision Functions. Raghu Raj Bahadur, University of Chicago and Delhi University, India.

The first part of the paper contains certain characterizations of sufficiency. These results are then used to show that the justification for the use of sufficient statistics in statistical methodology which was described in an informal way by P. R. Halmos and L. J. Savage in the final section of their work on sufficiency ("Application of the Radon-Nikodym theorem to the theory of sufficient statistics," *Annals. of Math. Stat.*, Vol. 20 (1949), pp. 225–241) is valid whenever the decision space may be taken to be a subset of Euclidean k-space. This justification is proved first for the case of an arbitrary but fixed sample space, and then generalized to sequential sample spaces. The result for the sequential case may be outlined as follows. Let $x_1, x_2, \cdots$ be a sequence of chance quantities having a joint probability distribution p belonging to a family P. For each $m = 1, 2, \cdots$, let $t_m(x_1, x_2, \cdots, x_m)$ be a statistic which is sufficient when the sample space consists of points $(x_1, x_2 \cdots, x_m)$. Then corresponding to any sequential decision function ξ based on $x_1, x_2, \cdots$ there exists a sequential decision function η based on $t_1(x_1), t_2(x_1, x_2), \cdots$ such that the joint probability distribution of the sample size and the terminal decision is the same under ξ and η for each p in P. This result holds without restriction (other than measurability) on the sampling scheme of ξ, so that in the special case of point estimation with a convex loss function it leads to an enlargement of the domain of Blackwell's Theorem and its generalizations.

20. **A Two Sample Test Procedure.** DONALD B. OWEN, University of Washington.

In testing hypotheses the standard procedure is to specify a test based on a single set of observations. Sequential analysis introduced a new concept: that of making a decision after each observation, either to accept the hypothesis, or to reject it, or to take another observation. Here an approach is worked out that lies somewhat between these: an initial set of observations is taken. Then a decision is made to accept, reject, or take one more set of observations. After this second set of observations, a decision on the hypothesis must be made.

The problem is to formulate these decision rules at the two stages of the process, to optimize them if possible, and to evaluate the performance of the tests. These depend on various parameters and it is pertinent to inquire how these parameters affect the answers to the questions noted. More precisely, the problem is to maximize (or minimize) with respect to a, b, c, d expressions of the type: $\int_a^b f(x)\,dx$, $\int_c^d g(x)\,dx$, subject to side conditions (which are also expressed as integrals). The functions f and g are probability density functions: here those associated with the normal probability density function. There are two main sections, the basis of division being whether the final decision is based only on the second sample or on the whole set of observations. The second procedure is the more economical, but mathematically it is much more difficult. Much less complete results are given in this section. In the direction of finding the optimum of this type of rule, the results are chiefly negative. Several theorems are given which show the difficulties in obtaining a solution to this problem For the rules given the performance of the tests is evaluated and various theorems worked out concerning them. Some of the lemmas have interest in their own right as properties pertaining to important probability density functions.

21. **A Combinatorial Central Limit Theorem.** WASSILY HOEFFDING, University of North Carolina.

Let $(Y_{n1}, \cdots, Y_{nn})$ be a random vector which takes on the $n!$ permutations of $(1, \cdots, n)$ with equal probabilities. Let $c_n(i, j)$, $i, j = 1, \cdots, n$, be n^2 real numbers. Two sufficient conditions for the asymptotic normality of $S_n = \sum_{i=1}^{n} c_n(i, Y_{ni})$ are given. In the special case $c_n(i, j) = a_n(i)b_n(j)$, which was considered by Wald and Wolfowitz, the first condition generalizes a condition given by Noether ("On a theorem by Wald and Wolfowitz," *Annals of Math. Stat.*, Vol. 20 (1949), pp. 455–458). The second condition is slightly stronger but simpler as it involves not an infinity of limiting relations but only a single one. Applications to the theory of nonparametric tests are indicated. (Work sponsored by the Office of Naval Research.)

22. Necessary Conditions for the Existence of a Symmetrical Group Divisible Design. R. C. Bose and W. S. Connor, Jr., University of North Carolina.

