step towards the possibility of using Wilcoxon's test for samples from *any* population.

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CORRECTION TO "ON CERTAIN METHODS OF ESTIMATING THE LINEAR STRUCTURAL RELATION"

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We are indebted to Professor J. Wolfowitz for calling our attention to a blunder in our paper under the above title (*Annals of Math. Stat.*, Vol. 22 (1951), pp. 352–361). In the statement of Theorem 3 on page 358 the symbols ξ_{p_1} and ξ_{1-p_2} should be replaced by X_{p_1} and X_{1-p_2}, respectively. It will be noticed that this change does not affect the proof nor the implications of the theorem.

ABSTRACTS OF PAPERS

(Abstracts of papers presented at the Washington meeting of the Institute, October 26–27, 1951)

1. On the Law of Propagation of Error. (Preliminary Report.) Churchill Eisenhart and I. Richard Savage, National Bureau of Standards.

In the main the results presented in this paper are not new, being at most minor extensions of known results. The aim is a unified treatment of the "law of propagation of error," with emphasis on the practical meaning of the formulas, and attention to the details of their rigorous derivation.

2. Multivariate Orthogonal Polynomials. (Preliminary Report.) L. W. Cooper and D. B. Duncan, Virginia Polytechnic Institute.

It is well known that the work of fitting a regression function, which is a polynomial in one variate, viz., (1) $y = \Sigma_{i=0}^{r} b_i x^i$ can be greatly simplified by the use of orthogonal polynomials of the form (2) $\epsilon_i' = \Sigma_{j=0}^{i} k_j x^j$. It is sometimes required to fit a regression function of the more complex multivariate polynomial form

(3) $$y = \sum_{\substack{i=0,1,\cdots,r\\ j=0,1,\cdots,s\\ k=0,1,\cdots,t}} b_{i,j,\cdots,k}\, x^i y^j \cdots z^k.$$

Following suggestions of Tyler (TYLER, G. W., "The experimental evaluation of definite integrals," unpublished thesis, Virginia Polytechnic Institute, Blacksburg, Va., 1949) and DeLury (DELURY, D. B., *Values and Integrals of the Orthogonal Polynomials up to n = 26*, University of Toronto Press, 1950), polynomials can be defined as $\epsilon_{i,j,\cdots,k} = \epsilon_i' \cdot \epsilon_j' \cdots \epsilon_k'$, which effects the same simplicity for fitting the functions (3). These are termed multivariate orthogonal polynomials. Their properties are investigated and short methods for using them are developed.

3. **An Analysis of Variance for Paired Comparisons.** HENRY SCHEFFÉ, Columbia University.

In a paired comparison test of m brands of a product each of the $\frac{1}{2}m(m - 1)$ pairs is presented to $2r$ judges: to r in one order, and to r in the other. An analysis of variance is developed for the case in which the judges' preferences are expressed on a 7- or 9-point scale. Account is taken of the effects of order of presentation. Main effects are defined for the brands. The hypothesis of subtractivity, analogous to the hypothesis of additivity in a two-way layout, states roughly that the results for any pair, after order effects are eliminated, can be attributed entirely to the difference of the main effects of the two brands in the pair. F-tests for the hypothesis of subtractivity and for the significance of the main effects are given, as well as estimates of various parameters and their standard errors. A numerical example illustrates the method. (Work sponsored by the office of Naval Research.)

4. **Statistical Theory of Fatigue Failures.** E. J. GUMBEL, Consultant, Stanford University, AND A. M. FREUDENTHAL, Columbia University.

The interpretation of the results of fatigue tests is made difficult by the fact that progressive damage, which finally leads to fatigue failure, is a highly structure-sensitive process. It produces, therefore, a very wide scatter of the test results for the rational analysis of which the character of the statistical distribution must be known. If n specimens are submitted to repeated stress cycles of amplitudes S they break at varying numbers N of cycles. The interpretation of the relative ranks as cumulative frequencies of survival for N cycles at different stress amplitudes leads to criteria for the consistency of the observed series. The frequencies of survival are reproduced by the third asymptotic probability of smallest values, and it is assumed in first approximation that the probability of survival reaches unity only for $N = 0$. If the decimal logarithms of N are traced on the extremal probability paper in descending scale at the plotting positions $m/(n + 1)$, the two parameters in the survivorship function which depend upon S may be estimated in the same way as for the first asymptotic probability function of extreme values. The fit of the theoretical straight lines obtained for copper, steel, and aluminum specimens is very good. Extrapolations give the number of cycles for which the probability of survival for a given stress level differs from unity by any desired small value. (Work done in part under the sponsorship of the Office of Naval Research.)

5. **Analysis of Chain Block Designs. (Preliminary Report.)** W. S. CONNOR, JR., AND W. J. YOUDEN, National Bureau of Standards.

One and a fraction replications can be arranged in incomplete blocks so that there is a carry over of two (or more) treatments from one block to the next block. Estimates of the treatment and block effects and the analysis of variance have been obtained for these chain-block designs.

6. **An Approximation Theorem. (Preliminary Report.)** I. RICHARD SAVAGE, National Bureau of Standards.

The distribution of a function of a sample mean is studied and the rapidity at which this function's distribution approaches normality is investigated. Berry's theorem about the distributions of sample means is used. The theorem proved gives a remainder term in the case where the function is increasing and has a continuous second derivative. The remainder is the same as Berry's, plus another term which depends on the second derivative of the function and is of size $1/\sqrt{n}$.

7. **Testing Multiparameter Hypotheses.** E. L. LEHMANN, Stanford University and University of California, Berkeley.

Let the distributions of some random variables depend on real parameters $\theta_1, \cdots, \theta_s$ and consider the hypothesis $H: \theta_i \leqq c_i$ for $i = 1, \cdots, s$. It is shown under certain regularity assumptions that unbiased tests of H do not exist. Tests of minimum bias and other types of minimax tests are derived under suitable monotonicity conditions. The two-sided hypotheses $H': a_i \leqq \theta_i \leqq b_i, i = 1, \cdots, s$ are discussed as well as certain related multi-decision problems.

8. **Analysis of a Certain Random Walk by the Monte Carlo Method.** ROBERT MIRSKY, Cornell Aeronautical Laboratory.

