41. **On Sets of Parameter Points where It Is Possible to Achieve Superefficiency of Estimates.** LUCIEN L. LECAM, University of California, Berkeley.

Let $X$ be a random variable with probability density $f(x \mid \theta)$ depending on a parameter $\theta \in \Omega$; $\Omega$ being a measurable set of points on the real line. Let $X^{(n)} = (X_1, X_2, \cdots, X_n)$ be a sample of $n$ independent observations on $X$. A sequence $\{T_n(X^n)\}$ of measurable functions is called a consistent asymptotically normal (c.a.n.) estimate of $\theta$, with asymptotic variance $\{\sigma_n^2(\theta)\}$, if for every $\theta \in \Omega$, and for every $t$, $\lim_{n\to\infty} p\{(T_n[X^{(n)}] - \theta)/(\sigma_n(\theta)) < t \mid \theta\} = 1/(\sqrt{2\pi}) \int_{-\infty}^{t} e^{-\frac{1}{2}x^2}\, dx$. Assume Cramér's regularity conditions which imply consistency and asymptotic normality of the maximum likelihood estimate of $\theta$ (*Mathematical Methods of Statistics*, Princeton University Press, 1946, p. 500). Let $\{\alpha_n^2(\theta)\}$ be the asymptotic variance of the M.L. estimate. As $n \to \infty$, let $\beta(\theta) = \limsup [\sigma_n(\theta)/\alpha_n(\theta)]$ and $\gamma(\theta) = \lim [\sigma_n(\theta)/\alpha_n(\theta)]$ if this limit exists. An estimate $\{T_n[X^{(n)}]\}$ is called superefficient on $S \subset \Omega$ if it is c.a.n. and if $\beta(\theta) \leqq 1$, for $\theta \in \Omega$ and $\beta(\theta) < 1$ for $\theta \in S$. This set $S$ is called the set of superefficiency. J. L. Hodges produced examples of superefficient estimates. His method of construction will be denoted by $(H)$. THEOREM 1. *Whatever* $\epsilon$, $0 \leqq \epsilon < 1$ *and whatever the closed and reducible set* $S_0 \subset \Omega$, *it is possible to construct superefficient estimates of* $\theta$ *with* $\beta(\theta) \leqq \epsilon$ *on* $S_0$. The method of construction is $(H)$. THEOREM 2. *The set* $S$ *of superefficiency has Lebesgue measure zero.* THEOREM 3. *If* $\gamma(\theta)$ *exists for all* $\theta \in S$ *then, whatever be* $\epsilon$, $0 \leqq \epsilon < 1$, *the subset of* $S$ *where* $\gamma(\theta) \leqq \epsilon$ *is closed and nondense.* THEOREM 4. *Whatever the denumerable set* $S \subset \Omega$, *it is possible to construct* $\{T_n[X^{(n)}]\}$ *c.a.n. on* $\Omega - S$, *with asymptotic variance* $\{\alpha_n^2(\theta)\}$ *and such that for every* $\theta \in S$, *the limit law of* $[T_n - \theta]/\alpha_n(\theta)$, *as* $n \to \infty$, *is more concentrated than the corresponding law of the M.L. estimates.*

42. **Relative Precision of Least Squares and Maximum Likelihood Estimates of Regression Coefficients.** JOSEPH BERKSON, Mayo Clinic.

Three "estimators" of the parameters $\alpha$ and $\beta$ of the logistic function $P_i = 1/(1 + e^{-(\alpha+\beta x_i)})$ as used in bioassay were compared for three equally-spaced values of the dose $x_i$, 10 at each dose: (1) maximum likelihood, (2) minimum (Pearson classic) $\chi^2$, (3) minimum logit $\chi_i$, the first two requiring iterative procedures for evaluation, the last obtainable directly. With central dose at the L.D. 50, the three estimates are unbiased; the variance is smallest for the minimum logit $\chi^2$, next larger for the minimum $\chi^2$, and largest for the maximum likelihood estimate. For dosage arrangements not symmetrical around the L.D. 50, the three estimates are biased, the maximum likelihood estimate positively, the $\chi^2$ estimates negatively; the mean square error is smallest for the minimum logit $\chi^2$, next larger for the minimum $\chi^2$, and largest for the maximum likelihood estimate. For all dose arrangements, the mean square error of the maximum likelihood estimate is larger than $1/I$, those of the $\chi^2$ estimates are less than $1/I$, in accordance with the Cramér inequality for the mean square error. Each of the estimators is sufficient.

