$(S_1S_2S_3S_4)$, such that $ad_{ii} = \sum_{k=1}^{4} {}_ks_{ii}^2$, where ${}_ks_{ii}$ is the entry in the ith column of S_k. This can always be done by the 4 square problem ([2], p. 235). Let $B = a^{1/2}(P^T)^{-1}B_1$, then B has integral entries, since B_1 and $a^{1/2}(P^T)^{-1}$ have integral entries. Then

$$P^TB = a^{1/2}B_1, \qquad P^TBB^TP = aB_1B_1^T = a^2D,$$

Thus $BB^T = a(P^T)^{-1}a^{1/2}Da^{1/2}P^{-1}$. But $A = (P^T)^{-1}DP^{-1}$. Thus $BB^T = a^2A$.

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ABSTRACTS OF PAPERS

(Abstracts of papers presented at the Iowa City Meeting of the Institute, November 26–27, 1945)

1. The Distribution of the Number of Components of a Random Mapping Function. (Preliminary Report.) LEO KATZ, Michigan State College, and JAY E. FOLKERT, Michigan State College and Hope College.

A set H, of N elements, is mapped into itself by a function f. The most general function takes each point into an arbitrary number of points of the set. A function is said to be random if the r_i images of the point x_i may with equal probability be any subset of r_i points of H. A subset h of H is a *component* of the mapping if it is a minimal subset such that $f(h) \subset h$ and $f^{-1}(h) \subset h$. Every mapping f decomposes H into a number of disjoint components. The probability distribution of the number of components of a random mapping, where only the numbers of images of each point are known in advance, is obtained. The probability distribution of the number of components is also obtained for a variant case in which the mapping is hollow in the sense that no point maps into itself. The two distributions are obtained through a modification of the King-Jordan-Frechét formula. For each case two specializations are considered; first, one in which the multiplicity of images is the same for each point of the set, and second, where this common multiplicity is unity (so that the function f is single-valued). Numerical examples and approximations to the exact distribution are considered. This work was supported by the Office of Naval Research.

2. Approximate Sequential Tests for Hypotheses about the Proportion of a Normal Population to One Side of a Given Number. WILLIAM KRUSKAL, University of Chicago.

It is sometimes of interest to test the hypothesis that the proportion of a given population exceeding a given number U is p_0 against the hypothesis that this proportion is p_1. This testing situation has been called that of testing for one-sided fraction defective. If the population is normal then the problem is to test the hypothesis $(U - \mu)/\sigma = K_0$ against the hypothesis $(U - \mu)/\sigma = K_1$. (Here μ is the mean, σ^2 the variance, and K_i the unit-normal deviate exceeded with probability p_i.) A simple translation puts this in the form: $\mu/\sigma = K_0$ vs. $\mu/\sigma = K_1$. If a sequential test is desired, it is very reasonable to base it on the sequence of Student t values computed from the first n observations. Application of the Wald sequential probability-ratio method to this sequence gives a procedure that may be called the WAGR test (after Wald, Arnold, Goldberg, and Rushton). Another sequential test procedure for this problem has been suggested by Wallis. A sequence of successive approximations to the WAGR test is derived; of these the Wallis procedure is the final approximation. When the true value of $(U - \mu)/\sigma$ is distant from K_0 and K_1 any of these sequential procedures may lead to unduly large average sample sizes. A suggestion is made to overcome this difficulty.

3. A Note on Some Limit Theorems in Probability Theory. John Gurland, Iowa State College.

A theorem by Cramér on the limiting distribution function of a sequence of random variables is extended by widening the class of functions to which the theorem applies. A theorem by Mann and Wald is modified by relaxing the continuity restrictions. The present note utilizes the continuity theorem for the limit of a sequence of characteristic functions in establishing the results stated.

4. Uniqueness of Latin Square Association Schemes for Partially Balanced Incomplete Block Designs. (Preliminary Report.) D. M. Mesner, Purdue University.

In Latin square type partially balanced incomplete block designs with i constraints, first associates are defined by means of a square array of the n^2 varieties with $i - 2$ mutually orthogonal Latin squares. $n_1 = i(n - 1)$ and $p_{11}^2 = i(i - 1)$, determining n_2 and the remaining p_{km}^i. For $i = 1$ the designs reduce to a special case of Group Divisible, for which Bose and Connor proved a uniqueness theorem (*Ann. Math. Stat.*, Vol. 23 (1952), p. 368). The analogous theorem here, that a design with the given parameter values implies the existence of the square array, is proved for $i = 2$. In this case, an arbitrary pair of first associates have as common first associates $n - 2$ additional varieties; these n varieties are assigned to one row of the array. Investigation of the incidence matrix of first associates shows that they are pairwise first associates. This is then used to show that the remaining association relations are those defined by the remainder of the array. The cases $i \geqq 3$ are being investigated and partial results have been obtained. That a complete generalization is impossible is shown by examples for which designs exist but the Latin squares do not. One example corresponds to the non-existent 6×6 Graeco-Latin square.

5. On Bounds for the Normal Integral. J. T. Chu, University of North Carolina.

Let $v(x) = \int_0^x (2\pi)^{-\frac{1}{2}} e^{-t^2/2}\, dt$, and $u(x, a) = \frac{1}{2}(1 - e^{-ax^2})^{\frac{1}{2}}$, for all $x \geqq 0$. Then $u(x, a) \leqq v(x) \leqq u(x, b)$ if and only if $0 \leqq a \leqq \frac{1}{2}$, and $b \geqq 2/\pi$. Therefore $u(x, \frac{1}{2})$ and $u(x, 2/\pi)$ are respectively the best possible lower and upper bounds for $v(x)$ of the type $u(x, a)$. The ratio $v/u(\frac{1}{2})$ is a steadily decreasing function of x and has an upper bound $2/\pi^{\frac{1}{2}}$. If $w(x, a) = \frac{1}{2}[ax^2/(1 + ax^2)]^{\frac{1}{2}}$, then $v(x) \geqq w(x, a)$ if and only if $0 \leqq a \leqq 2/\pi$, and $w(x, 2/\pi)$ is the best possible one of the type $w(x, a)$. Comparisons are made of the two new lower bounds for $v(x)$ and several known ones converted from inequalities for Mills' ratio, obtained by R. D. Gordon, Z. W. Birnbaum, and R. F. Tate (*Ann. Math. Stat.* (1941), 364–366; (1942), 245–246; (1953), 132–134). Of the three: $w(x, 2/\pi)$, $u(x, \frac{1}{2})$, and Tate's $(\frac{1}{4} + (e^{-x^2}/2\pi x^2))^{\frac{1}{2}} - (e^{-x^2/2}/x(2\pi)^{\frac{1}{2}})$, it is found that approximately the first is the best for $0 \leqq x \leqq 1$, the second, for $1 \leqq x \leqq 1.01$, and the last, for $x \geqq 1.01$.