A point object $B(x, y)$ starting from $(d, 0)$ moves toward its ultimate destination $T(0, 0)$ in the following manner: every τ seconds B takes a sight reading on T and attempts to follow the rectilinear course until the next reading is taken; the speed of B is a constant v. Because of imperfections of the sight reading instruments, the actual course followed at each time deviates from the true line of sight BT so that in general a zigzag path is determined. The angular errors $\alpha_1, \alpha_2, \cdots$, are assumed to be symmetric about BT, independent, and to have the same distribution at each time τ. This random walk was studied by M. Kac who obtained several results which are compared in this paper with results obtained by Monte Carlo sampling. It is shown that while statistical analysis can be used to check the accuracy of the Monte Carlo process, Monte Carlo results can at the same time be used to determine the validity of analytic formulas whose derivation involved simplifying assumptions or approximations. (This work was carried out under the sponsorship of the Office of Naval Research.)

9. **On Certain Estimators Based on Large Samples of Extremes. (Preliminary Report.)** JULIUS LIEBLEIN, National Bureau of Standards.

E. J. Gumbel and B. F. Kimball have given estimators for the parameters of the asymptotic distribution of largest values, Prob $\{X \leqq x\} = \exp[-e^{-\alpha(x-u)}]$, which have been applied in the analysis of data in large samples. The present paper applies the theory of order statistics to the problem of seeking more efficient estimators which are at the same time simpler to compute. Several large-sample estimators are found which, with one exception, appear to have greater efficiency than those derived from the methods of Gumbel and Kimball, yet require much less effort in computation. If punch cards are used the work can by handled by a mechanical sorter which ranks the observations in order of size and then selects a small number of them with predetermined ranks. (This work was sponsored by the National Advisory Committee for Aeronautics.)

10. The Use of Previous Experience in Reaching Statistical Decisions. J. L. Hodges, Jr., University of California, Berkeley, and E. L. Lehmann, Stanford University and University of California, Berkeley.

Instead of minimizing the maximum risk it is proposed to restrict attention to decision procedures whose maximum risk does not exceed the minimax risk by more than a given amount. Subject to this restriction one may wish to minimize the average risk with respect to some guessed a priori distribution suggested by previous experience. It is shown how Wald's minimax theory can be modified to yield analogous results concerning such restricted Bayes solutions. A number of examples are discussed.

11. Results of Some Tests of Randomness on Pseudo-random Numbers. (Preliminary Report.) J. M. Cameron, National Bureau of Standards.

A method for generating random numbers on automatic computing machines (such as the SEAC) has been developed by Dr. Olga Taussky-Todd. Using this method about 2^{40} pseudo-random numbers can be generated by taking residues (mod 2^{42}) of successive powers of 5^{17}. The results of some tests of the randomness applied to these numbers are presented. The evidence from these tests is in agreement with the hypothesis of randomness.

(Abstracts of papers presented at the Boston meeting of the Institute, December 26–29, 1951)

12. Two Rank Order Tests Which Are Most Powerful against Specific Parametric Alternatives. Milton E. Terry, Jr., Virginia Polytechnic Institute.

The most powerful rank order tests of the hypotheses that two samples come from the same population and that in each of k groups of two samples the two samples came from a common population are considered, and most powerful rank order tests against certain normal alternatives are derived. For the two tests of immediate practical importance, asymptotic, approximate, and (for certain small sample sizes) exact distributions are given. The relationship of these tests with others are investigated.

13. Partially Balanced Designs with $k > r = 3$, $\lambda_1 = 1$, $\lambda_2 = 0$. R. C. Bose and W. H. Clatworthy, University of North Carolina.

Incomplete block designs with a few replications are of practical importance to experimenters. Partially balanced designs with $k > r = 2$ have been studied by one of the authors (R. C. Bose, "Partially balanced incomplete block designs with two associate classes involving only two replications," *Calcutta Stat. Assoc. Bull.*, Vol. 3 (1951), pp. 120–125). The present paper extends this investigation to the case $k > r = 3$, $\lambda_1 = 1$, $\lambda_2 = 0$. It has been shown that only three types of partially balanced designs belong to this class, viz. (a) designs obtained by dualizing balanced incomplete block designs with $k = 3$, $\lambda = 1$; (b) lattice designs with $r = 3$; (c) partially balanced designs belonging to the series $v = (t + 2)(2t + 3)$, $b = 3(2t + 3)$, $r = 3$, $k = t + 2$, $n_1 = 3(t + 1)$, $n_2 = 2(t + 1)^2$, $p_{11}^1 = t$. It has been shown that for the series (c) the only combinatorially possible design for which $k > 3$, is the design with $t = 3$. The cases $t = 0$ and $t = 1$, though not belonging to the class $k > r$, are combinatorially possible cases. Designs corresponding to all other values of t have been shown to be impossible.

14. Some Observations on the F-Test in Analysis of Variance. S. N. Roy, University of North Carolina.

It is well known that analysis of variance deals with a class of problems which can be reduced to a problem of testing of a (composite) linear hypothesis (within a certain model) and for which the test in common use is the F-test; and within the last few years several optimum properties of this test have been brought out by different workers. It was also noted in 1948 by Lehmann and Stein (E. LEHMANN AND C. STEIN, "Most powerful tests of composite hypotheses I. Normal distributions," *Annals of Math. Stat.*, Vol. 19 (1948), pp. 495–516) and the author (S. N. ROY, "Notes on testing of composite hypotheses—II," *Sankhyā*, Vol. 9 (1948), pp. 19–38) that for this problem of testing of a (composite) linear hypothesis it is possible to construct an infinite class of similar region tests among which there is a most powerful test against a specific (composite) alternative (which differs from alternative to alternative). The corresponding statistic (whose structure depends upon the alternative) has the t distribution, when the hypothesis to be tested is true; and the test itself could be properly called a generalized t-test, since the ordinary t-test could be shown to be a special case of it. This paper works out a connection between the ordinary F-test and this generalized t-test, showing that the F-test at a level of significance, say α, could be derived in a certain manner from the infinite class of (most powerful) t-tests of the hypothesis at a level $\beta(<\alpha)$ against the infinite class of possible specific (composite) alternatives.

15. **The Neyman-Pearson Lemma Factor Functions.** L. M. COURT, American Power Jet Company, New York.