## NEWS AND NOTICES

*Readers are invited to submit to the Secretary of the Institute news items of interest.*

### Personal Items

Dr. Kurt W. Back, formerly at Washington, D. C., is now acting as Social Research Analyst on the Air Force Contract for the Bureau of Applied Social Research, a Columbia University project.

Professor Z. W. Birnbaum is on leave of absence from the University of Washington and is Visiting Professor of Statistics at Stanford University for the academic year 1951–52.

Dr. Edward P. Coleman, formerly at Columbia University, is a Mathematical Statistician for Hughes Aircraft Company, Culver City, California, and is teaching engineering statistics at the University of California at Los Angeles.

Dr. William S. Connor, Jr., who has been doing graduate work at the University of North Carolina at Chapel Hill, has joined the Statistical Engineering Laboratory of the National Bureau of Standards, Washington, D. C.

Dr. John T. Dailey has left the Human Resources Research Center, Lackland Air Force Base, San Antonio, Texas, to replace Dr. Brundage as Chief of Classification and Survey Research, Bureau of Naval Personnel, Washington 25, D. C.

Mr. John F. Hofmann, formerly with the Statistical Laboratory, Iowa State College, has taken a position as Mathematical Statistician at Dugway Proving Ground, Tooele, Utah.

Mr. Elvin A. Hoy, formerly Head (Section A), Applied Sciences and Mathematics, Navy Training Publications Center, U. S. Navy, will serve as Analytical Statistician in the Industrial Factors Section, Planning Research Branch, Program Review and Analysis Division, Office of the Comptroller of the Army.

Dr. Eric R. Immel has accepted an instructorship in the Department of Mathematics, University of Wisconsin.

Dr. H. P. Mulholland has accepted an appointment as Lecturer in Pure Mathematics, University of Birmingham, England.

Dr. M. G. Neurdenburg has left the statistical division of the Municipal Medical and Public Health Department of Amsterdam and has been appointed Medical Officer of Public Health in the Netherlands State Public Health Department. His duties will be to start the Cancer Registration in the Netherlands, and to act as Head of the newly created Division of Public Health Statistics in the same Department.

Mr. Don C. Price has accepted a position in the Institute for Cooperative Research, the Johns Hopkins University, Baltimore, Maryland.

Dr. P. Ratoosh has resigned his position as Instructor at the University of Wisconsin and has been appointed Assistant Professor in the Department of Psychology at Ohio State University.

Mr. David Rubinstein, who has been a Research and Teaching Assistant at the University of California, has been appointed as Supervisory Survey Statistician in the Department of Biometrics at the USAF School of Aviation Medicine, Randolph Field, Texas.

Miss Marion Sandomire, formerly employed in the Bureau of the Census, Washington, D. C., has accepted a position as Statistician, Production Division, New York Operations Office of the Atomic Energy Commission.

Dr. S. S. Shrikhande, who is on leave of absence from the College of Science, Nagpur, India, has been appointed Visiting Assistant Professor of Mathematical Statistics at the University of Kansas, Lawrence, for the academic years 1951 to 1953.

Mr. Waldo A. Vezeau has been promoted to Associate Professor of Mathematics at St. Louis University.

Mr. Harry Weingarten has accepted a position with the Quality Control Division, Bureau of Ordnance, Navy Department, Washington, D. C.

Dr. R. K. Zeigler, formerly Associate Professor at Bradley University, Peoria, Illinois, has accepted a position as Statistician with the Atomic Energy Commission, Oak Ridge, Tennessee.

Dr. P. V. Sukhatme, Chief of the Statistics Branch, Food and Agriculture Organization of the United Nations, will be Visiting Professor of Statistics at Iowa State College during the Spring Quarter, beginning March 27, 1952; he will give lectures in advanced survey sampling.

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A copy of the first issue of *Veröffentlichungen des Deutschen Aktuarvereins* has been received. The publication includes actuarial, probability and statistical papers related to insurance.