6. Rules for Determining Error Terms in Hierarchical and Partially Hierarchical Models, Mary Lum, Wright Air Development Center.

In the classical analysis of variance procedure a linear mathematical model is used, together with certain well-known assumptions, as a basis for determining proper test factors for the F-test of significance. This paper is concerned with models for which one or more factors are of the "within" or "nesting" type, with complete replication for the other factors. A set of rules is given for obtaining the appropriate test factors (error terms) for all main effects and interactions of the general n-factor partially hierarchical model. These rules take into account models which are fixed, mixed, or random in nature. The usual assumptions of the analysis of variance are made, with one exception: a weaker set of restrictions is imposed concerning the independence of the interaction effects. The resulting test factors are compatible with those which result from the randomization theory approach of Kempthorne and Wilk. To illustrate the convenience and usability of these rules, two examples are given.

(Abstracts of papers presented at the Annual Membership Meeting, December 27–30, 1954)

7. Maximum Likelihood Estimates of Monotone Parameters. H. D. Brunk, University of Missouri.

The maximum likelihood estimators of distribution parameters subject to certain order relations are determined for simultaneous sampling from a number of populations, when (i) the order relations may be specified by regarding the distribution parameters, of which one is associated with each population, as values at specified points of a function of n variables monotone in each variable separately; and (ii) the distributions of the populations from which sample values are taken belong to the exponential family (cf. Girshick and Savage, "Bayes and minimax estimates for quadratic loss functions," *Proc. Second Berk. Symposium*, Univ. of Calif. Press, 1951, pp. 53–73; p. 65). The results of the present paper generalize those of Ayer, Brunk, Ewing, Reid, and Silverman, "An empirical distribution function for sampling with incomplete information," to appear in *Ann. Math. Stat.*, which treats the special case in which $n = 1$ and the distribution is binomial. This paper also represents a specific application of results obtained by Brunk, Ewing, and Utz on minimizing integrals in certain classes of monotone functions (to be offered for publication). This research was supported by the United States Air Force, through the Office of Scientific Research of the Air Research and Development Command.

8. Extension of Certain Classes of Contagious Distributions. John Gurland, Iowa State College.

Beall has recently shown that Neyman's types A, B, and C of contagious distributions are members of a certain general class of contagious distributions. The present paper provides alternative ways of extending this class of distributions, and discusses the interrelations which exist among various classes. The methods employed are similar to those in recent articles on the subject.

9. Recurrent Markov Processes II. (Preliminary Report.) T. E. Harris, The Rand Corporation.

All sets considered are Borel measurable. For X a real set, let $P^n(x, E) = \text{Prob}\,(x_n \in E) \mid x_0 = x)$ for $n = 0, 1, \cdots$ be the iterates of a temporally homogeneous Markov transition function; m is a sigma-finite measure. Condition 1: $m(B) > 0$ implies $\text{Prob}\,[x_n \in B\ i.o. \mid x_0] = 1$, for all $x_0 \in X$. Condition 1 implies (abstract, Nov. 27, 1954 AMS meeting, Los Angeles) existence of a sigma-finite measure π where $\pi(X) = \infty$ is allowed, such that m is absolutely continuous with respect to π, with $\pi(E) = \int_X \pi(dx)P^1(x, E)$, for all $E \subset X$. Next, Condition 1 being satisfied, suppose $A \subset X$, with $0 < \pi(A) < \infty$; suppose the Markov process, obtained by observing only those x_n which are in A, satisfies condition

D_0 (Doob, *Stochastic Processes*, p. 221). Then

$$\lim_{N\to\infty} \Sigma_{n=1}^{N} P^n(x, E)/\Sigma_{n=1}^{N} P^n(y, F) = \pi(E)/\pi(F)$$

for all x, y, E, F in A, provided $\pi(F) > 0$.

10. The Role of Perks' Distributions in the Theory of Wiener's Stochastic Function. JOSEPH TALACKO, Marquette University and University of California.

In "Wiener's Random Function, and other Laplacian Random Functions," by Paul Lévy, *Second Berkeley Symposium on Math. Stat. and Prob.*, 1951, pp. 171–187, Wiener's function has been applied to the study of the Brownian plane curve. For the area between an arc and its chord particular solutions of the relative transition probability are found. This paper, using the method of characteristic functions, shows that the solutions are all non-Gaussian and from the Perks family of functions $f(x) = c(e^{\pi x} + k + e^{-\pi x})^{-1}$, where $0 \leqq k \leqq 2$.

11. On the Orthogonality of Measures and the Existence of Consistent Tests. CHARLES KRAFT, University of California.

Let $(\mathfrak{X}, \mathfrak{A}) = \Pi_{i=1}^{\infty} (\mathfrak{X}_i, \mathfrak{A}_i)$ and let $\{P\}$ and $\{Q\}$ be any two families of measures on $\mathfrak{A}$. $\{P\}$ is called orthogonal to $\{Q\}$ when there is a set A in $\mathfrak{A}$ such that $P(A) = 1$ and $Q(A) = 0$ for all $P \varepsilon \{P\}$ and $Q \varepsilon \{Q\}$. When the families each contain one element, their orthogonality is equivalent to the existence of a consistent test between the simple hypotheses P and Q. Further, the orthogonality can be characterized in terms of "distances" between the n-dimensional distributions. This characterization is a generalization of the one given by Kakutani for product measures. It is shown that the orthogonality of the two families of measures and the existence of a consistent test for the corresponding hypotheses are identical as long as the families are countable, but that this is not necessarily the case when the families are not countable. The methods used for these problems are then applied to the problem of consistency for likelihood ratio test and maximum likelihood estimates for cases when the observations are dependent.

12. The Minimax Character of the Neyman-Pearson Critical Region. L. M. COURT, Diamond Ordnance Fuze Laboratories.

In his "Statistical Decision Functions", Wald indicates that the Neyman-Pearson theory of testing statistical hypotheses is a special case of Decision Theory for which $d(0) \varepsilon D^e = (1, \cdots, N)$ and $d(x \mid s_1) \varepsilon D^t = (d_1^t, d_2^t)$. By a further natural restriction of the class of decision functions to be considered, the minimax character of the Neyman-Pearson critical region is easily established. Since a minimax solution of a decision problem is essentially also a Bayes' solution with respect to a least favorable a priori distribution, the Neyman-Pearson critical region also corresponds to a Bayes' solution.