According to the Neyman-Pearson lemma, recently proved necessary as well as sufficient by Dantzig and Wald ("On the fundamental lemma of Neyman and Pearson," *Annals of Math. Stat.*, Vol. 22 (1951), pp. 87–93), the optimum critical region w of size α for testing $p_0(x) \equiv p_0(x_1, \cdots, x_n)$ against $p_1(x)$ is given by w: $p_1(x) \geqq kp_0(x)$, where k is a suitable constant (factor). There obviously is a relationship between the size α and the factor k, i.e., $\alpha = \tau(k)$ or $k = \tau^{-1}(\alpha)$, where the functions τ and τ^{-1} need not be as naive as the ones encountered in the elementary calculus. The writer establishes four properties of $\tau(k)$ and $\tau^{-1}(\alpha)$: (1) to any value of k there corresponds in general a closed interval of α-values (which may reduce to a point); (2) to any value of α, there corresponds in general an interval of k-values, open at the bottom and closed at the top; (3) $\tau(k)$ is a nonincreasing function of k in a generalized sense; and (4) $\tau^{-1}(\alpha)$ is a nonincreasing function of α in a generalized sense. These properties are extended, with some restrictions, to the functions $\alpha_i = \tau_i(k_1, \cdots, k_m)(i = 1, \cdots, m)$ and their inverses, where $k_1, \cdots, k_m$ are the factors resulting when a region w is sought maximizing $\int_w f_{m+1}(x)\, dx$ subject to $\int_w f_i(x)\, dx = \alpha_i(i = 1, \cdots, m)$, this region being characterised by $f_{m+1}(x) \geqq k_1p_1(x) + \cdots + k_mp_m(x)$—the general Neyman-Pearson lemma. (Every $f_i(x)$ is assumed nonnegative throughout R_n.)

16. **The Probability of a Correct Ranking. (Preliminary Report.)** ROBERT E. BECHHOFER, Columbia University.

Let X_{ij} be normally and independently distributed with mean μ_i and unit variance $(i = 1, 2, \cdots, k; j = 1, 2, \cdots, N)$. Let $\mu[1], \mu[2], \cdots, \mu[k]$ be the ordered μ_i; let $\bar{X}(i)$ be the sample arithmetic mean associated with the population having mean $\mu[i](i = 1, 2, \cdots, k)$. The μ_i are unknown; it is not known which population is associated with $\mu[i]$. Specify positive integers $r_1, r_2, \cdots, r_s(s \leqq k)$ such that $\Sigma_{i=1}^{s} r_i = k$. Define "indifference regions" δ_i^* as the smallest differences, $\mu[i+1] - \mu[i]$, $(i = 1, 2, \cdots, k-1)$, which it is desired to detect. Define the symbol $S_j = \Sigma_{i=0}^{j} r_i$. We wish to determine $P_N(\delta_1^*, \delta_2^*, \cdots, \delta_{k-1}^*) = \text{Inf for } \delta_i > \delta_i^*(i = 1, 2, \cdots, k-1)$ of Prob $[\text{Max}\{\bar{X}(S_j + 1), \cdots, \bar{X}(S_{j+1})\} < \text{Min}\{\bar{X}(S_{j+1} + 1), \cdots, \bar{X}(S_{j+2})\}$ simultaneously for $j = 0, 1, 2, \cdots, s-2$ with $r_0 = 0]$. The required probability is expressed as a volume under a $(k-1)$-variate normal surface. For given $\beta(0 < \beta < 1)$, the desired N is the smallest one for which

$P_N(\delta_1^*, \delta_2^*, \cdots, \delta_{k-1}^*) \geqq 1 - \beta$. An analogue of power is defined. A method of making confidence statements is described. It is shown that many analysis of variance (Model I) problems can be more meaningfully formulated as problems involving multiple ranking of means, and how experimental designs (randomized blocks, Latin squares, etc.) can be used to increase the probability of a correct ranking. A procedure similar to the one described above is applied to the ranking of variances when the population means are known or unknown, the required probabilities being expressed as volumes under $(k - 1)$-variate generalized F-distribution surfaces. Other directions of generalization are indicated. (Part of this work was done under the sponsorship of the ONR.)

17. A Nonparametric Analogue Based upon Ranks of One-way Analysis of Variance. William Kruskal, University of Chicago.

Given independent random variables $\xi_j^{(i)} (i = 1, \cdots, C; j = 1, \cdots, n_i; \Sigma n_i = N)$ with continuous distribution functions $\Pr\{\xi_j^{(i)} \leq x\} = F_i(x)$, one may wish to test the null hypothesis: $F_1 = F_2 = \cdots = F_C$ against alternatives of form $F_i(x) = F(x - \theta_i) (i = 1, \cdots, C)$ with not all the θ's equal. The following test (essentially proposed by Wallis) is discussed: replace the ξ's by their ranks in the N-fold sample, $X_j^{(i)}$, and compute $H = [12/(N(N + 1))]\Sigma_{i=1}^{C}(1/n_i)(R_i - \frac{1}{2}n_i(N + 1))^2$, where $R_i = \Sigma_{j=1}^{n_i} X_j^{(i)}$; reject if H is too large. This test is a generalization of the symmetrical two-tail version of the Wilcoxon-Mann-Whitney test, and is also equivalent to the use of the standard F-test for one-way analysis of variance after replacement of observations by ranks. H is shown to be asymptotically chi-square with $C-1$ degrees of freedom. A condition for consistency is stated and given an intuitive interpretation; the translation alternatives mentioned above satisfy this condition, but so do many others. The variance of H under the null hypothesis and the maximum value of H are obtained explicitly and their use in approximating the distribution of H is suggested. The possibly discontinuous case is considered, and a method for handling ties proposed by Wallis is discussed.

18. A Series of Group Divisible Designs for Two-way Elimination of Heterogeneity. S. S. Shrikhande, University of Kansas.

From the affine resolvable design $v = s^2, b = s^2 + s, r = s + 1, k = s, \lambda = 1$ omit a complete replication, and from the remaining blocks omit the treatments lying in any $n(\leq s - 1)$ blocks of the omitted replication. We get a group divisible design with $v = s\ (s - n), b = s^2, r = s, k = s - n$, where the v treatments can be divided into $s - n$ groups of s each where any two treatments of the same group do not occur together in any block, whereas any two treatments coming from different groups occur together in just one block. This design can be used for two-way elimination of heterogeneity by suitably interchanging the positions of treatments in the various blocks if necessary. All treatment comparisons are made with at most two accuracies.

19. A Test of the Uniformity of a Circular Distribution. (Preliminary Report.) J. Arthur Greenwood, Manhattan Life Insurance Co., and David Durand, National Bureau of Economic Research.