In January 1950 there was created in the Superior Council of Scientific Research of Spain the Department of Statistics (Calle Serrano, 123, Madrid), whose Director is Professor Sixto Ríos, Professor of Mathematical Statistics at the University of Madrid. The purpose of this department is scientific research in the various branches of statistics and its applications. It publishes every four months the review *Trabajos de Estadística*, and in this department seminars have been conducted by Professors Fréchet, Cramér, Wold, and Mahalanobis. Professor Herman Wold, Director of the Institute of Statistics at the University of Uppsala, conducted during November 1951 a series of seminars on statistical inference and stochastic processes.

**Biostatistics Conference, June 16–July 23, 1952, Iowa State College, Ames, Iowa**

A biostatistics conference has been scheduled for June 16 to July 23, 1952, at Iowa State College, Ames, Iowa. It is sponsored by faculty members working in agriculture, biology, and statistics at Iowa State College and by the Biometric Society (ENAR). The plan of the program is that each morning a biologist will present a problem, outline the objectives, describe techniques suitable for the experiment and analysis. A paired statistician will discuss suitable experimental designs and statistical and mathematical methods for attacking the problem. These speakers will preside at a general discussion period of the same topic the same afternoon.

The program is tentatively arranged in five somewhat separate weekly units as follows:

*First week:* Development of Quantitative Biology
*Second week:* Specification of Populations and Their Processes
*Third week:* The Estimation of Populations
*Fourth week:* Individual Growth
*Fifth week:* Biomathematical Mechanisms Within the Individual and Species.

Invited speakers include: Edgar Anderson, Geoffrey Beall, Joseph Berkson, Chester I. Bliss, P. W. Bridgman, Samuel Brody, C. West Churchman, William G. Cochran, C. C. Cockerham, Jerome Cornfield, Charles W. Cotterman, James F. Crow, S. L. Crump, D. B. DeLury, Gordon E. Dickerson, Harold F. Down, A. M. Dutton, W. T. Federer, Robert P. Gage, C. B. Godbey, A. A. Hasel, P. G. Homeyer, John W. Hopkins, H. Hotelling, S. Isaacson, O. Kempthorne, H. H. Kramer, Warren Leonard, Howard Levene, H. L. Lucas, S. E. Luria, J. L. Lush, William G. Madow, Walter M. Meyer, Lloyd Miller, H. J. Muller, H. C. Murphy, Jerzy Neyman, H. W. Norton, Thomas Park, N. Rashevsky, A. S. Rosenblueth, L. W. Scattergood, Franz Schrader, John P. Scott, G. W. Snedecor, G. W. Stewart, P. C. Tang, D. J. Thompson, John W. Tukey, A. S. Wiener, Norbert Wiener, Frank Wilcoxon, Edwin B. Wilson, Sewall Wright, M. R. Zelle, J. A. Zoellner.

Rooms will be available in the college dormitories at the usual rates. For more detailed information write: T. A. Bancroft, Director, Statistical Laboratory, Iowa State College, Ames, Iowa.

### Summer Sessions in Berkeley, California

This year's summer program at the Statistical Laboratory of the University of California, Berkeley, California, consists of two sessions, June 23–August 2 and August 4–September 13. The program includes 2 of the usual undergraduate courses, one in each session, as well as one new course in each session. In the first session the new course being offered is called: "Statistical methods of searching for causal relationships." The course is designed to acquaint the students with statistical methods of approaching practical problems with particular reference to correlation and causality and to the pitfalls which studies of this kind frequently involve.

The faculty of the summer sessions will include Dr. Grace E. Bates, Assistant Professor, Department of Mathematics, Mt. Holyoke College, South Hadley, Mass.; Professor J. Neyman, Dr. E. Fix, Dr. G. Kallianpur, Mr. L. LeCam, of the Statistical Laboratory, University of California. Professor J. Neyman will be available for consultations on work leading to higher degrees.

## New Members

*The following persons have been elected to membership in the Institute*

(August 23, 1951 to December 1, 1951)

**Barberi, Benedetto,** Ph.D., Director General of the Central Institute of Statistics, Via Balbo 16, Rome, Italy.

**Bates, Grace E**, Ph.D. (Univ. of Ill.), Assistant Professor of Mathematics, Mount Holyoke College, South Hadley, Massachusetts.