13. On the Existence of Linear Regressions. T. S. FERGUSON, University of California.

Let ξ and η be independent nondegenerate random variables with zero means. If the regression of ξ on $X = a\xi + \eta$ is linear for two values, a_1 and a_2, of a with $|a_1| \neq |a_2|$, then ξ and η are both semistable in the sense of P. Lévy, with the same characteristic exponent. Hence if second moments are assumed, both ξ and η are normal. If the regression of ξ on X is linear for three values a_1, a_2, and a_3 of a, with $\log |a_1/a_2|$ and $\log |a_1/a_3|$ incommensurable, then both ξ and η are stable. This extends the results of E. Fix: "Distributions which lead to linear regressions," *Proc. Berkeley Symposium on Math. Stat. and Prob.*, Univ. of Calif. Press, 1949. Two further results are obtained. (1) If the regression of ξ on $X_1 = a_1\xi + \eta_1$ and $X_2 = a_2\xi + \eta_2$ is linear where ξ, η_1, and η_2 are independent nondegenerate random variables with zero means, and $a_1 \neq 0 \neq a_2$, then ξ, η_1, and η_2 are each normal. (2) Let $X = A\xi + \eta$ where ξ and η are independent s- and n-dimensional random vectors, respectively, with zero means, and A is an $n \times s$ matrix of constants. The components of η are assumed independent and at least one of them nondegenerate. If the regression of $b\xi$ on X is linear for all values of the matrix A and the vector b, then for $n = 1$ both ξ and η are multivariate stable and for $n > 1$ both ξ and η are multivariate normal.

14. On the Asymptotic Distribution of U-Statistics Modified by the Introduction of Estimates of Parameters. (Preliminary Report.) B. V. Sukhatme, University of California.

Let $X_1, \cdots, X_n$ be n independent identically distributed r.v.'s with c.d.f. $F(x - \theta)$. Let $\hat{\theta}_n(X_1, \cdots, X_n)$ be an estimate of θ such that given $\epsilon > 0$, there exists a number b such that for n sufficiently large, $P\{|\hat{\theta}_n - \hat{\theta}| \geqq b/\sqrt{n}\} \leqq \epsilon$. The paper investigates the asymptotic behavior of U-statistics when each observation is centered at $\hat{\theta}_n$ and shows that under certain mild restrictions, they are asymptotically normally distributed. Conditions are also given under which these statistics have the same asymptotic normal distribution as the original U-statistics. These results have been extended to random vectors, to functions of several U-statistics, and to the type of statistics that occur in testing the hypothesis that two populations differ only in location.

15. Essentially Complete Classes of Experiments. Sylvain Ehrenfeld, Columbia University.

An experimenter has available a family of independent random variables Y_x depending on a parameter $\theta \,\varepsilon\, \Omega \subseteq E^{(p)}$ and where $x \,\varepsilon\, A \subseteq E^{(k)}$, with A compact and $E^{(p)}$ and $E^{(k)}$ Euclidian spaces. This paper deals with choosing the x's if the experimenter is restricted to a finite number of observations. Suppose Y_x is normally distributed with mean $\psi(\theta, x)$ and variance σ^2. Let $\mathcal{E}_N(B)$ be the class of experiments with x restricted to $B \subseteq A$, and let $V_{x_1,\ldots,x_N}(\widehat{t'\theta})$ be the variance of the maximum likelihood estimate of $t'\theta$ when $Y_{x_1},\ldots,Y_{x_N}$ are observed. $\mathcal{E}_N(B)$ is said to be essentially complete if for any x_j $(j = 1, \cdots, N)$ with $x_j \,\varepsilon\, A$ and any unknown $\theta \,\varepsilon\, \Omega \subseteq E^{(p)}$ there exists a set x_j^* $(j = 1, \cdots, s)$ with $s \leqq N$ and $x_j^* \,\varepsilon\, B \subseteq A$ such that $V_{x_1^*,\ldots,x_s^*}(\widehat{t'\theta}) \leqq V_{x_1,\ldots,x_N}(\widehat{t'\theta})$ for *all* $t \,\varepsilon\, E^{(p)}$. The notion of asymptotic essential completeness is defined and several classes are shown to have the above property for this general case. In case $\psi(\theta, x) = x'\theta$ (the linear Hypothesis) stronger results can be proven without the normality assumption. Let $r(A) \subseteq A$ be defined as follows: $r(A) = \{\lambda(x)x \mid x \,\varepsilon\, A\}$ where $\lambda(x)$ is for each $x \,\varepsilon\, A$ $(x \neq 0)$ a scalar such that $\lambda(x) \geqq 1$ and $\lambda(x)x \,\varepsilon\, A$, while if $\lambda' > \lambda(x)$ then λ' non-ε A. It is shown that: (a) $\mathcal{E}_N(r(A))$ is essentially complete for all N; (b) if $b_1, \cdots, b_s$ are s vectors in $r(A)$ such that their convex closure contains $r(A)$, then $\mathcal{E}_N(b_1, \cdots, b_s)$ is asymptotically essentially complete; (c) with $b_1, \cdots, b_s$ as above $\mathcal{E}_{N+s}(b_1, \cdots, b_s)$ is essentially complete with respect to $\mathcal{E}_N(A)$ for all N.