Let $X_1, \cdots, X_n$ be a sample from an unknown distribution on the circumference of a circle. To test the hypothesis that the distribution is uniform, the use of the statistic A is proposed, where $n^2A^2 = (\Sigma_{i=1}^{n} \cos X_i)^2 + (\Sigma_{i=1}^{n} \sin X_i)^2$. The distribution of A is essentially given by the solution of Pearson's random walk problem (K. Pearson, "Mathematical contributions to the theory of evolution. XV. A mathematical theory of random migration," *Drapers' Co. Res. Mem. Biometric Series 3*, Cambridge University Press, 1906) and is $P\{A \leq a\} = \int_0^\infty [J_0(t)]^n \, na \, J_1(nat) \, dt$; this distribution is also obtained readily by the use of characteristic functions. For the alternatives given by the cyclical distribution of von Mises (Über die "Ganzzahlbarkeit" der Atomgewichte und verwandte Fragen," *Physik. Zeitschr.*, Vol. 19 (1918), pp. 490–500) the method of characteristic functions gives the power function in terms of similar but less tractable integrals.

20. A Method for Limit Theorems in Markov Chains. T. E. Harris, The Rand Corporation.

The following Markov process has been used by Mosteller in a psychological application. Let $x_0, x_1, \cdots$ be the random variables of the process, $0 \leq x_i \leq 1$. Let $S(x) = \sigma x$ and $T(x) = \alpha + (1 - \alpha)x$, $0 < \alpha, \sigma < 1$, be two linear functions of x, and let $f(x) = a + bx$ be a third linear function such that $0 < f(x) < 1$ when $0 \leq x \leq 1$. The transition law is as follows. Suppose x_n is given. Then $x_{n+1} = S(x_n)$ with probability $f(x_n)$ and $x_{n+1} = T(x_n)$ with probability $1 - f(x_n)$. It is shown that the distribution of x_n approaches a limiting distribution independent of x_0 as $n \to \infty$. The method can be modified to give a proof of the "ergodic" theorem for Markov processes with discrete states and also, in the generalization of renewal theory by Chung and Pollard, to random variables with positive and negative values, to remove the restriction to distributions with an absolutely continuous component.

21. On Tests of Certain Hypotheses about Multivariate Normal Populations. S. N. Roy, University of North Carolina.

Large classes of problems in multivariate analysis can be brought under one or other of the three problems of testing of the composite hypotheses of (i) equality of the dispersion matrices for two p-variate normal populations; (ii) equality of the means (of each separate variate) for $K(\geq 2)$ p-variate normal populations; (iii) independence between two sets of variates p_1 and p_2 under a $(p_1 + p_2)$-variate normal distribution. It is partly shown in previous papers by different workers and more fully shown here that, if we take over and carry through the idea behind discriminant analysis, we would get in each case a test which, for case (i), would be based on the largest and smallest roots, and, for cases (ii) and (iii), on the largest root of certain determinantal equations (different for the three cases). Such tests would have all the known desirable properties of other possible similar region tests (including the likelihood ratio tests) of the composite hypotheses concerned. The main purpose of the present paper, however, is to show that the three tests given here (for the three cases), at any level, say α, could each be derived in a certain manner from an infinite class of most powerful tests at another level $\beta(<\alpha)$ against different possible (composite) alternatives.

22. An Inequality for Orthogonal Arrays of Strength 2. S. S. Shrikhande, University of Kansas.

A matrix $A(a_{ij})$ with l rows and N columns where each element a_{ij} is one of the n integers $1, 2, \cdots, n$ is called an orthogonal array of strength 2 if for every pair (i_1, i_2) of two rows, the pairs (a_{i_1j}, a_{i_2j}), $j = 1, 2, \cdots, N$ contain each of the n possible pairs exactly μ times $(N = \mu n^2)$. The array is said to be of size N, l constraints and n levels. It is known that $l_{\max} \leq I((\mu n^2 - 1)/(n - 1))$, where $I(x)$ is the largest integer contained in x. If n and t are integers $(n \geq 2, t \geq 0)$, then with $N = n^2((n - 1)t + 1)$, where n is the number of levels, the above inequality gives $l_{\max} \leq n^2t + n + 1$. Using a result due to Plackett and Burman ("The design of optimum multifactorial experiments," *Biometrika*, Vol. 33 (1946), pp. 305–325) it can be shown that $l_{\max} \leq n^2 t + n$ in the three cases of impossibility of affine resolvable balanced incomplete block designs announced at the thirteenth summer meeting of the Institute of Mathematical Statistics in September, 1951.

23. The Distribution of the Range in Samples from a Compound Normal Population. KENNETH H. KRAMER, Youngstown Sheet and Tube Company.

The distribution of the range, R, in samples of n observations from a population having the distribution function $F(x) = p\Phi(x; m, \sigma) + q\Phi(x; 0, 1)$, where $\Phi(x; m, \sigma) = 1/(\sqrt{\pi 2}\sigma) \int_{-\infty}^{x} \exp\{-(t - m)^2/(2\sigma^2)\}\, dt$, $0 \leq p \leq 1$, is considered. This type of distribution provides a good model for many industrial processes, and the range is used extensively in industrial statistics. For $n = 2, 3$, the range distribution function $G_n(R) = n \int_{-\infty}^{\infty} [F(x + R) - F(x)]^{n-1}\, dF(x)$ is integrated, giving $G_2(R)$ in terms of integrals of the normal frequency function, and $G_3(R)$ in terms of integrals of the bivariate normal frequency function over rectangular regions. These expressions for $G_2(R)$ and $G_3(R)$ are then used to derive similar expressions for the mean range, $\bar{R}$, and the standard deviation of the range, σ_R. To compute $G_n(R)$ for $n > 3$, an expression by Hartley, (E. S. PEARSON, "The probability integral of the range in samples of n observations from a normal population. I. Foreword and tables," *Biometrika*, Vol. 32 (1942), pp. 301–308. H. O. HARTLEY, "The probability integral of the range in samples of n observations from a normal population. II. Numerical evaluation of the probability integral," *Biometrika*, Vol. 32 (1942), pp. 309–310. H. O. HARTLEY, "The range in random samples," *Biometrika*, Vol. 32 (1942), pp. 334–348, especially pp. 341, 342) is generalized. Tables given by Hastings, Mosteller, Tukey, and Winsor, ("Low moments for small samples: a comparative study of order statistics," *Annals of Math. Stat.*, Vol. 18 (1947), pp. 413–426) provide a basis for approximate formulas for $\bar{R}$ and σ_R (all n); these formulas are asymptotic in m, and in the case of $\bar{R}$ give a lower bound on the exact value. Tables and graphs of $G_n(R/\sigma_c)$ and $\bar{R}/\sigma_c$, where $\sigma_c^2 = p\sigma^2 + q + pqm^2$ is the variance of the compound normal population, are given for $p = 0, \frac{1}{4}, \frac{1}{2}, m = 0, 1, 2, 2, \sigma = 1, 2, \Delta R = 1, n = 2, 3, \cdots, 20$. The tables are then used to construct power curves for the Shewhart control chart for ranges under various types of alternatives.

