In Theorem 2 change $\mathfrak{D}^{(b)}$ to $\mathfrak{D}^{(a)}$ (in the proof the subscript of M should be F). In the last Corollary add $\mathfrak{D}^{(a)}\{\alpha, y(\alpha); -\}$, delete "$\mathfrak{D}^{(c)}$ is closed," and change $\mathfrak{D}^{(c)}\{\alpha, y(\alpha); -\}$ to $\mathfrak{D}'\{\alpha, y(\alpha); -\}$ with $\mathfrak{D}'$ any closed subset of $\mathfrak{D}^{*(b)}$ or $\mathfrak{D}^{*(c)}$, $\mathfrak{D}^*$ denoting the set of all possible decision procedures (In the proof the first paragraph should be deleted, and $\mathfrak{D}^{(b)}$ changed to $\mathfrak{D}^{(a)}$.)

In the penultimate paragraph of Section 2 change $\mathfrak{D}^{(b)}$ to $\mathfrak{D}^{(a)}$ and $\mathfrak{D}^{(c)}$ to $\mathfrak{D}'$, where $\mathfrak{D}'$ is a closed convex subset of $\mathfrak{D}^{(0)(b)}$ or $\mathfrak{D}^{(1)(c)}$ satisfying (v). The enumeration of exceptions in the next paragraph should read

"$\mathfrak{D}_{y_0}$, $\mathfrak{D}\{-; \beta, z(\beta)\}$, and $\mathfrak{D}'$ and its subclasses for $\mathfrak{D}' \subset \mathfrak{D}^{(c)}$."

ABSTRACTS OF PAPERS

(Abstracts of papers presented at the Washington meeting of the Institute, December 27–30, 1953)

1. Confidence Intervals of Fixed Length for the Poisson Mean and the Difference Between Two Poisson Means. Allan Birnbaum, Columbia University.

1. To construct an estimate λ' of the unknown parameter λ of a Poisson process $X(t)$ such that with probability at least $1 - \alpha$, $|\lambda' - \lambda| \leqq \epsilon$, where α and ϵ are given positive constants, let n be a positive integer. Observe T_n, the waiting time required for the occurrence of n events. Let $c = \alpha\epsilon^2/2n$. Perform additional observation of the process for $1/2cT_n$ units of time; let X be the number of events observed in this period. Set $\lambda' = 2cT_nX$. 2. To construct an estimate Δ' of $\Delta = \lambda_2 - \lambda_1$, where λ_1, λ_2 are the unknown parameters of two Poisson processes, such that with probability at least $1 - \beta$, $|\Delta' - \Delta| \leqq \eta$, where β and η are given positive constants, we may set $\Delta' = \lambda_2' - \lambda_1'$, where λ_1' and λ_2' are obtained as above, taking $\epsilon = \eta/2$ and $(1 - \alpha)^2 = 1 - \beta$. In case the two processes can be observed simultaneously, a more efficient estimate can be given. At least for λ exceeding some lower bound, c can be replaced by $c^* = \epsilon^2/k$, where $\Pr\{z \geqq k\} = \alpha$ if z is the product of independent chi-square variates with 1 and $2n$ d.f.

2. Convexity Properties of the alpha-beta-set Under Composite Hypotheses. Herman Rubin and Oscar Wesler, Stanford University.

Suppose one is presented with a statistical decision problem of the following kind. A random variable X is observed and it is desired to test whether the (not necessarily finite-dimensional) parameter of the distribution of X is in Ω_1 or in Ω_2. Define, as usual, $\alpha(\varphi)$ to be the supremum of the probability of an error of the first kind and $\beta(\varphi)$ to be the supremum of the probability of an error of the second kind when the random decision procedure φ is used. If Ω_1 and Ω_2 consist of one point each, it is known that the set S of all points $(\alpha(\varphi), \beta(\varphi))$ is convex and symmetric about $(\frac{1}{2}, \frac{1}{2})$. It is shown that the subset T of S lying on or below the line $\alpha + \beta = 1$ is convex, and that if the set of distributions under consideration is dominated by a σ-finite measure, the lower boundary of T belongs to T. It is also shown that the symmetric image of T, and possibly more, belongs to S. An example is given to show that this "more" can destroy convexity.