16. A Theorem on Uniform Convergence of Families of Sequences of Random Variables. Emanuel Parzen, Columbia University.

In the author's paper "On uniform convergence of families of sequences of random variables" (*Univ. Calif. Publ. Statist.*, Vol. 2 (1954), pp. 23–54), a theorem (18D) was stated which is a uniform analogue of theorem 20.6 in Cramér's *Mathematical Methods of Statistics*

(Princeton, 1946). In this note we state a theorem, more general than our previous theorem, which pays attention to UC^* convergence, and does not impose unnecessarily broad equicontinuity conditions on the limit distribution functions. This theorem applies to the multivariate case as well as to the univariate case, and may be proved as an immediate consequence of the Uniform Continuity Theorem. ASSUMPTIONS: Let r-dimensional random vectors $\mathbf{X}^{(n)}(\theta) \xrightarrow{UC^*} \mathbf{X}(\theta)$, and s-dimensional random vectors $\mathbf{Y}^{(n)}(\theta) \xrightarrow{P} \mathbf{a}(\theta)$, constants, uniformly in θ. For $j = 1, \cdots, m$, let $u_j(\mathbf{x}, \mathbf{y}, \theta)$ be a family of real-valued Borel functions, such that, for each $\mathbf{x}$, $u_j(\mathbf{x}', \mathbf{y}, \theta) \to u_j(\mathbf{x}, \mathbf{a}(\theta), \theta)$ uniformly in θ as $\|\mathbf{x}' - \mathbf{x}\| \to 0$ and $\|\mathbf{y} - \mathbf{a}(\theta)\| \to 0$, and, for each $\mathbf{x}$, $|u_j(\mathbf{x}, \mathbf{a}(\theta), \theta)|$ is bounded uniformly in θ. Define $\mathbf{u}(\mathbf{x}, \mathbf{y}, \theta) = (u_1(\mathbf{x}, \mathbf{y}, \theta), \cdots, u_m(\mathbf{x}, \mathbf{y}, \theta))$. Let $\mathbf{Z}^{(n)}(\theta) = \mathbf{u}(\mathbf{X}^{(n)}(\theta), \mathbf{Y}^{(n)}(\theta), \theta)$ and $\mathbf{Z}(\theta) = \mathbf{u}(\mathbf{X}(\theta), \mathbf{a}(\theta), \theta)$. CONCLUSION: $\mathbf{Z}^{(n)}(\theta) \xrightarrow{UC^*} \mathbf{Z}(\theta)$.

17. Decision Procedures for the Comparison of Exponential Populations. F. S. McFEELY and J. E. FREUND, Virginia Polytechnic Institute.

Minimax procedures are applied to the comparison of two exponential populations to determine optimum sample size if one of two alternatives must be accepted. Interpreting the first r out of n ordered observations from each population as failures in a life test, the loss function is taken as

$$L = c_1(1 - \theta_1/\theta_2) \Pr\{\hat{\theta}_1 > \hat{\theta}_2\} + 2c_2 r + 2c_2'(n - r) + c_3\theta_2 \textstyle\sum_{j=1}^{r}(n - j + 1)^{-1}$$

assuming that $\theta_2 > \theta_1$. In order, the terms stand for losses due to wrong decision, to items used, and to cost of experimentation. Boundedness is obtained by considering L to be split into components L_1 and L_2 consisting of the first two and last two terms of L respectively. Minimax procedures applied to L_1 yield an r which is subsequently used in minimizing L_2 for n. Values of r and n are tabulated for selected values of the constants in L.

18. The Efficiency of Some Nonparametric Competitors of the t-test. J. L. HODGES, JR. and E. L. LEHMANN, University of California.

Consider samples from continuous distributions $F(x)$ and $F(x - \theta)$. We may test the hypothesis $\theta = 0$ by using the two-sample Wilcoxon test. It is shown that its asymptotic efficiency (in the sense of Pitman), relative to the t-test, never falls below 0.864. This lower bound is attained when $F'(x)$ is parabolic. A number of alternative notions of asymptotic efficiency are compared. In particular, the limit of the power efficiency function of Dixon at a fixed alternative is derived for the sign test relative to the t, for normal F.

19. Multi-level Continuous Sampling Plans. GERALD LIEBERMAN, Stanford University, and HERBERT SOLOMON, Teachers' College, Columbia University.

In 1943, Dodge (*Ann. Math. Stat.*, Vol. 14 (1943), pp. 264–279) published a sampling inspection plan for continuous production for production in statistical control. Wald and Wolfowitz (*Ann. Math. Stat.*, Vol. 16 (1945), pp. 30–49) later gave a continuous sampling plan which guaranteed a prescribed Average Outgoing Quality Limit (AOQL) even when production is not in statistical control. An inconvenient feature of both plans is the abrupt change between 100 percent inspection and partial inspection. Multi-level continuous sampling plans are introduced to provide smoother transitions between partial inspection and 100 percent inspection. The Dodge plan is a special case occurring when there is one sampling level. The Average Outgoing Quality (AOQ) function is derived for a k-level plan. Contours of constant AOQL are developed for two-level sampling plans and infinite level sampling plans. An approximation is given for contours of constant AOQL for other levels. The choice of a specific multi-level plan is discussed in terms of minimum average fraction inspected and local stability.

20. Numerical Values for Covariances of Order Statistics for Samples of Size Twenty and Less from the Normal Distribution. D. TEICHROEW, University of California.

This paper gives the covariances to more than ten decimal places. The method of computation is given and auxiliary tables obtained in the process are described. These tables include the normal density function, the normal probability integral and its first nineteen powers. All these functions are now available to twenty-five decimal places for the argument increasing in steps of .02.

The table of covariances is the end result of a project proposed by W. J. Dixon to the Institute for Numerical Analysis of the National Bureau of Standards. The project was started before and completed after the Institute for Numerical Analysis had become Numerical Analysis Research of the University of California, Los Angeles. The project was supported by the Office of Naval Research.

21. Estimation of the Parameters of the Rectangular and the Exponential Populations from Singly and Doubly Censored Samples. A. E. SARHAN, University of North Carolina.

The best linear estimates of the parameters of the rectangular population $f(y) = 1/\theta_2$, $\theta_1 - \frac{1}{2}\theta_2 \leqq y \leqq \theta_1 + \frac{1}{2}\theta_2$, and of the exponential population $f(y) = \sigma^{-1} \exp [-(y - \mu)/\sigma]$, where $\mu \leqq y \leqq \infty$, and the variances and relative efficiencies of these estimates are worked out for singly and doubly censored samples of size n. The corresponding quantities are also derived for the special cases $f(y) = 1/\theta_2$, where $0 \leqq y \leqq \theta_2$, and $f(y) = 0/\sigma^{-1}e - y/\sigma$, where $0 \leqq y \leqq \infty$.

22. Estimation from "Censored" Samples of Extreme Data. JULIUS LIEBLEIN, National Bureau of Standards.

In a previous report (NACA Technical Note 3053), the author has obtained unbiased minimum-variance order statistics estimators of an arbitrary linear function of the two parameters of the extreme-value distribution with c.d.f. $P(x) = \exp\{-\exp[-(x - u)/\beta]\}$. These estimators are "optimum" within the class of all linear functions of the order statistics of a sample of given size n. The present paper extends these methods, together with the necessary tables, to the case of a "censored" sample, defined as one where the total number of observations is known but full information is not available with regard to some of them. For example, in fatigue testing, the test may be discontinued before all test specimens have failed, so that the endurance lives for the "run-outs" are not available. The method is applied to an example of fatigue life-testing of ball bearings.