24. On the Operating Characteristics of Certain Quality Control Tests. JOHN E. WALSH, U. S. Naval Ordnance Test Station, Pasadena.

This paper presents values of the operating characteristic (OC) function for a common type of quality control test and for two possible substitutes for this test. The situation considered is that of small samples from a normal population. The common type of quality control test investigated is based on the sample mean and the sample standard deviation (using $n - 1$). One of the substitute tests is based on the t-statistic and the sample standard deviation. The other substitute test is based on the sample mean and the estimate of the population standard deviation obtained by using the mean of the population which determined the control limits. Each of the three types of tests is found to have regions where its operating characteristics are poor. No one type of test has uniformly better operating characteristics than the others. For each type of test there exist regions where its operating characteristics are superior to those of the other two. On the whole, however, the tests based on the t-statistic and the sample standard deviation appear to be inferior to the other two types, which are roughly equivalent. An extensive OC function analysis is presented for the common type of quality control test. This analysis furnishes a fairly comprehensive picture of the operating characteristics for this kind of test.

25. Operating Characteristic of the Control Chart for Sample Means. (Preliminary Report.) Edgar P. King, Carnegie Institute of Technology.

A study is made of the Type I and Type II errors of the control chart for sample means in the case where process standards are unspecified. Under the null hypothesis, the distribution of the process is $N(\mu, \sigma^2)$, where μ and σ are unknown constants. Under the alternative, the process mean is a random variable with a $N(\mu, \theta^2\sigma^2)$ distribution ($\theta > 0$). The two types of errors are tabulated for cases ranging from 2 samples of size 2 to 4 samples of size 10. Bounds on these errors are tabulated for cases ranging from 5 samples of size 2 to 25 samples of 10. The effect of altering the traditional "3-sigma" limits is investigated and the power is compared with that of the corresponding analysis of variance test.

26. Joint Sampling Distribution of the Mean and Standard Deviation for Frequency Functions of the Second Kind. Melvin D. Springer, U. S. Naval Ordnance, Indianapolis.

The joint sampling distribution of $\bar{x}$ and s is derived for frequency functions of the second kind, i.e., for frequency functions defined on the interval $(0, \infty)$. The joint distribution has the integral form

$$F(\bar{x}, s) = \iint \cdots \int f(x_1)f(x_2) \cdots f(x_{n-2}) \cdot f(\tfrac{1}{2}[n\bar{x} - \Sigma_1^{n-2} x_i - \Omega_1^{(n-2)}])f(\tfrac{1}{2}[n\bar{x} - \Sigma_1^{n-2} x_i + \Omega_1^{(n-2)}])$$

$2n^2 s/\Omega_1^{(n-2)} dx_{n-2} \cdots dx_2\, dx_1$, where the limits of integration of $x_{n-r}(r = 2, 3, \cdots, n-1)$ are given in terms of $\bar{x}$, s, and $x_{n-r-i}(i = 1, 2, \cdots, n-r-1)$ and depend largely upon which of the intervals $I_j: (\sqrt{j/\{n-j\}}\bar{x}, \sqrt{\{j+1\}/\{n-(j+1)\}}\bar{x})$, $j = 0, 1, \cdots, n-2$, contains s. The limits of integration of $x_{n-r}(r = 2, 3, \cdots, n-1)$ also involve $\Omega_m^{(k)}$, $m = 1, 2, \cdots, n-1$, $k = 0, 1, \cdots, n-2$, where $\Omega_m^{(k)} = [-m(m+2)\Sigma_1^k x_i^2 - 2m\Sigma_{i=1}^{k-1}\Sigma_{j=i}^{k-1} x_i x_{j+1} + 2nm\bar{x}\Sigma_1^k x_i - mn(n-m-1)\bar{x}^2 + (m+1)mns^2]^{\frac{1}{2}}$. As an example, $F(\bar{x}, s)$ is evaluated when $f(x)$ is a chi-square universe. Throughout this paper n represents sample size.

27. Statistical Theory of Droughts. E. J. Gumbel, Consultant, Stanford University.

The droughts, x, the annual minima of discharges, are analyzed by the asymptotic theory of smallest values of a positive statistical variate and, the extremal probability paper used up to now for the floods is used for the logarithms of the droughts. If the observed cumulative frequencies are scattered about a straight line and, in particular, not bent downward toward the end, the limiting value of the droughts may be assumed to be zero. In this case the scale parameter $1/\alpha$ and the location parameter u are estimated by the methods used for the floods. If the observed points approach a curve which is bent downward toward the end, the lower limit ϵ exceeds zero, and the asymptotic probability function contains three parameters. They may be estimated by the method of moments which leads to Gamma functions depending only upon the scale parameter $1/\alpha$. The three parameters $1/\alpha$, u, ϵ, are then obtained with the help of a table calculated by Gladys Garabedian. A statistical criterion is given which decides whether the lower limit may be assumed to be zero or not. The droughts of 13 rivers analyzed by this procedure show a very good fit between theory and observations. The theoretical curves can be used to estimate the most severe drought to be expected within a given number of years, provided that the basic conditions will prevail. This procedure may be important for solving problems arising for storage and irrigation.

28. Some Tests Based on the First r Ordered Observations Drawn from an Exponential Distribution. Benjamin Epstein and Milton Sobel, Wayne University.