3. Critical Regions in Terms of Lower Dimensional Critical Regions. L. M. Court, Diamond Ordnance Fuze Laboratory.

Let $p_1(x \mid \theta) \equiv p_1(x_1, \cdots, x_{n_1} \mid \theta_1, \cdots, \theta_{m_1})$ and $p_2(y \mid \phi) \equiv p_2(y_1, \cdots, y_{n_2} \mid \phi_1, \cdots, \phi_{m_2})$ be two independent density distributions and $p(x, y \mid \psi) = p_1(x \mid \theta)p_2(y \mid \phi)$ the joint distribution formed by their multiplication. Let R_1, R_2, R be critical regions of size α, β, γ for $p_1(x \mid \theta)$, $p_2(y \mid \theta)$, $p(x, y \mid \psi)$ respectively. Expressions are derived for R in terms of R_1 and R_2 in the cases in which the critical regions are determined by 1) the method of maximum likelihood, 2) the likelihood ratio and 3) the Neyman-Pearson theory.

4. A Stochastic Model of Traffic Congestion. C. B. Winsten, Cowles Commission, University of Chicago.

A simplified model is presented to represent the behavior of traffic at an intersection controlled by a stop sign or repeated cycle traffic lights. An important property of such traffic is that it is spaced out, both on arriving at the intersection and on leaving it. To take account of this properly the model is set up with discrete time points at each of which at most one car can arrive or leave. For the stop sign case, cars in the minor road wait till there is at least a safe interval of w time units till the next car in the major road is due. The arrivals in the minor and major roads are taken as binomial, with probabilities α, β, respectively. From a discussion of the queueing process in the minor road, the condition for the system to settle down to equilibrium is $\alpha < (1 - \beta)^w$, and in this case the mean waiting time per car is shown to be $[1 - (1 + w\beta)(1 - \beta)^w]/\beta[(1 - \beta)^w - \alpha]$. A method for calculating mean waiting time for the equal cycle traffic light problem is also given.

5. On a Property of Certain Linear Functions of Order Statistics from some Normal Populations. K. C. Seal, University of North Carolina.

Suppose there are n normal populations $N(\mu_i, \sigma_i^2)$, $i = 1, 2, \cdots, n$, and that one random observation from each of these n populations is given. It is not known which population any particular observation came from. Let $x_{(1)}, x_{(2)}, \cdots, x_{(n)}$ denote the n observations written in an increasing order of magnitude. It is shown that the expectation of any linear function $c_1x_{(1)} + \cdots + c_nx_{(n)}$ of the $x_{(i)} (i = 1, \cdots, n)$ with nonnegative coefficients at least one of which is positive, is a monotonically increasing function of each of the population means $\mu_i (i = 1, \cdots, n)$.

6. A Historical Note on the Relation Between Extreme Values and Tensile Strength. Julius Lieblein, National Bureau of Standards.

It appears to be commonly believed that the statistical treatment of the "weakest-link" hypothesis and the use of extreme-value methods in connection with strength of test specimens originated with F. T. Peirce in an article published in 1926. The discovery by the writer of a pair of long-forgotten articles in two engineering journals of the 1880's shows that we must push the date of first application of extreme values to breaking strength back nearly 50 years, at least. These articles show a statistical viewpoint ahead of their time, and might well be read by anyone interested in the development of statistical thought. The articles, both by W. S. Chaplin, are "The Relation Between the Tensile Strengths of Long and Short Bars," *Van Nostrand's Engineering Magazine*, December 1880, and "On the Relative Tensile Strengths of Long and Short Bars," *Proceedings of the Engineers' Club*, 1882.

7. A Note on Statistics and sigma-subfields. R. R. Bahadur, Columbia University.

Let there be given a Borel set X of m-dimensional Euclidean space $(1 \leqq m \leqq \infty)$, and let S be the class of Borel measurable subsets of X. If f is a function on X into a set Y, let $S_f = \{f^{-1}(A): A \subseteq Y, f^{-1}(A) \varepsilon S\}$. Then, for each f, S_f is a σ-subfield of S. It is shown in this note that corresponding to any σ-subfield S^* and any probability measure p on S there exists an f (depending in general on p) such that $S^* = S_f$ in the sense that corresponding to each set in one σ-subfield there exists a set in the other such that the symmetric difference of the two sets is of p-measure zero. This result is obtained by means of the theory of sufficiency, and has in turn certain applications in that theory; for example, if f is a necessary and sufficient statistic for a dominated set P of measures on S then S_f is a necessary and sufficient σ-subfield for P. It is shown by an example that the converse of the last stated result is false.