An incomplete block design with v treatments each replicated r times in b blocks of size k is said to be group divisible if the treatments can be divided into m groups each with n treatments, so that the treatments of the same group occur together in λ_1 blocks, and treatments of different groups occur together in λ_2 blocks, $\lambda_1 \neq \lambda_2$. The combinatorial properties and the methods of construction for these designs have been studied by the authors elsewhere (cf. Abstract No. 4, *Annals of Math. Stat.*, Vol. 22 (1951), p. 311). An incomplete block design is said to be symmetrical if the number of treatments v equals the number of blocks b, and in consequence $k = r$. If N is the incidence matrix of a symmetrical group divisible design, the Hasse invariate $C_p(NN')$ of the quadratic form with matrix NN' (where N' is the transpose of N, and p is any odd prime) has been obtained in a simple form. Its value is $C_p(NN') = (P, \lambda_2)_p(P, n)_p^m(P, -1)_p^{m(m-1)/2}(Q, n)_p^m(Q, -1)_p^{v(v-1)/2+m(m-1)/2}$, where $P = r^2 - v\lambda_2$, $Q = r - \lambda_1$, and $(a, b)_p$ is the Hilbert norm residue symbol. For the existence of a symmetrical group divisible design $C_p(NN') = +1$ for all odd primes p, and $P^{m-1}Q^{m(n-1)}$ must be a perfect square. This shows that necessary conditions for the existence of a symmetrical group divisible design are (i) if m is even then P must be a perfect square, and if further $m = 4t + 2$ and n is even then $(Q, -1)_p = +1$ for any odd prime p; (ii) if m is odd and n is even then Q must be a perfect square and $((-1)^\alpha n\lambda_2, P)_p = +1$ for any odd prime p, where $\alpha = m(m - 1)/2$; (iii) if m and n are both odd then $((-1)^\alpha n\lambda_2, P)_p((-1)^\beta n, Q)_p = +1$ for any odd prime p, where $\alpha = m(m - 1)/2$ and $\beta = n(n - 1)/2$. The impossibility of a large number of symmetrical group divisible designs can be proved by using these conditions.

23. On a Problem of Mapping of One Space on Another with Applications in Sampling Distributions. S. N. Roy, University of North Carolina.

Denoting a $p \times n$ matrix M by $M(:p \times n)$, a triangular matrix (with the upper right hand corner zero) by $\tilde{T}$ (with elements t_{ij}), a diagonal matrix with diagonal elements $\theta_1, \theta_2, \cdots, \theta_p$ by D_θ, and a $p \times p$ unit matrix by $I(p)$, consider the transformations

(i) $x(:p \times n) = \tilde{T}(:p \times p)\, \mathfrak{L}\, (:p \times n)$, where $p \leq n$; x is of rank p; $\mathfrak{L}\mathfrak{L}' = I(p)$; $t_{ii} > 0$, $i = 1, 2, \cdots, p$.

(ii) $x(:p \times n) = M(:p \times p)D_{\sqrt{\theta}}(:p \times p)\mathfrak{L}(:p \times n)$, where $MM' = M'M = \mathfrak{L}\mathfrak{L}' = I(p)$; $p \leq n$; x is of rank p; the first row of M consists of positive elements; θ stands for the p positive characteristic roots of xx', and $\sqrt{\theta}$ stands for the positive square root of θ.

These transformations have proved extremely useful for almost the entire range of problems on sampling distributions based on multivariate normal populations. In (i), by virtue of $\mathfrak{L}\mathfrak{L}' = I(p)$, we could choose from $\mathfrak{L}$, in various alternative ways, a set of $pn - p(p + 1)/2$ independent elements to be called $\mathfrak{L}_I$, and in (ii), by virtue of $MM' = I(p)$, a similar set of $p(p - 1)/2$ from M to be called M_I, and by virtue of $\mathfrak{L}\mathfrak{L}' = I(p)$ a similar set of $pn - p(p + 1)/2$ from $\mathfrak{L}$ to be called $\mathfrak{L}_I$. In this paper is discussed the nature of the transformations (i*a*), from x to $\tilde{T}\mathfrak{L}$, under $\mathfrak{L}\mathfrak{L}' = I(p)$; (i*b*) from x to $\tilde{T}\mathfrak{L}_I$; (ii*a*) from x to $\theta M\mathfrak{L}$, under $MM' = M'M = \mathfrak{L}\mathfrak{L}' = I(p)$; and (ii*b*) from x to $\theta M_I\mathfrak{L}_I$. In this connection certain problems are also posed for mathematical statisticians which the author has not been able to solve so far.