In this paper we study statistical problems which arise when the observations become available in an ordered manner. There exist many practical test situations, e.g., life testing, fatigue testing, and other kinds of destructive test situations where the data occur in order of magnitude (i.e., the weakest item fails first, the second weakest item fails next, etc.). It seems that in such cases we can if we choose, discontinue experimentation after the first r ($r \leq n$, the number of items tested) failures in a life test have occurred. Two principal advantages stem from the fact that the observations occur in an ordered manner. These are that we may be able to reach a decision in a shorter average time or with fewer observations on the average or both than if we were to utilize a procedure which has the same risks of making wrong decisions but which involves taking all m out of m observations (and thus in effect disregards the basic fact that information is becoming available in an ordered manner). The problem is explored in some detail for the special case where the life X is a random variable whose probability law is given by the pdf $f(x, \theta) = e^{-x/\theta}/\theta$, $x > 0$, $\theta > 0$. A number of procedures meeting either one or both of the desirable objectives mentioned above are given in connection with various simple and composite tests on the parameter θ. (Research supported by the Office of Naval Research under Contract No. Nonr-451(00).)

29. Some Theorems Relevant to Life Testing. MILTON SOBEL AND BENJAMIN EPSTEIN, Wayne University.

A set S of N independent exponential random variables $X_i (i = 1, 2, \cdots, N)$ is considered, the density of X_i being $f(x_i) = e^{-(x_i - \alpha_i)/\theta}/\theta$, $x_i > \alpha_i$, $\theta > 0$, where θ is the common unknown parameter to be estimated. One of the cases considered is $\alpha_i = \alpha (i = 1, 2, \cdots, N)$, where α is known and positive. All possible ways are considered of breaking up the set S into subsets. A total of R observations are taken subject to the condition that within each subset the observations are ordered. For each of these ways the distribution of the maximum likelihood estimate $\hat{\theta}$ of θ is the same, namely a distribution of Type III. Hence they are all equivalent relative to any properties depending only on the distribution of $\hat{\theta}$, e.g. the variance of $\hat{\theta}$. A replacement procedure is also considered in which the experimenter can only work with a maximum of one set of $n (0 \leq n \leq N - R + 1)$ random variables. After each observation he takes a new random variable to replace the item that failed. If R observations are taken the distribution of $\hat{\theta}$ is again the same as above. Some results on the average time required are also obtained. (Research supported by the Office of Naval Research under Contract No. Nonr-451(00).)

30. A Method of Reducing the Time Required to Complete Certain Fatigue Tests. LEONARD G. JOHNSON, General Motors Corporation, Detroit.

If it is assumed that the form of a specimen's life distribution is known, and that there is but one unknown parameter, the author shows that the distribution of the maximum likelihood estimate of the parameter based on the r first failures out of a sample of n is independent of n. As a result, it can be concluded that the testing time required to fail the first r specimens, r being fixed, can be made as small as desired simply by making n sufficiently large.

31. On the Multivariate Poisson Distribution. HENRY TEICHER, Purdue University.

The joint distribution of correlated Poisson variables $X_1, X_2, \cdots, X_p$ may be derived from a multinomial population and involves a $(2^p - p - 1)$-fold summation as well as $2^p - 1$ parameters. Its cf is $\phi(t_1, \cdots, t_p) = C \exp \{\Sigma_{i=1}^p a_i z_i + \Sigma_{i<j} a_{ij} z_i z_j + \cdots + a z_1 z_2 \cdots z_p\}$, where $z_j = e^{it_j}$, the a_i, a_{ij}, a_{ijk} etc. are nonnegative parameters and C is such that

$\phi(0, \cdots, 0) = 1$. For $p = 2$, one has the known result $\Pr\{X = x, Y = y\} = \Sigma_{k=0}^{w} (\mu_1 - \mu)^{x-k}(\mu_2 - \mu)^{y-k}\mu^k/[(x - k)!(y - k)!k!] e^{-(\mu_1+\mu_2-\mu)}$, where $w = \min(x, y)$, $\mu_1 = E(X)$, $\mu_2 = E(Y)$, $\mu = \text{Cov}(X, Y)$. The cases $p = 2$ and $p = 3$ are considered in detail and the preceding distribution is generated from three simple postulates. The limiting distribution and some properties are discussed.

32. Formulas for Approximating the Hypergeometric and Binomial by the Poisson Distribution. Irving W. Burr, Purdue University.

Let a sample of n be drawn from a lot of m objects of which d are of one sort; let d/m be p and np be λ. Then if the hypergeometric, binomial and Poisson probabilities of exactly x in the sample are respectively $h(x; n, m, d)$, $b(x; n, p)$, and $p(x; \lambda)$ we have approximately $h(x; n, m, d) = b(x; n, p)[1 + (x - (x - \lambda)^2)/(2mp(1 - p))]$; $b(x; n, p) = p(x; \lambda)[1 + (x - (x - \lambda)^2)/(2n)]$. The former is to within terms of the order $1/(m^2p^2)$ while the latter is to within terms ofthe order of $1/n^2$. Since the second terms in the brackets are approximate relative errors, they may be added in going from $p(x; \lambda)$ to $h(x; n, m, d)$. Using the formulas as a correction to tabulated values of the Poisson distribution, we get excellent approximations to hypergeometric probabilities.

33. Distributions of Ranges from an Arbitrary Discrete Population. Irving W. Burr, Purdue University.

The exact sampling distribution for the range, R, for small samples from any discrete population (with finite range) may be obtained from formulas involving combinations of sums of nth powers of certain sums of consecutive probabilities. The calculation is not at all prohibitive for samples of five or less if probabilities are taken to the nearest .01 or .005.

34. Sufficient Statistics and Selection Depending on the Parameter. D. A. S. Fraser, University of Toronto.

For a class of density functions with respect to a fixed measure, "functional sufficiency" or "f-sufficiency" is defined by the density factorization usually associated with sufficiency. Conditions are immediately available under which sufficiency and f-sufficiency are equivalent. A minimal f-sufficient statistic is defined and proved to be essentially unique; its construction is given. The minimal f-sufficient statistic is shown to be equivalent to the combination of a "statistic of selection" and the minimal f-sufficient statistic for a class of densities for which the region of positive density is fixed. Subject to mild continuity conditions, sufficient statistics in this latter case have been treated by B. O. Koopman. If the parameter is a parameter of selection from a fixed distribution, then the statistic of selection is the minimal f-sufficient statistic. If in addition the regions of positive density are monotone and are indexed monotonely by a real parameter, then the statistic of selection is sufficient according to the Halmos and Savage definition.