8. Completeness, Similar Regions, and Unbiased Estimation, Part II. (Preliminary Report.) E. L. Lehmann and Henry Scheffé, University of California, Berkeley.

Continuation of Part I (*Sankhyā*, Vol. 10 (1950), pp. 305–340) to obtain theorems about the generation of complete families of measures from other complete families, application of these results (i) to the Pitman-Koopmans-Darmois family to prove certain tests concerning this family uniformly most powerful unbiased, and (ii) to some nonparametric problems.

9. On Linear Regression Analysis when the Dependent Variable is Rectangular. E. G. Olds, Carnegie Institute of Technology.

Assume chance variables y_i, rectangular on $\alpha + \beta(x_i - \bar{x}) \pm c$, with c known but with α and β unknown. Values of y_i are observed for a fixed set of x-values and an estimator $a + b(x_i - \bar{x})$ is to be determined. In general, the maximum likelihood estimator is not unique. Furthermore, the method of least squares sometimes yields an estimator which might be called inadmissible since, for one or more values of x the value of the estimator differs from the observed value of y by more than c. The present paper gives a convenient method of obtaining a modified least squares' estimator which is admissible in the sense implied above. The proposed estimator belongs to the convex set of maximum likelihood estimators and is unbiased. Also included in the paper is a method for finding the largest and smallest admissible estimates corresponding to any specified value of x.

10. Some Further Results in Simultaneous Confidence Interval Estimation. S. N. Roy, University of North Carolina.

In continuation of previous work in this line (S. N. Roy and R. C. Bose, "Simultaneous Confidence Interval Estimation," *Ann. Math. Stat.*, Vol. 24 (1953), pp. 513–536) simultaneous confidence bounds have been obtained for each of the following sets: (1) (a) elementary symmetric functions of the characteristic roots of the covariance matrix Σ for one multi-variate normal population, (b) the same for the matrix $\Sigma_1\Sigma_2^{-1}$ for two multivariate normal populations and (2) certain simple functions of the canonical regressions between two subsets of a multivariate normal set. It has been possible to obtain the bounds for 1(a) in terms of similar functions of the characteristic roots of the sample matrix S, for 1 (b) in terms of similar functions based on $S_1S_2^{-1}$, and for (2) in terms of sample canonical regressions.

11. A Method for Generating Random Variates on Electronic Computers. D. Teichroew, National Bureau of Standards.

Some of the methods by which values of random variates are obtained for use in high-speed automatically-sequenced computers are: (i) using previously computed values, such as tables of normal deviates, (ii) determining the value for which the probability integral has a given (random) value, and (iii) accepting or rejecting random values on the basis of other random values, for example, a random value, x, may be accepted if another random value is less than the value of the density function at the point x. In many cases the most serious objection to these methods is that they take too much computing time. A method is proposed which appears useful for very fast computers with a relatively small amount of storage. The method consists of two computations. The machine first computes a random variate y and then transforms y into x, a variate with the desired distribution. Computation time is minimized by selecting a y which can be computed quickly and which also permits the transformation to have a simple form.

12. Theory of Successive Multiphase Sampling. (Preliminary Report.) B. D. Tikkiwal, Columbia University.

Suppose there are k characters of an infinite population under consideration and the ith $(i = 2, \cdots, k)$ character is always studied on the part of the sample taken for the $(i - 1)$st character on each of the occasions observed up to a certain period. A best estimate and its variance for each of the k characters on each occasion were obtained under a certain pattern of correlation between the various characters occurring on the various occasions, [paper read by the author before the annual meeting of the Indian Society of Agricultural Statistics, 1951]. Now it has been further noted that if the infinite population studied on each of the successive occasions is replaced by a finite population of size N, the best estimates for the various k characters remain the same. However, the variance of the best estimate is decreased in each case by a quantity σ^2/N where σ^2 is the population variance on the hth occasion of the character under consideration. Thus in particular it can be noted that the best estimate given by Patterson [*J. Roy. Stat. Soc. Suppl.*, Vol. 12 (1950), pp. 241–255] for the study of one character remains unaffected, when the infinite population is replaced by a finite population of size N, but its variance is decreased by σ^2/N.

