The order of integration and summation may be interchanged. From the definition of L it is clear that L is real so that only the real part of this expression need be integrated. The only nonvanishing contributions to the integral occur when

$$(7) \qquad 2j + k + l = 0, \qquad l = 2n$$

where n is an integer. The result is that

$$(8) \qquad L = 2\pi \sum_{j=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} (-1)^n I_j(ar^2) I_{2n+2j}(br) I_{2n}(cr).$$

To reduce equation (8) to a single summation the following special case of an addition theorem given in Watson [7] is applied to the summation over n;

$$(9) \quad I_n(\sqrt{Z^2 + z^2}) \cos n\psi = \sum_{r=-\infty}^{\infty} (-1)^r I_{n+2r}(Z) I_{2r}(z), \ \tan \psi = z/Z.$$

The result leads directly to equation (2).

REFERENCES

[1] F. N. Frenkiel, "Frequency distributions of velocities in turbulent flow," *J. Meteorol.*, Vol. 8, (1951), pp. 316–320.

[2] J. L. Lawson and G. E. Uhlenbeck, *Threshold Signals*, McGraw-Hill Book Co., (1950), p. 195.

[3] P. B. Patnaik, "The non-central χ^2 and F-distributions and their applications," *Biometrika*, Vol. 36 (1949), pp. 202–232.

[4] Herbert Robbins and E. J. G. Pitman, "Application of the method of mixtures to quadratic forms in normal variates," *Ann. Math. Stat.*, Vol. 20 (1949), pp. 552–560.

[5] Wasao Shibagaki, "Tables of the Modified Bessel Functions," Kyushi University Physics Department, 1946.

[6] P. C. Tang, "The power function of the analysis of variance tests with tables and illustrations of their use," *Statistical Research Memoirs, II*, University of London, (1938), pp. 126–149.

[7] G. N. Watson, *A Treatise on the Theory of Bessel Functions*, 2nd ed., Macmillan, 1948, Equation 7, p. 361.

[8] *Tables of the Bessel-Functions*, Part II, Vol. 10, "Functions of positive integer order," British Association for the Advancement of Science, 1952.

CORRECTION TO "ON CERTAIN CLASSES OF STATISTICAL DECISION PROCEDURES"*

By H. S. Konijn

University of California, Berkeley

I am indebted to Dr. L. Le Cam for pointing out an error in the above-named paper (*Annals of Math. Stat.*, Vol. 24 (1953), pp. 440–448). Let

$$\mathfrak{D}^{(a)} = \{\delta \varepsilon \mathfrak{D}: r(F, \delta) \text{ is bounded by a function of } F\}.$$

* Received 11/16/53.

In Theorem 2 change $\mathfrak{D}^{(b)}$ to $\mathfrak{D}^{(a)}$ (in the proof the subscript of M should be F). In the last Corollary add $\mathfrak{D}^{(a)}\{\alpha, y(\alpha); -\}$, delete "$\mathfrak{D}^{(c)}$ is closed," and change $\mathfrak{D}^{(c)}\{\alpha, y(\alpha); -\}$ to $\mathfrak{D}'\{\alpha, y(\alpha); -\}$ with $\mathfrak{D}'$ any closed subset of $\mathfrak{D}^{*(b)}$ or $\mathfrak{D}^{*(c)}$, $\mathfrak{D}^*$ denoting the set of all possible decision procedures (In the proof the first paragraph should be deleted, and $\mathfrak{D}^{(b)}$ changed to $\mathfrak{D}^{(a)}$.)

In the penultimate paragraph of Section 2 change $\mathfrak{D}^{(b)}$ to $\mathfrak{D}^{(a)}$ and $\mathfrak{D}^{(c)}$ to $\mathfrak{D}'$, where $\mathfrak{D}'$ is a closed convex subset of $\mathfrak{D}^{(0)(b)}$ or $\mathfrak{D}^{(1)(c)}$ satisfying (v). The enumeration of exceptions in the next paragraph should read

"$\mathfrak{D}_{y_0}$, $\mathfrak{D}\{-; \beta, z(\beta)\}$, and $\mathfrak{D}'$ and its subclasses for $\mathfrak{D}' \subset \mathfrak{D}^{(c)}$."

ABSTRACTS OF PAPERS

(Abstracts of papers presented at the Washington meeting of the Institute, December 27–30, 1953)

1. Confidence Intervals of Fixed Length for the Poisson Mean and the Difference Between Two Poisson Means. Allan Birnbaum, Columbia University.

1. To construct an estimate λ' of the unknown parameter λ of a Poisson process $X(t)$ such that with probability at least $1 - \alpha$, $|\lambda' - \lambda| \leqq \epsilon$, where α and ϵ are given positive constants, let n be a positive integer. Observe T_n, the waiting time required for the occurrence of n events. Let $c = \alpha\epsilon^2/2n$. Perform additional observation of the process for $1/2cT_n$ units of time; let X be the number of events observed in this period. Set $\lambda' = 2cT_nX$. 2. To construct an estimate Δ' of $\Delta = \lambda_2 - \lambda_1$, where λ_1, λ_2 are the unknown parameters of two Poisson processes, such that with probability at least $1 - \beta$, $|\Delta' - \Delta| \leqq \eta$, where β and η are given positive constants, we may set $\Delta' = \lambda_2' - \lambda_1'$, where λ_1' and λ_2' are obtained as above, taking $\epsilon = \eta/2$ and $(1 - \alpha)^2 = 1 - \beta$. In case the two processes can be observed simultaneously, a more efficient estimate can be given. At least for λ exceeding some lower bound, c can be replaced by $c^* = \epsilon^2/k$, where $\Pr\{z \geqq k\} = \alpha$ if z is the product of independent chi-square variates with 1 and $2n$ d.f.

2. Convexity Properties of the alpha-beta-set Under Composite Hypotheses. Herman Rubin and Oscar Wesler, Stanford University.

Suppose one is presented with a statistical decision problem of the following kind. A random variable X is observed and it is desired to test whether the (not necessarily finite-dimensional) parameter of the distribution of X is in Ω_1 or in Ω_2. Define, as usual, $\alpha(\varphi)$ to be the supremum of the probability of an error of the first kind and $\beta(\varphi)$ to be the supremum of the probability of an error of the second kind when the random decision procedure φ is used. If Ω_1 and Ω_2 consist of one point each, it is known that the set S of all points $(\alpha(\varphi), \beta(\varphi))$ is convex and symmetric about $(\frac{1}{2}, \frac{1}{2})$. It is shown that the subset T of S lying on or below the line $\alpha + \beta = 1$ is convex, and that if the set of distributions under consideration is dominated by a σ-finite measure, the lower boundary of T belongs to T. It is also shown that the symmetric image of T, and possibly more, belongs to S. An example is given to show that this "more" can destroy convexity.

3. Critical Regions in Terms of Lower Dimensional Critical Regions. L. M. Court, Diamond Ordnance Fuze Laboratory.

Let $p_1(x \mid \theta) \equiv p_1(x_1, \cdots, x_{n_1} \mid \theta_1, \cdots, \theta_{m_1})$ and $p_2(y \mid \phi) \equiv p_2(y_1, \cdots, y_{n_2} \mid \phi_1, \cdots, \phi_{m_2})$ be two independent density distributions and $p(x, y \mid \psi) = p_1(x \mid \theta)p_2(y \mid \phi)$ the joint dis-