35. On a Problem Suggested by Blackwell. (Preliminary Report.) Charles Stein, University of Chicago.

If M, M' are probability measures concentrated on a finite set of points in n-space, we say that $M \supset M'$ if for every convex function ϕ, $\int \phi \, dM \geq \int \phi \, dM'$, and we say that $M > M'$ if there exists a stochastic matrix (p_{ij}) (one with $p_{ij} \geq 0$, $\Sigma_j p_{ij} = 1$) such that $\lambda_j' = \Sigma \lambda_i p_{ij}$, $\lambda_j' y_j' = \Sigma \lambda_i y_i p_{ij}$, where M, M' are concentrated on y_i, y_j' respectively and $M(y_i) = \lambda_i$, $M'(y_j') = \lambda_j'$. It is obvious that $M > M'$ implies $M \supset M'$ and here the converse is proved.

The following lemma is used. If ϕ is a convex function defined on the convex set $[y_1, \cdots, y_k]$ generated by the points $y_1, \cdots, y_k$ in a real vector space, then there exists a convex function ϕ_1 on $[y_1, \cdots, y_k]$ such that $\phi_1 \geq \phi$, $\phi_1(y_i) = \phi(y_i)$ for $i = 1 \cdots k$, and there exists a decomposition of $[y_1, \cdots, y_k]$ into simplexes with vertices among $y_1, \cdots, y_k$ such that ϕ_1 is linear on each of these simplexes. It is proved that the set of all M'' such that $M \supset M'' > M'$ ordered by $>$ has a maximal point. It is proved that any maximal M'' must be concentrated on the set of points to which M assigns positive probability. The proof is completed by the additivity of $>$. The significance of this result for statistical decision theory is explained by Blackwell ("Comparison of experiments," *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability*, University of California Press, 1951, pp. 93–102).

36. On a Class of Infinitely Divisible Distributions. GOPINATH KALLIANPUR, University of California, Berkeley, AND HERBERT ROBBINS, University of North Carolina.

Let $\Phi(x)$ be a non-lattice df having a finite moment of order $2 + \delta(\delta > 0)$ and let $\phi(t)$ be its characteristic function (cf). Set $\phi'(0) = ia$ and $\phi''(0) = -b^2$. A study is made of logarithms of cf's (lcf) which are given by one of the representations,

$$\text{(A)} \qquad g(t) = i\gamma t + \int_{-\infty}^{\infty} [\phi(tu) - 1 - iatu/(1 + u^2)][(1 + u^2)/u^2] \, dG(u),$$

$$\text{(B)} \qquad g(t) = i\gamma t + \int_{-\infty}^{\infty} [\phi(tu) - 1 - iatu][1/u^2] \, dG(u),$$

where γ is a real constant and $G(u)$ is a nondecreasing function of B.V. with $G(-\infty) = 0$. The following results are proved. (1) The limit of a sequence of lcf's of type (A) is also the lcf of type (A). (2) The formula (A) uniquely determines γ and $G(u)$. (3) A necessary and sufficient condition for a sequence $g_n(t)$, $(n = 1, 2, \cdots)$ of lcf's of type (A) to converge to the lcf $g(t)$ is that as $n \to \infty$, $\gamma_n \to \gamma$ and $G_n(u) \to G(u)$ at points of continuity of the latter. Analogous results for lcf's of type (B) are stated. Lcf's of types (A) and (B) occur naturally in connection with the investigation of limiting distributions (as $\lambda \to \infty$) of r.v.'s of the form (*) $\int_\alpha^\beta R_\lambda(u) \, dX(u) - A_\lambda$, where (i) α, β are finite, (ii) $R_\lambda(u)$ is for $\lambda > 0$ a continuous, monotone function in (α, β), (iii) A_λ is a constant depending on λ only, and (iv) $X(u)$ is a generalized Poisson process, with λ as the Poisson parameter. Using the above results, it is proved that the class of limit laws of (*) is the class of distributions whose lcf's have the representation (A).

37. Almost Sure Estimability of Linear Structures in n Dimensions. T. A. JEEVES, University of California, Berkeley.

Let B denote an $n \times n$ matrix of rank r of sure numbers. Let X and U be independent unobservable n-dimensional random row vectors, and Y be an observable n-dimensional random row vector, such that $Y = XB + U$. Assume that U has a multinormal distribution and that B is identifiable. The problem considered is to use a sequence of N independent observations of Y to construct an almost sure estimate S_N of S, the space spanned by the row vectors of B. Let $\Delta_N = \max_{a_N} \min_{a} | a_N - a |$ for all unit vectors a_N in S_N, and all unit vectors a in S. Then S_N is said to be an almost sure estimate of S if the probability of Δ_N converging to zero is unity. The estimate S_N defined below generalizes the procedure of Neyman for the case of two dimensions. Let Z_N be the $N \times n$ matrix of N observations on Y. A function $G_N(Z_N, K)$ of Z_N and the row vector K is obtained which converges with probability one to $G(K)$, a function which is zero if and only if K is perpendicular to S. Let K_{N1} be the vector which minimizes $G_N(Z_N, K)$, and let K_{Ni} (for $i = 2, 3, \cdots, n - r$) be the vector which minimizes $G_N(Z_N, K)$, subject to the restraint that K_{Ni} is perpendicular to K_{Nj} for $j = 1, 2, \cdots, i - 1$. S_N is the space orthogonal to the vectors K_{Ni} for $i = 1, 2, \cdots, n - r$.

38. Completeness of the Class of Admissible Decision Procedures. HERMAN RUBIN, Stanford University.

Let a space Ω of distributions and a space Φ of actions be given, and for each action ϕ and distribution θ let a risk (ϕ, θ) be given. For every sequence ϕ_i such that for all θ, $(\phi_i, \theta) \geq (\phi_{i-1}, \theta)$ for all i, let there be a ψ such that $(\phi_i, \theta) \geq (\psi, \theta)$ for all i. Then if (ϕ, θ) is continuous in θ for each ϕ in some topology in which Ω has a countable dense set, or if (ϕ, θ) is lower semicontinuous in some topology in which every subset of Ω has a countable dense subset, then the admissible elements of Φ form a complete class. An example of the applicability of this theorem is as follows. Let a sequential decision problem be given with a finite set of terminal actions. Then if the density of the first n observations is continuous for each n in some topology in which every subset of Ω has a countable dense subset (in particular, if each observation can have only countably many values), the admissible procedures form a complete class.