13. Estimation of the Size of a Stratified Population. Douglas G. Chapman and C. O. Junge, Jr., University of Washington and Washington State Department of Fisheries.

The estimation, by marking methods, of the size of a population which has a variable stratification, is studied. It is noted that no unbiased estimate of this parameter exists. Conditions are found under which the standard estimate (which is constructed without reference to the stratification) and an estimate given by Schaeffer are consistent. A new estimate is obtained, which is consistent not only under wider conditions, which are more likely to be fulfilled in actual marking experiments, but also under different sets of assumptions. The asymptotic variance of this estimate is derived. Tests are suggested for determining which of these estimates should be used. The results may also be adapted to evaluating the stratification changes that occur within a population.

14. Sequential Life Tests in the Exponential Case. Benjamin Epstein and Milton Sobel, Wayne University and Cornell University.

In this paper a sequential life test procedure is worked out in detail. As in previous work devoted to nonsequential methods, it is assumed that the underlying p.d.f. is exponential. An interesting feature of the test is that decisions are made continuously in time. Various useful formulae and tables are given. (Sponsored in part by the Office of Naval Research and the Office of Ordnance Research of the U. S. Army.)

15. Distributions of Some Integrals of Certain Gaussian Stochastic Processes and the Limiting Distributions of Some "Goodness of Fit" Criteria. T. W. Anderson, Columbia University.

To test the hypothesis that a sample of N observations has been drawn from a population with a specified continuous cumulative distribution function $F(x)$ one can compare the empirical cumulative distribution $F_N(x)$ with $F(x)$ by means of

$$W_N = \int [F_N(x) - F(x)]^2 \psi[F(x)] \, dF(x)$$

and $V_N = \iint [F_N(x) - F(x)][F_N(y) - F(y)]l[F(x), F(y)]\, dx\, dy$. The limiting distribution of V_N under the null hypothesis is the distribution of $V = \iint X(u)X(v)l(u, v)\, du\, dv$, where $X(u)(0 \leqq u \leqq 1)$ is a certain Gaussian stochastic process with mean zero, and the characteristic function of V is shown to be $\Pi\ (1 - 2it \mid \mu_j)^{-\frac{1}{2}}$, where μ_j are the eigenvalues of $\int k(u, w)l(w, u)\, dw$ and $EX(u)X(w) = k(u, w)$. The characteristic functions of $T = \int [X(u) + k(u)]^2\psi(u)\, du$ and $S = \iint [X(u) + k(u)][X(v) + k(v)]l(u, v)\, du\, dv$ are shown to be the products of the characteristic functions of $W = \int X^2(u)\psi(u)\, du$ and V, respectively, and certain exponentials, the exponents being integrals of $k(u)$. If the sample of N is drawn from $H_N(x)$ and if $H_N(x)$ approaches $F(x)$ in a certain way, then the limiting distribution of V_N is the distribution of T.

16. Some Sampling Results on the Power of Nonparametric Tests Against Normal Alternatives. W. J. Dixon and D. Teichroew, National Bureau of Standards.

This report contains some of the results of sampling investigations of the distribution of several nonparametric two-sample tests under the null hypothesis and under alternative normal hypotheses. The sampling was carried out at the University of Oregon in 1949–50 and at the National Bureau of Standards, Los Angeles, in 1952. In both cases the number of samples was not large. The sampling performed, however, is sufficiently extensive to give a general indication of the relative power of the different tests and to indicate the range of alternatives to be sampled further to gain more precise determination of power. The tests, listed in order of their power to reject the alternative hypotheses, are: (i) rank sum test, (ii) maximum deviation test, (iii) median test, and (iv) run tests. The results for the rank sum test indicate that one does not lose much power if the rank sum test is used instead of the t test in cases where the distributions are actually normal.