24. **On a Theorem in Jacobians with Statistical Applications.** S. N. ROY, University of North Carolina.

If $F_i(y_1, \cdots, y_m, x_1, \cdots, x_{m+n}) = 0$ $(i = 1, 2, \cdots, m + n)$ are such that we could select any set of m x_i's (to be called without any loss of generality $x_1, x_2, \cdots, x_m$) and could find real values of $(y_1, \cdots, y_m)$ and of $(x_{m+1}, \cdots, x_{m+n})$ for real values of $(x_1, \cdots, x_m)$, then, assuming that the numerator and the denominator on the right-hand side of the equation below are nonvanishing, and assuming certain other restrictions, we would have $J(y_1, y_2, \cdots, y_m : x_1, x_2, \cdots, x_m) = -[\partial(F_1, \cdots, F_{m+n})/\partial(x_1, \cdots, x_{m+n})]/[\partial(F_1, \cdots, F_{m+n})/\partial(y_1, \cdots, y_m, x_{m+1}, \cdots, x_{m+n})]$, where absolute values of the determinants are to be taken. Important special cases of this general theorem with various statistical applications are discussed in this paper.

25. **The Inventory Problem.** A. DVORETZKY, J. KIEFER, AND J. WOLFOWITZ, Cornell University.

The inventory problem is the general problem of what quantities of goods to stock in anticipation of future demand, where loss is caused by inability to supply demand or by stocking goods for which there is no demand. Let x_i be the initial stock of a given commodity in the ith interval $(i = 1, \cdots, N)$ before any ordering is done, and y_i the starting stock after an amount $y_i - x_i \geq 0$ has been ordered and instantaneously received by the stocking agency. The amount demanded in the ith interval is a chance variable with known distribution function F_i. $W_i(x_i, y_i, d_i)$ is the loss incurred in the ith interval when x_i is the starting stock, y_i the initial stock, and d_i is the amount demanded in this interval. F_i, W_i, and the expected value of W_i with respect to the demand may also be functions of the "past history" as given by $\beta_i = \{x_j, y_j, d_j : j < i\}$. An ordering policy Y is a set of functions $Y_i(x_i, \beta_i)$ $(i = 1, \cdots, N)$, where one orders an amount $Y_i(x_i, \beta_i) - x_i$ in the ith interval. With each Y and x_1 there is associated a quantity $A(Y \mid x_1)$ which is the total expected loss over all intervals (the loss in the ith interval being discounted by a factor $1 - \alpha_i$) when Y is used and x_1 is the initial stock in the first interval. An optimal (ϵ-optimal) policy is one which minimizes this quantity (within ϵ) for every x_1. A method for constructing such policies is given. The case of an infinite number of intervals is similarly treated. Analogous results are obtained in more general cases, e.g., when there is a time lag between the ordering and delivery of goods, when there are several commodities, etc.

The second part of the paper deals with the case when the set of distributions F_i is known only to be a member of a certain class Ω. Constructive methods for obtaining Bayes solutions and complete classes are given. (This research was sponsored by the Office of Naval Research.)

NEWS AND NOTICES

Readers are invited to submit to the Secretary of the Institute news items of interest

Personal Items

Mr. Fred C. Andrews, Teaching Assistant and Research Assistant, Statistical Laboratory, University of California, Berkeley, was promoted to Lecturer and Research Assistant effective July 1, 1951.

Dr. Dorothy S. Brady, formerly Professor of Economics at the University of