23. On the Probability Integral Associated with a Certain Multivariate Test. K. V. RAMACHANDRAN, University of North Carolina.

The acceptance region $c_0 \leqq$ all $c(S_1S_2^{-1}) \leqq c_0'$ has been proposed by Roy for testing the hypothesis H_0: $\Sigma_1 = \Sigma_2$, where Σ_1 and Σ_2 stand for the dispersion matrices of two p-variate normal populations, S_1 and S_2 for those of two random samples from them, $c(S_1S_2^{-1})$ for the characteristic roots of $S_1S_2^{-1}$, and c_0 and c_0' are two preassigned positive constants. In this paper a technique for obtaining the probability $P[c_0 \leqq$ all $c(S_1S_2^{-1}) \leqq c_0'|H_0]$ is developed which is an extension of the previous methods due to Roy, Nanda and Pillai for obtaining $P[\text{all } c(S_1S_2^{-1}) \leqq c_0' \mid H_0]$ or $P[c_0 \leqq \text{all } c(S_1S_2^{-1}) \mid H_0]$. The actual expressions for the probability are given for $p = 2, 3, 4$ and 5.

24. Some Uses of Quasi-ranges. J. T. Chu, University of North Carolina.

Confidence intervals for, and tests of hypotheses about, the difference of two quantiles (of the same distribution) are obtained, using the difference (called quasi-range) of two properly chosen order statistics. Let $0 < p < q < 1$ be given and ξ_p and ξ_q be the pth and qth quantiles of a certain distribution. Let x_r and x_s, with $r \leqq s$, be the rth and sth order statistics in a sample of size n. Then

$$P\{(x_s - x_r) \geqq (\xi_q - \xi_p)\} \geqq B_n(s-1, q) - B_n(r-1, p) = L,$$

where $B_n(r, p) = \sum_{i=0}^{r} \binom{n}{i} p^i (1-p)^{n-i}$. Corresponding to every integer k, with $0 \leqq k \leqq n - 1$, define $r = [(n - k)p / (1 - c)] + 1$, and $s = r + k$, where $[x]$ is the integral part of x, and $c = q - p$. Then L is a nondecreasing function of k. For a given α (and sufficiently large n), choose the least integer k such that $L \geqq 1 - \alpha$. Then $P\{(x_s - x_r) \geqq (\xi_q - \xi_p)\} \geqq 1 - \alpha$. If the parent distribution satisfies certain continuity conditions, then $x_s - x_r$ is a consistent estimate of $\xi_q - \xi_p$ and variance $O(n^{-1})$. Confidence lower bounds and intervals for, and tests of hypotheses about, $\xi_q - \xi_p$ can be obtained in similar ways. Applications are given to the standard deviation of a normal distribution, quantiles, and the proportion of defectives in a lot.

25. Inadmissibility of the Usual Estimate for the Mean of a Multivariate Normal Distribution. Charles M. Stein, Stanford University.

If $X_1, \cdots, X_n$ are independently normally distributed with unknown means $\xi_1, \cdots, \xi_r$ and variance 1, the usual estimate of ξ_i is X_i. If the loss is the sum of squares of the errors, this estimator is admissible for $n \leqq 2$, inadmissible for $n \geqq 3$. In the latter case, a better estimator is $[1 - (n-2) / (c + \Sigma X_i^2)]X_i$, with c sufficiently large. The constant c should be chosen to increase with n a little faster than $\sqrt{n}$, and the estimate should be replaced by 0 if the correction factor is negative. As $\Sigma \xi_i^2 \to \infty$, this yields asymptotically the best possible improvement over the usual estimator among all estimators having spherical symmetry about the origin. For large $\Sigma \xi_i^2$ or large n, the mean squared error of the proposed estimator is nearly $n - (n-2)^2 / (\Sigma \xi_i^2 + n)$. For large n, it is nearly a Bayes' estimator with respect to any spherically symmetric a priori distribution. For extremely large n, it seems desirable to represent the parameter space as an orthogonal direct sum and apply the method separately in each of the spaces entering into the direct sum. In many experimental designs there is a natural way of doing this, for example by the order of the interaction. Better approximations are needed before this method can be applied freely. It seems likely that the present results can be extended to a large class of problems in which one is interested in estimating many parameters with a single overall measure of the error.

26. A Locally Optimal Test for the Independence of Two Poisson Variables. Mohamed S. Ahmed, University of California.

Let (x_i, y_i), for $i = 1, 2, \cdots, n$, be independent observations on a bivariate Poisson with respective expectations λ and μ. It is shown that no test for the independence of x and y can be both similar in λ, μ and also uniformly most powerful. However, a test of the form $\Sigma_1^n (x_i - \bar{x})(y_i - \bar{y}) \geqq c_n(\bar{x}\bar{y})^{\frac{1}{2}}$ is similar and locally most powerful. Using a theorem on the convergence to a normal of the conditional distribution of $\Sigma_1^n (x_i - \bar{x})(y_i - \bar{y})$, given $\bar{x}$ and $\bar{y}$, asymptotic formulae for the values of the c_n's corresponding to a given level are derived. Furthermore, it is shown that the asymptotic power of the test can be obtained from a normal approximation.

27. Statistical Estimation of the Endurance Limit. E. J. GUMBEL, Columbia University.

The theoretical scheme for fatigue failures and life tests is the probability $l_S(N)$ of surviving N cycles under a stress S. Within any such scheme all specimens tested are expected to fail after a finite number of cycles. However, numerous observations show cases where n' out of n pieces tested survived 10^7 or 10^8 cycles, which is taken as infinity. A stress S_0 so small that the probability of surviving an infinity of cycles is equal to unity is called the endurance limit. It can exist provided there exist minimum lives $N_{0,S}$, that is numbers of cycles before which fracture cannot occur under the stress S. To explain the existence of an endurance limit, the probability of survival is written $l_S(N) = l_S(\infty) + 1 - l_S(\infty)\, L_S(N)$ where $L_S(N)$ stands for the probability of survival up to N cycles of the broken pieces and $l_S(\infty)$ is the probability of permanent survival. A first estimate for $l_S(\infty)$ is the quotient $(n - n') \,/\, (n + 1)$. If the probability $L_S(N)$ is interpreted as the asymptotic probability of the smallest values of a variate limited to the left by $N_{0,S}$, the value $l_S(\infty)$ can be estimated by the method of moments and successive approximations. Finally, the probability $l_S(\infty)$ is again interpreted by the same asymptotic theory of smallest values with the lower limit S_0, which may be estimated by a procedure similar to the estimation of the minimum lives $N_{0,S}$. The few reliable observations on the frequencies of permanent survival which are available lead to a good fit of the theory. (The work was done in part under the support of the Office of Ordnance Research.)