17. On the Large Sample Power of Rank Order Tests in the Two-Sample Problem. Meyer Dwass, Northwestern University.

Let the random variables $X_1, \cdots, X_N$ be independent, and let $R_1, \cdots, R_N$ be their ranks. Let $S_N = \Sigma a_{Ni}f(R_i/N)$, where $a_{N1} = \cdots = a_{Nm}$, $a_{Nm+1} = \cdots = a_{NN}$, $\Sigma a_{Ni} = 0$, $\Sigma a_{Ni}^2 = 1$. Let H_0 be the hypotheses that the X_i are identically distributed. Let $H_1(\theta)$ be the alternative that the X_i are independent, but that the first m have one density function $g_1(x, \theta)$, and the remaining $N - m$ have another density function $g_2(x, \theta)$, where θ is a one-dimensional parameter and where $g_1(x, 0) = g_2(x, 0)$. The following is shown subject to certain regularity conditions. 1) If f is a polynomial, S_N is asymptotically normal when H_0 is true and when $H_1(\theta)$ is true. 2) Consider the test which rejects H_0 when S_N is too large. For f a polynomial, we approximate the large sample power against $H_1(\theta)$ for such tests whose significance levels approach α as $N \to \infty$, in the following sense: c is determined such that the power differs from $1 - \Phi(\lambda - \theta\sqrt{N}c)$ by less than any preassigned $\epsilon > 0$ for N sufficiently large, where Φ is the normal (0, 1) c.d.f. and $1 - \Phi(\lambda) = \alpha$. 3) It is shown how to choose that polynomial of a given order which maximizes the large sample power. A polynomial f can be chosen of sufficiently high order so that the large sample power of the test based upon S_N is arbitrarily close to the large sample power of the classical likelihood ratio test.

18. Multiple Tests and Intersection Region Procedures. David L. Wallace, Massachusetts Institute of Technology.

A statistical hypothesis is often expressed as the logical intersection of several component hypotheses. If tests of the component hypotheses are available, a natural test for the full hypothesis is defined to reject whenever one or more of the component hypotheses are rejected. If an indexed family of such hypotheses is tested, confidence regions for the index are obtained from each of the families of component tests. The region defined by the family of multiple tests is the intersection of the component regions. Properties of the multiple test and intersection region are discussed. Of special interest is the pth order general linear hypothesis with its p single linear component hypotheses. Intersection region procedures are useful in obtaining "usable" confidence regions for the location of the vertex in quadratic regression.

19. Some Significance Test Procedures for Multiple Comparisons. H. O. Hartley, Iowa State College.

In the comparison of k experimental means arising from an 'Analysis of Variance' of data one of the procedures for deciding on the significance of all the $\frac{1}{2}k(k - 1)$ differences is the so-called 'Newman-Keuls' procedure. Some properties concerning the error of the first kind and of the power involved in this procedure are proved. A similar sequential procedure for testing the significance of k mean squares is then suggested. This is based on the distribution of the largest of k F-ratios obtained from k treatment mean squares s_i^2, respectively based on ν_i degrees of freedom and all divided by the same error mean square based on ν degrees of freedom. This procedure is first developed for the case of equal ν_i, then generalised to differing ν_i and shown to have properties similar to the Newman-Keuls procedure.

20. A Sequential Test of Randomness Against Linear Trend. Gottfried E. Noether, Boston University.

Given observations $X_1, X_2, \cdots$, let $Z_{gh} = 0$ or 1 depending on whether X_{2gj+h} is smaller or greater than $X_{(2g+1)j+h}$, $j \geqq 1$, $g = 0, 1, 2, \cdots$, $h = 1, 2, \cdots, j$. Under the alternative hypothesis of the linear trend $F(x_1, x_2, \cdots, x_n) = \Pi_{i=1}^{n} F(x_i + i\theta)$, $P(Z_{gh} = 1) = P(X_1 > X_{j+1}) = p_1$, say, while under the hypothesis of randomness, $P(Z_{gh} = 1) = \frac{1}{2}$. The hypothesis of randomness can then be tested by means of the usual sequential procedure for testing the hypothesis that for a binomial distribution $p = \frac{1}{2}$ against the alternative $p = p_1$. There exists an optimum value of j in the sense that the expected number of observations required by the test corresponding to this j is not larger than that for any other j. This sequential test is compared with tests of randomness based on runs up and down and also with Mann's T-test. It turns out that on the average the sequential test requires fewer observations than these other tests, at least for sufficiently small values of θ. (Research sponsored by Air Research and Development Command.)

21. Further Results in the Theory of Quality Control Charts. Leo A. Aroian, Hughes Research and Development Laboratories.

The type of decision for a quality control chart, the case of erratic production for two or more charts, and a special case of a control chart by attributes versus control charts by variables, are investigated. Examples illustrate the theory for the control of the mean and the standard deviation for one chart used alone or two charts used together. The theory extends results given in a previous paper, "The Effectiveness of Quality Control Charts," by L. A. Aroian and H. Levene, *J. Amer. Stat. Assn.*, Vol. 45 (1950), pp. 520–529.