**Bignardi, Francesco,** Teacher of Social Statistics, University of Palerme, *Viale Albertazzi 16 III, Bologna, Italy.*

**Brenna, Svein,** M.A. (Univ. of Oslo), Consultant, Central Bureau of Statistics, *Ole Moe's vei 35, Nordstrandshogda, Norway.*

**Breny, Henri,** Ph.D. (Univ. of Liège), Lecturer at the École Superieure des Textiles, Verviers, *528, Rue Haveigne, Fraipont, pce le Liège, Belgium.*

**Brownlee, Kenneth A.,** M.A. (Cambridge), Chief, Test Design Branch, Plans and Evaluation Office, Technical Operations, Dugway Proving Ground, Tooele, Utah.

**Cox, Edwin L.,** M.S. (Virginia Polytech. Inst.), Mathematical Statistician, Plans and Evaluation Office, Technical Operations, Dugway Proving Ground, Tooele, Utah.

**Garner, Norman R.,** M.S. (North Carolina State Coll.), Analytical Statistician, *Apt. 4-C, Woodlawn Court Apartments, Aldan, Del. County, Pennsylvania.*

**Garrett, John E.,** B.S. (A&M College of Texas), Process Engineer, Film Research and Development Department, Olin Industries, Inc., P.O. Drawer 906, New Haven, Connecticut.

**Grings, William W.,** Ph.D. (Univ. of Iowa), Associate Professor of Psychology, University of Southern California, *8020 Agnew Avenue, Los Angeles 45, California.*

**Homeyer, Paul G.,** M.S. (A&M College of Texas), Professor of Statistics, Statistical Laboratory, Iowa State College, Ames, Iowa.

**Luders, Johann D.,** Ph.D. (Univ. of Bonn), Chief, Statistical Methods Section, Regional Statistical Office of North Rhine-Westphalia, Statistisches Landesamt, Haroldstrasse 37, Düsseldorf, Germany.

**Malmquist, Sten,** Fil. Dr. (Univ. of Uppsala), Docent, Institute of Statistics, University of Uppsala, Uppsala, Sweden.

**Moan, Obert B.,** M.S. (Univ. of Minnesota), Quality Control Analyst, Julius Hyman and Company, Denver 1, Colorado.

**Powell, James H.,** M.A. (Mich. State College), Graduate Assistant, *904-C Maple Lane, East Lansing, Michigan.*

**Rosenberry, L. Porter,** B.S. (Pa. State College), Mathematical Physicist, Mathematical and Theoretical Physics Section, Explosives and Physical Sciences Division, Bureau of Mines, 4800 Forbes Street, Pittsburgh 13, Pennsylvania.

**Rosenblatt, Murray,** Ph.D. (Cornell Univ.), Research Associate and Assistant Professor, Committee on Statistics, Eckhart Hall, University of Chicago, Chicago, Illinois.

**Shaw, Donald J.,** B.A. (Hope College, Holland, Mich.), Quality Control Engineer, E. I. duPont de Nemours & Co., Inc., *230 Isle Avenue, Waynesboro, Virginia.*

**Singleton, Marcus G.,** M.S. (Univ. of North Carolina), Member of Engineering Staff, North American Aviation, Inc., *1455 West 97th Street, Los Angeles 47, California.*

**Stange, Kurt,** Ph.D. (Göttingen), Dozent for applied mathematics and mechanics and acting for Director, Institute für Mathematik und ihre technischen Anwendungen, Technische Hochschule (17a), Karlsruhe, Germany.

**Terry, Milton E.,** Ph.D. (Univ. of North Carolina), Associate Professor of Statistics, Virginia Polytechnic Institute, Blacksburg, Virginia.

**Venkatasubbiah, Gargeswari,** B.T. (Mysore Univ.), Consulting Actuary and Professor of Statistics and Actuarial Science, B.M. College of Commerce, *894 Amrai Camp, Poona 4, India.*

**Yamauti, Ziro,** Ph.D. (Tokyo Imperial Univ.), Professor, Faculty of Engineering, University of Tokyo, *Iogi-cho, 2-chome, No. 1, Suginami-ku, Tokyo, Japan.*

## REPORT OF THE WASHINGTON MEETING OF THE INSTITUTE

The forty-ninth meeting of the Institute of Mathematical Statistics was held in Washington, D. C., at the National Bureau of Standards on October 26–27, 1951, in conjunction with the October, 1951, meetings of the American Mathematical Society and of the Washington Section of the American Society for