39. Moment-Problem Solutions with Continuous Derivatives. (Preliminary Report.) LIONEL WEISS, University of Virginia.

Given a finite sequence of moments $\mu_0, \mu_1, \cdots, \mu_n$ so that there is at least one cumulative distribution function on a given interval $[a, b]$ with these moments, conditions are known under which one can find an infinite number of cumulative distribution functions over the given interval with these moments. In these cases, further conditions are given so that at least one of the functions shall have a derivative whose square is integrable, and that function (say $F(x)$) is sought whose derivative $f(x)$ has the property that $\int_a^b f^2(x)\, dx$ is as small as possible. The solution is essentially unique, and $f(x)$ is continuous and is equal to a polynomial of at most the nth degree wherever it is greater than zero. The results can be extended in various directions.

40. Significance Consistency of the Basic Neyman-Pearson Test. L. M. COURT, American Power Jet Company, New York.

In a recent note ("A property of some tests of composite hypotheses," *Annals of Math. Stat.*, Vol. 22 (1951), pp. 475–476) C. Stein pointed out that for most of the common tests, a result that is significant at the 1% level is significant at the 5% level. Having said this, he uses the fundamental Neyman-Pearson lemma to construct an example for which this is not so. (A similar example for Neyman-Pearson Type A regions is given by Chernoff, "A property of some Type A regions," *Annals of Math. Stat.*, Vol. 22 (1951), pp. 472–474.) In Stein's example, a composite hypothesis (2 elements) is tested against a simple alternative, all three distributions being entirely discrete. The writer shows that it is essentially the discreteness that is responsible for this undesirable behaviour. He goes on to show that when the hypothesis and alternative are both simple and absolutely continuous distributions (i.e., density functions exist everywhere) and the Neyman-Pearson lemma is used to determine critical regions, this phenomenon cannot arise. If the hypothesis is composite but a Bayesian approach (Wald) is possible, i.e, there is a least favorable distribution for the parameter over the range specified by the hypothesis in the Lehmann-Stein sense, this conclusion can be extended to it.

41. **On Sets of Parameter Points where It Is Possible to Achieve Superefficiency of Estimates.** LUCIEN L. LECAM, University of California, Berkeley.

Let X be a random variable with probability density $f(x \mid \theta)$ depending on a parameter $\theta \varepsilon \Omega$; Ω being a measurable set of points on the real line. Let $X^{(n)} = (X_1, X_2, \cdots, X_n)$ be a sample of n independent observations on X. A sequence $\{T_n(X^n)\}$ of measurable functions is called a consistent asymptotically normal (c.a.n.) estimate of θ, with asymptotic variance $\{\sigma_n^2(\theta)\}$, if for every $\theta \varepsilon \Omega$, and for every t, $\lim_{n\to\infty} p\{(T_n[X^{(n)}] - \theta)/(\sigma_n(\theta)) < t \mid \theta\} = 1/(\sqrt{2\pi}) \int_{-\infty}^{t} e^{-\frac{1}{2}x^2}\, dx$. Assume Cramér's regularity conditions which imply consistency and asymptotic normality of the maximum likelihood estimate of θ (*Mathematical Methods of Statistics*, Princeton University Press, 1946, p. 500). Let $\{\alpha_n^2(\theta)\}$ be the asymptotic variance of the M.L. estimate. As $n \to \infty$, let $\beta(\theta) = \limsup [\sigma_n(\theta)/\alpha_n(\theta)]$ and $\gamma(\theta) = \lim [\sigma_n(\theta)/\alpha_n(\theta)]$ if this limit exists. An estimate $\{T_n[X^{(n)}]\}$ is called superefficient on $S \subset \Omega$ if it is c.a.n. and if $\beta(\theta) \leqq 1$, for $\theta \varepsilon \Omega$ and $\beta(\theta) < 1$ for $\theta \varepsilon S$. This set S is called the set of superefficiency. J. L. Hodges produced examples of superefficient estimates. His method of construction will be denoted by (H). THEOREM 1. *Whatever ϵ, $0 \leqq \epsilon < 1$ and whatever the closed and reducible set $S_0 \subset \Omega$, it is possible to construct superefficient estimates of θ with $\beta(\theta) \leqq \epsilon$ on S_0.* The method of construction is (H). THEOREM 2. *The set S of superefficiency has Lebesgue measure zero.* THEOREM 3. *If $\gamma(\theta)$ exists for all $\theta \varepsilon S$ then, whatever be ϵ, $0 \leqq \epsilon < 1$, the subset of S where $\gamma(\theta) \leqq \epsilon$ is closed and nondense.* THEOREM 4. *Whatever the denumerable set $S \subset \Omega$, it is possible to construct $\{T_n[X^{(n)}]\}$ c.a.n. on $\Omega - S$, with asymptotic variance $\{\alpha_n^2(\theta)\}$ and such that for every $\theta \varepsilon S$, the limit law of $[T_n - \theta]/\alpha_n(\theta)$, as $n \to \infty$, is more concentrated than the corresponding law of the M.L. estimates.*

42. **Relative Precision of Least Squares and Maximum Likelihood Estimates of Regression Coefficients.** JOSEPH BERKSON, Mayo Clinic.

Three "estimators" of the parameters α and β of the logistic function $P_i = 1/(1 + e^{-(\alpha+\beta x_i)})$ as used in bioassay were compared for three equally-spaced values of the dose x_i, 10 at each dose: (1) maximum likelihood, (2) minimum (Pearson classic) χ^2, (3) minimum logit χ_i, the first two requiring iterative procedures for evaluation, the last obtainable directly. With central dose at the L.D. 50, the three estimates are unbiased; the variance is smallest for the minimum logit χ^2, next larger for the minimum χ^2, and largest for the maximum likelihood estimate. For dosage arrangements not symmetrical around the L.D. 50, the three estimates are biased, the maximum likelihood estimate positively, the χ^2 estimates negatively; the mean square error is smallest for the minimum logit χ^2, next larger for the minimum χ^2, and largest for the maximum likelihood estimate. For all dose arrangements, the mean square error of the maximum likelihood estimate is larger than $1/I$, those of the χ^2 estimates are less than $1/I$, in accordance with the Cramér inequality for the mean square error. Each of the estimators is sufficient.

NEWS AND NOTICES

Readers are invited to submit to the Secretary of the Institute news items of interest.

Personal Items

Dr. Kurt W. Back, formerly at Washington, D. C., is now acting as Social Research Analyst on the Air Force Contract for the Bureau of Applied Social Research, a Columbia University project.