22. Minimum Life in Fatigue. E. J. Gumbel and A. M. Freudenthal, Columbia University.

The probability of survival at N cycles for constant stress levels S was analysed with the help of the asymptotic theory of smallest values of a nonnegative variate. In this case two parameters exist, the characteristic number of cycles to failure V_S and a scale parameter $1/\alpha_S$. The available fatigue test results for copper, nickel and aluminum specimens at high stress levels show that this approach is justified in first approximation assuming that the "minimum life," that is the number of cycles $N_{0,S}$ below which no specimen breaks at the stress level S, is practically zero. However, test results on the same metals at low stress levels and for steel show a significant positive minimum life. Therefore the theory is generalized by the introduction of this value as a third parameter. The exponential function and four pseudo-symmetrical cases where two averages coincide or the skewness is zero, are special cases of this survivorship function. The three parameters are estimated by the method of moments which requires the calculation of the sample mean, the standard deviation and skewness. The estimate of the scale parameter depends only on the sample skewness which has a high degree of variation. The estimates of the characteristic number of cycles to failure and the minimum life depend upon all three statistics and are obtained without any successive approximations. The theory leads also to an upper bound, a large number of cycles at which the probability of survival becomes infinitesimally small. Observations on copper, nickel and aluminum at high stress levels and on steel traced on the logarithmic extremal probability paper lead to curved survivorship functions which are very well reproduced by this theory. (Work sponsored in part by the Office of Ordnance Research.)

23. **Bounds for the Distribution Function of a Sum of Independent, Identically Distributed Random Variables.** Wassily Hoeffding and S. S. Shrikhande, University of North Carolina and University of Nagpur.

The problem is considered of obtaining bounds for the cumulative distribution function of the sum of n independent, identically distributed random variables with k prescribed moments and given range. For $n = 2$ it is shown that the best bounds are attained or arbitrarily closely approached with discrete random variables which take on at most $2k + 2$ values. Explicit bounds are obtained for the case of nonnegative random variables with given mean when $n = 2$; for arbitrary values of n bounds are given which are asymptotically best in the "tail" of the distribution. Some of the results contribute to the more general problem of obtaining bounds for the expected value of a given function of independent, identically distributed random variables when the expected values of certain functions of the individual variables are given.

24. **Sequential Rank Sum Tests. (Preliminary Report.)** Chia Kuei Tsao, Wayne University.

Let $f(x)$ be a continuous p.d.f. defined over a space S. To test a simple hypothesis H_0: $f(x) = f_0(x)$ against an alternative hypothesis $H_1 : f(x) = f_1(x)$, we divide S into three mutually exclusive sets S_0, S_1 and S_2. For $i = 0, 1$, the set S_i is subdivided into k_i subsets $S_{i1}, S_{i2}, \cdots, S_{ik_i}$. Random observations are drawn successively. At each stage, count the number of observations falling in each of the $k_0 + k_1 + 1$ sets. For each $m(m = 1, 2, \cdots)$ denote by m_{ij} the number of observations falling in the set S_{ij}, $j = 1, 2, \cdots, k_i$; $i = 0, 1$. Let $s_i = \sum_{j=1}^{k_i} jm_{ij}(i = 0, 1)$. Let a_0 and a_1 be two positive integers. Continue to draw observations as long as $s_i < a_i(i = 0, 1)$. The experiment is discontinued as soon as $s_0 \geqq a_0$ or $s_1 \geqq a_1$. The hypothesis H_i is accepted if $s_i \geqq a_i$ $(i = 0$ or $1)$. The $k_0 + k_1 + 1$ sets are determined so that the following four conditions are satisfied: (1) $S_{0j} : c_{0j} \leqq f_1(x)/f_0(x) \leqq c_{0,j-1}$, $j = 1, 2, \cdots, k_0$; (2) $S_{1j} : c_{1,j-1} \leqq f_1(x)/f_0(x) \leqq c_{1j}$, $j = 1, 2, \cdots, k_1$; (3) $\Pr(X \varepsilon S_{ij} \mid H_0) = (\Pr(X \varepsilon S_i \mid H_0))/k_i$, $j = 1, 2, \cdots, k_i$, $i = 0, 1$; (4) the pair (c_{00}, c_{10}), where $c_{00} \leqq c_{10}$, is so determined that the test satisfies certain requirements. In this paper, the distribution and the m.g.f. of the sample size, the power function of the test and the ASN function are obtained. Other properties are also studied. Applications to the parametric and nonparametric problems are discussed.