28. On the Efficiency of Estimates in Successive Multiphase Sampling. (Preliminary Report.) B. D. TIKKIWAL, University of North Carolina.

The estimates given in a preliminary report, (*Ann. Math. Stat.*, Vol. 25 (1954), pp. 174), on sampling on successive occasions for K characters from a finite population of size N, are further seen to satisfy the necessary and sufficient conditions for the efficiency of an estimate given by Patterson (1950), in view of the following lemma. Let ${}_lX_{ij}$ represent the ith unit observed for the jth character on the lth occasion and ${}_l\hat{X}_j$ the best estimate, with corresponding variances ${}_l\sigma_j^2$ and $V_N({}_l\hat{X}_j)$. The correlation between the units for the same character on lth and mth occasions is $P_{l,m}$, and the correlation between units for ith and jth characters on the same occasion is P'_{ij}. Then

$$\mathrm{Cov}({}_lX_{ij}\ {}_m\hat{X}_{j'}) = \begin{cases} P'_{j,j'}({}_l\sigma_j/{}_m\sigma_{j'})\ \mathrm{Cov}({}_lX_{ij'}\ {}_m\hat{X}_{j'}), & j > j'; \\ P'_{j,j'}({}_m\sigma_{j'}/{}_l\sigma_j)\ \mathrm{Cov}({}_lX_{ij}\ {}_m\hat{X}_j), & j < j'; \\ \psi\ P_{l,m}\ {}_l\sigma_j\ {}_m\sigma_j\,, & j = j'. \end{cases}$$

When $l < m$, $\psi = \Pi({}_t\phi_j)[V_\infty({}_lX_j) \,/\, {}_l\sigma_j^2] + \Sigma_{i=1}^{j-1}\, [P'^2_{ij}\, V_\infty({}_l\hat{X}_i) \,/\, {}_l\sigma_i^2]\, [\Pi({}_t\phi_i) - \Pi({}_t\phi_{i+1})] - 1/N$, where limits on Π are $t = l + 1$ to $t = m$ and ${}_t\phi_j$ is the weight associated with the estimate for jth character on tth occasion. When $l \geqq m$, $\psi = V_N({}_m\hat{X}_j) \,/\, {}_m\sigma_j^2$. If ${}_lX_{ij}$ is not in the sample, $\psi = -1/N$. The estimates in an infinite population are seen to satisfy the conditions of Patterson by putting $N = \infty$.

29. Identification of a Certain Stochastic Structure. HENRY TEICHER, Purdue University.

The structure characterized by $x_i = u_i + \alpha u_{i-1}$, with the u_i identically and independently distributed, is analyzed with respect to the identifiability of the parameter α and the distribution of u. Under simple conditions both are identifiable.

30. Detection and Hypothesis Testing by Linear Methods. EMANUEL PARZEN, Columbia University.

The "measurement signal-to-noise ratio" considered by various authors, and considered as a detection criterion by E. Parzen and N. Shiren (*An analysis of a general system for the detection of amplitude modulated noise*, to be published) is shown to be a useful generalization of the notion of power signal-to-noise ratio, and to be unambiguously defined in cases where the latter is not. It is also shown that the detection criterion provides a possible risk function on which to base a theory of statistical hypothesis testing restricted to rejection regions determined by linear functionals in the manner of estimation by linear methods. The test so obtained for the mean value function of a random time function coincides with the test obtained for a Gaussian process by the Neyman-Pearson theory.

31. Probability of Indecomposability of a Set Under Random Mapping. LEO KATZ, Michigan State College.

A set Ω of N (finite) points is mapped *into* itself by a single-valued function, $f(x)$. The mapping, in the general case, is random if the N^N sample points of $f(x)$ have uniform probability. Each function decomposes the set into a number, k, of disjoint, non-null, *minimal* invariant subsets, as $\Omega = w_1 + w_2 + \cdots + w_k$, with $f(w_i) \subset w_i$ and $f^{-1}(w_i) \subset w_i$. If $k = 1$, the set is indecomposable. In the "hollow" case, of some importance in social psychology, no point maps into itself and the $(N - 1)^N$ sample points have uniform probability. For both cases, the probability that the set is indecomposable under the random mapping is obtained. A table of the probabilities is given for $N = 2(1)20(2)40(5)100$. Asymptotic expressions for the probabilities indicate that indecomposability is more likely in the hollow case by a factor of e, approximately. This last finding is significant in that it contradicts standard sociometric folklore, which holds that the hollow case may be approximated by the general one, for large N. (This work was supported by the Office of Naval Research.)

32. On the Near Monotonic Character of the Power Function of a Certain Multivariate Test. S. N. ROY and K. V. RAMACHANDRAN, University of North Carolina.

In previous papers by one of the authors the acceptance region $c_0 \leqq c_1 \leqq c_p \leqq c_0'$ is proposed for the hypothesis $H_0 : \Sigma_1 = \Sigma_2$, and some of its operating characteristics are studied, where Σ_1 and Σ_2 stand for the dispersion matrices of two p-variate normal populations, S_1 and S_2 for those of two random samples from them, c_1 and c_p for the smallest and largest characteristic roots of $S_1S_2^{-1}$, and $\gamma_1 \leqq \cdots \leqq \gamma_p$ for the characteristic roots of $\Sigma_1\Sigma_2^{-1}$. The quantities c_0 and c_0' are supposed to be given by (i) $P(c_0 \leqq c_1 \leqq c_p \leqq c_0' \mid H_0) = 1 - \alpha = P_0$ (say); and

$$\text{(ii)} \qquad \left[\frac{\partial P}{\partial \gamma_i}(c_0 \leqq c_1 \leqq c_p \leqq c_0' \mid H)\right]_{\gamma_1 = \cdots = \gamma_p = 1} = 0, \qquad i = 1, \cdots, p.$$

It is to be noted that (a) $H_0 \Leftrightarrow \gamma_1 = \cdots = \gamma_p = 1$ and $H \neq H_0$; (b) $P(c_0 \leqq c_1 \leqq c_p \leqq c_0' \mid H) = P(c_0 \leqq c_1 \leqq c_p \leqq c_0' \mid \gamma_1, \cdots, \gamma_p \neq 1) = P$ (say); and also that (c) the equations (ii) are equivalent to just one equation. In this paper it is proved (1) that if all the γ's are equal and stay equal, in other words, if $\Sigma_1\Sigma_2^{-1} = \gamma I(p)$, or $\Sigma_1 = \gamma\Sigma_2$, then P or the second kind of error monotonically decreases, that is, the power monotonically increases as γ tends away from 1 (increasing or decreasing) and also (2) that when the γ_i's are not equal the power increases as each γ_i, separately, tends away from 1, provided that (i) all are greater than 1 or less than 1 to start with and (ii) the gap between the smallest and the largest one does not exceed a certain value.