25. **A Remark on the Geometrical Method of Construction of an Orthogonal Array.** Esther Seiden, University of Chicago.

R. C. Bose and K. A. Bush showed [*Ann. Math. Stat.*, Vol. 23 (1952), pp. 508–524] how one can make use of the maximum number of points, no three collinear, in finite projective spaces in order to construct orthogonal arrays. In particular, this method enabled them to construct an orthogonal array (81, 10, 3, 3). They proved, on the other hand, that in the case considered the maximum number of constraints cannot exceed 12 (Theorem 2C). Hence they state: "We do not know whether we can get 11 or 12 constraints in any other way." It is shown that no such way exists.

26. Some Contributions to the Theory of Markov Chains. (Preliminary Report.) Cyrus Derman, Columbia University.

Suppose that a collection of particles are moving about independently according to probabilities given by a Markov chain with transition matrix $P = \{p_{ij}\}\ i, j, = 0, 1, \cdots$. Let $A_n(i)$ denote the number of particles in state i at time n $(i, n = 0, 1, \cdots)$. Some sufficient conditions on P and on the distributions of the $A_0(i)$'s were found such that for any set $i_1, \cdots, i_r$, the joint distribution of $A_n(i_1), \cdots, A_n(i_r)$ tends as $n \to \infty$ to that of r independent Poisson distributions. Consider a recurrent Markov chain $\{X_n\}$. Let $N_n(i) = \{$the number of r's such that $X_r = i$ for $1 \leqq r \leqq n\}$ $i = 0, 1, \cdots$. The following theorem was proved. If $H(n)$ is any nondecreasing unbounded function and if i and j are any two states belonging to the same class, then with probability one the inequality $| (N_n(j) - a_{ij}N_n(i))/(N_n(i) + N_n(j))^{\frac{1}{2}}H(N_n(i) + N_n(j)) | > b_{ij}$, where a_{ij} and b_{ij} are certain constants, will be satisfied for infinitely many or at most finitely many n according as $\sum_{n=1}^{\infty} H(n) \exp(-H^2(n)/2)/n$ diverges or converges. Sufficient conditions were given such that for any $i_1, \cdots, i_r$ the distribution of $N_n(i_1), \cdots, N_n(i_r)$ properly normalized approaches a multivariate normal distribution. The asymptotic covariance matrix was computed.

27. Minimax Invariant Procedures for Estimating Cumulative Distribution Functions. Om P. Aggarwal, University of Washington.

Let $x_1 < x_2 < \cdots < x_n$ be the ordered observations on a chance variable with cumulative distribution function F. Let $\hat{F}$ denote an estimate of F based only upon the sample. The minimax invariant procedures of estimating F are obtained for two classes of loss functions $L(F, \hat{F})$. For $L(F, \hat{F}) = \int_{-\infty}^{\infty} | F(x) - F(x) |^r dx$, with integer $r \geqq 1$, the minimax invariant procedure is to estimate F by a step function $\hat{F}(x) = c_j$ for $x_j \leqq x < x_{j+1}$; $j = 0, 1, \cdots n$, where x_0 and x_{n+1} denote $-\infty$ and $+\infty$ and c_j is obtained as the root of an equation of degree $(n + r)$ when r is odd and of degree $(r - 1)$ when r is even. For the special case $r = 1$ the value of c_j is the median of a Beta distribution. For $r = 2$, one obtains $c_j = (j + 1)/(n + 2)$. For the class of loss functions $L(F, \hat{F}) = \int_{-\infty}^{\infty} [F(x) - \hat{F}(x)]^{2k}/F(x)[1 - F(x)]\, dx$, one again obtains for minimax invariant procedure step functions with c_j determined as root of an equation of degree $(2k - 1)$. In particular for $k = 1$ this optimum procedure turns out to be the usual sample cumulative function with $c_j = j/n$. (Work supported by the Office of Naval Research.)

NEWS AND NOTICES

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Personal Items

F. J. Anscombe of Cambridge, England has been appointed Research Associate in the Department of Mathematics, Princeton University, for the year 1953–1954.