33. Simultaneous Distribution of the Mean, Standard Deviation, and Range for Probability Density Functions of Doubly Infinite Range. (Preliminary Report.) Melvin D. Springer, U. S. Naval Ordnance.

The simultaneous distribution of the sample mean ($\bar{x}$), standard deviation (S), and range (R) is derived in integral form for probability density functions $f(x)$, for $-\infty < x < \infty$. This derivation is effected through the use of a transformation which transforms the sample probability element $\Pi_1^n f(x_i)dx_i$ into an element which is a function of $x_2, x_3, \cdots, x_{n-2}, \bar{x}, S, R$. After limits of integration of x_{n-r} are determined, this function is integrated with respect to x_{n-r} for $r = 2, 3, \cdots, n - 2$, to obtain the simultaneous distribution $F(\bar{x}, S, R)$. Integration of $F(\bar{x}, S, R)$ with respect to $\bar{x}$, S, and R, respectively, yields the joint distributions $G(S, R)$, $H(\bar{x}, R)$, and $K(\bar{x}, S)$, whose correlation properties are then considered. In particular, the regression functions, scedastic functions, and correlation ratios are presented in general (integral) form for both regression systems of each of the joint functions $G(S, R)$, $H(\bar{x}, R)$, and $K(\bar{x}, S)$. The methods are then applied to the normal density function to illustrate the procedures for determining $F(\bar{x}, S, R)$ and for analyzing the regression system of R on S.

34. A Class of Asymptotic Tests of Composite Hypothesis. J. Neyman, University of California.

Let H be a composite hypothesis asserting that all the observable r.v.'s of the sequence $\{X_i\}$ (possibly multivariate) are completely independent and have the same probability density $p(x|\theta)$ defined for $x \varepsilon \mathfrak{X}$ and depending on a parameter point $\theta = \{\theta_1, \theta_2, \cdots, \theta_s\}$. H fails to specify the value θ^0 of θ except that it must belong to an open set Θ. The problem considered is that of "asymptotically α-similar" regions for testing H. Let $Y_n = \{X_1, X_2, \cdots, X_n\}$ and w_n be a subset of $\mathfrak{X}^n$. We say that the sequence $\{w_n\}$ is an asymptotically α-similar critical region for testing H if $P\{Y_n \varepsilon w_n \mid \theta^0\} \rightarrow \alpha$ as $n \rightarrow \infty$ for all $\theta^0 \varepsilon \Theta$. Under suitable regularity conditions on $p(x|\theta)$ it is proved that, whenever for each $j = 1, 2, \cdots, s$ there is known a consistent estimate $\theta_i^*(Y_n)$ of θ_i^0 converging to θ_i^0 so fast that the product $\sqrt{n}\ |\theta_i^*(Y_n) - \theta_i^0|$ is bounded in probability, a family Φ of asymptotically α-similar regions can be constructed as follows. Let $\nu(\alpha)$ be an open set of numbers such that $N(0, 1)\ \varepsilon\ \nu(\alpha)$ with probability α. Let $f(x, \theta)$ be a sufficiently regular function on $\mathfrak{X}\times\Theta$, such that the expectation of $f(X, \theta^0)$ is zero and its variance unity for all $\theta^0 \varepsilon \Theta$. Let $\varphi_j(x, \theta) = \partial(\log p)/\partial\theta_j$, let $A_j(\theta^0)$ be the regression coefficient of $f(X, \theta^0)$ on $\varphi_j(X, \theta^0)$ and let $\sigma^2(\theta^0)$ be the variance of $f(X, \theta^0) - \Sigma_1^s A_j(\theta^0)\varphi_j(X, \theta^0)$. Finally, let w_n stand for the subset of $\mathfrak{X}^n$ defined by

$$\sum_{i=1}^{n} \{f[x_i; \theta^*(Y_n)] - \sum_{j=1}^{s} A_j[\theta^*(Y_n)]\varphi_j[x_i, \theta^*(Y_n)]\} \frac{1}{\sigma[\theta^*(Y_n)]\sqrt{n}} \quad \varepsilon \quad \nu(\alpha).$$

Then the sequence $\{w_n\}$ is asymptotically α-similar. The problem of selecting within Φ an "optimum" critical region for testing H is thus reduced to that of selecting $f(x, \theta)$ and $\nu(\alpha)$.

35. The Transitions of a Markovian Process from Set to Set. Bayard Rankin, Massachusetts Institute of Technology.

Let $(\Omega, \mathfrak{A}, P)$ be a probability space, $(\Gamma, \mathfrak{B})$ a measurable space and R^+ the space of positive real numbers. A measurable function $G_\omega(t)$ defined on $\Omega \times R^+$ to Γ will be called a stochastic process with range space Γ. If γ represents a measurable partition of Γ, it is possible to describe the transitions of a Markovian process $G_\omega(t)$ from set to set of γ in terms of a stochastic process $F_\omega(t)$ with range space γ. The process $F_\omega(t)$ is such that for any

$g \varepsilon \gamma$, $F_\omega(t) = g$ whenever $G_\omega(t) \varepsilon g$. In general $F_\omega(t)$ will be non-Markovian, but analytically it may be treated as a temporally inhomogeneous Markov process. In particular, if the transitions of $G_\omega(t)$ through the points of Γ can be described by a temporally homogeneous linear diffusion equation, the transitions of $F_\omega(t)$ through the points of γ can be described by a temporally inhomogeneous linear equation which differs from the latter only in the coefficients. The new coefficients will be weighted averages of the old, the weighting functions being the conditional distribution of $G_\omega(t)$ given that $G_\omega(t)$ belongs to some specified set of γ and given the initial conditions. Thus the "past" of the non-Markovian process is put into the coefficients. If the conditional distribution is arbitrarily assumed independent of time, this defines a new stochastic process $F_\omega(t)$ which is non-Markovian but both temporally homogeneous and linear. (This work was sponsored in part by the Office of Naval Research.)

36. Confidence Intervals for the Ratio and for the Difference of Two Probabilities. (Preliminary Report.) R. R. Bahadur and W. H. Kruskal, University of Chicago.

Let X_i for $i = 1, 2$ be the numbers of successes in binomial samples of sizes n_i with success probabilities θ_i. The authors propose specific exact randomized tests of composite null hypotheses such as $\theta_1/\theta_2 = 2$ and $\theta_1 - \theta_2 = .3$, more precisely of any linear null hypothesis of form H: $a\theta_1 + b\theta_2 + c = 0$, where a, b, and c are constants. In the usual manner, such tests may be used to generate confidence sets for parameter functions like θ_1/θ_2 and $\theta_1 - \theta_2$. The proposed tests are based on the observation that if Z is 1 or 0 with probability θ and $1 - \theta$, and if A and B have independent distributions on the same two points with probabilities for 1 of α and β respectively, then $AZ + B(1 - Z)$ also is 1 or 0 with probability for 1 equal to $\alpha\theta + \beta(1 - \theta)$. In this way null hypotheses of form H may be reached to form $\theta_1 = \theta_2$. The authors propose to study further the small sample and asymptotic properties of these tests and confidence sets. One of the authors (WHK) is studying similar methods in other areas, notably in components of variance problems.

37. Limit Theorems for Conditional Distributions. G. P. Steck, University of California.

Let $U_n = (U_{n1}, \cdots, U_{nL})$ and $V_n = (V_{n1}, \cdots, V_{nM})$ be random vectors. Given that the limiting distribution of (U_n, V_n) exists, conditions are given which insure that the limiting conditional distribution of U_n given V_n also exists. In this connection we have the following theorems: Theorem 1. *Let the conditional characteristic function of U_n given $V_n = v$ (v a possible value of V_n) be denoted by $w_n(v)$. If* (i) *(U_n, V_n) converges in law to the random vector (U, V), and* (ii) *the family $\{w_n(v)\}$ is equicontinuous on bounded sets, then $\{w_n(v)\}$ can be extended to a family $\{w_n^*(v)\}$ such that $w_n^*(v) \rightarrow w(v)$ uniformly for v in a bounded set, where $w(v)$ is the conditional characteristic function of U given $V = v$.* Theorem 2. *Let U_n and V_n be as indicated above. If* (i) *(U_n, V_n) has a limiting normal distribution, and* (ii) *V_{nk} is the normalized sum of independent random variables which are identically distributed on a lattice independently of n, then the limiting conditional distribution of U_n given V_n is also normal.* Theorem 2 can be extended to cases where the summands of V_{nk} are not identically distributed and have distributions depending on n. An application of Theorem 1 indicates that with mild restrictions on the cell probabilities, the limiting distribution of the Pearson χ^2 statistic (s observations, n cells) as $n \rightarrow \infty$ and $s \rightarrow \infty$ is normal if $s^2/n \rightarrow \infty$. Further, if the cells are equiprobable, then the limiting distribution is Poisson if $s^2/n \rightarrow c > 0$ and degenerate if $s^2/n \rightarrow 0$.

38. Tests of Contagion and Time Effect in Accident Proneness. (Preliminary Report.) HOWARD TUCKER, University of California.

This paper considers the case of linear contagion in the theory of accident proneness treated by Miss Grace E. Bates in "Contributions to the theory of Contagion" (to appear in *Ann. of Math. Stat.*). In her paper Miss Bates derived a UMPU test for zero contagion against non-zero contagion in the absence of time effect, based on the exact times at which accidents occur, and in the case of one type of exposure to accidents. In this paper a model is considered for r types of exposure, and the Bates test is extended for this model. For the situation considered by Miss Bates, a test is derived based on the number of accidents which occur in certain time intervals. The power of this test is lower than, but comparable to, that of the Bates test. This test is also extended for the multi-exposure model. Finally, for any fixed value of the contagion parameter, a test is derived for the hypothesis of zero time-effect against non-zero time effect, based on the number of accidents in certain time intervals.

39. Chi Square Test of Goodness of Fit for a Class of Cases of Dependent Observations. JOSEPH PUTTER, University of California.

In the ordinary multinomial situation, we have a sequence of vector r.v. $\{X_n, \cdots, X_{nm}\}$ such that, given $X_{n1}, \cdots, X_{n,k-1}$, the (binomial) r.v. X_{nk} is $\mathcal{B}(n - X_{n1} - \cdots - X_{n,k-1}, \; p_k)$; we know that $\Sigma_{k=1}^{m}[X_{nk} - (n - X_{n1} - \cdots - X_{n,k-1})p_k]^2 \,/\, (n - X_{n1} - \cdots - X_{n,k-1})p_k(1 - p_k)$ is asymptotically distributed as χ_m^2. This is extended to the case where p_k is replaced by $p_{nk}(X_{n1}, \cdots, X_{n,k-1})$, that is, the probability of an observation falling in the kth class depends on the number of observations in the preceding classes. In this case, under some restrictions on the functions p_{nk}, the corresponding statistic is still asymptotically a χ_f^2, but the number of d.o.f. is often reduced. For example, in testing the goodness of fit of a simple chain-binomial model of an epidemic, we get $f = 1$ no matter how many generations we observe.

40. Approximate Probability Values for Observed Number of Successes from Statistically Independent Binomial Events with Unequal Probabilities. JOHN E. WALSH, Lockheed Aircraft Corporation.

Let us consider a number of statistically independent binominal events with possibly unequal probabilities for "success". The principal problem considered is that of determining the probability that the observed number of "successes" equals one of a specified set of values. Given the "success" probabilities, this paper presents an approximate expression which ordinarily furnishes this probability to a reasonable accuracy and is not difficult to evaluate. This approximate expression is obtained by expanding the true expression in terms of the differences between the "success" probabilities and their arithmetic average. If the variation among the event probabilities has known bounds and the upper bound is not too large, usable results often can be obtained in terms of these bounds and the average of the event probabilities. Inverse use of the results for situations of this type sometimes will yield approximate confidence intervals and significance tests for the average of the event probabilities.