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NOTE ON AN APPLICATION OF THE SCHUMANN-BRADLEY TABLE

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Summary. In a recent paper [1] Schumann and Bradley present a table of the ratio of central variance ratios for use in comparing the sensitivity of experiments. The purpose of this note is to point out some other applications of the Schumann-Bradley Table I [1] which stem from a variance components model.

Model. The model employed in [1] is a "fixed effects" analysis of variance schema (henceforth abbreviated FEM). In this note a "random effects" model (REM) will be used. Let s_{ei}^2 and s_{ti}^2 be the mean squares for "error" and "treatments" in the ith experiment ($i = 1, 2$). Let n_{ei} and n_{ti} be the respective degrees of freedom, let σ_{ei}^2 and σ_{ti}^2 be the variance components, and let K_i be a design constant. Finally let χ_{ei}^2 and χ_{ti}^2 be central chi-squares with n_{ei} and n_{ti} degrees of freedom respectively. Then the usual REM is

$$n_{ei}s_{ei}^2 = \sigma_{ei}^2\chi_{ei}^2, \qquad n_{ti}s_{ti}^2 = (\sigma_{ei}^2 + K_i\sigma_{ti}^2)\chi_{ti}^2. \tag{1.01}$$

Let F_i be the ratio of the mean squares and let F_{ci} be a central F ratio with degrees of freedom corresponding to F_i (i.e., n_{ti}, n_{ei}). Then for independent mean squares,

$$F_i = \left(\frac{\sigma_{ei}^2 + K_i\,\sigma_{ti}^2}{\sigma_{ei}^2}\right)F_{ci}. \tag{1.02}$$

Let w be the ratio of F_1 to F_2, let w_c be the ratio of central variance ratios (i.e., F_{c1}/F_{c2}), and let

$$\psi = \frac{\sigma_{e1}^2 + K_1\,\sigma_{t1}^2}{\sigma_{e1}^2} \cdot \frac{\sigma_{e2}^2}{\sigma_{e2}^2 + K_2\,\sigma_{t2}^2}, \tag{1.03}$$

then

$$w = \psi w_c. \tag{1.04}$$

For the special case (C_0) where the two experiments have the same structure (i.e., $n_{e1} = n_{e2}$, $n_{t1} = n_{t2}$, $K_1 = K_2$) equation (1.04) leads immediately to exact significance tests based on Schumann-Bradley Table I [1]. In the nota-

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tion of that table $n_{e1} = n_{e2} = 2b$, $n_{t1} = n_{t2} = 2a$, and the tabular quantity, w_0, is such that:

$$(1.05) \qquad P(w_c < w_0) = 0.95.$$

Since

$$P(w < \psi w_0) = P(\psi w_c < \psi w_0) = P(w_c < w_0)$$

and since for the null hypothesis

$$H_0 : \sigma_{t1}^2 / \sigma_{e1}^2 = \sigma_{t2}^2 / \sigma_{e2}^2$$

the value of ψ is unity, it follows that:

$$(1.06) \qquad P(w < w_0 \mid H_0, C_0) = 0.95.$$

Equation (1.06) provides a one-tailed significance test at the 5% level. A two-tailed test at the 10% level is obtained by using the fact that

$$P(w < w_0 \mid H_0, C_0) = P(1/w_0 < w \mid H_0, C_0).$$

Corresponding confidence intervals on ψ are readily obtained.

In the event that the two experiments do not have the same structure there are both theoretical and practical difficulties. For $K_1 \neq K_2$ the value of ψ under H_0 depends on a nuisance parameter. If $n_{t1} \neq n_{t2}$ or $n_{e1} \neq n_{e2}$ then Table I [1] cannot be used and it is unlikely that a corresponding table will be worked out since it would be a quadruple entry table.

An approximate test in the general case can be obtained by letting

$$\phi = \frac{\sigma_{t1}^2}{\sigma_{e1}^2} \frac{\sigma_{e2}^2}{\sigma_{t2}^2}, \qquad \mu = \frac{\sigma_{e1}^2 + K_1 \sigma_{t1}^2}{\sigma_{e1}^2} + \frac{\sigma_{e2}^2 + K_2 \sigma_{t2}^2}{\sigma_{e2}^2} - 1.$$

An algebraic identity for ψ is

$$(1.07) \qquad \psi = \frac{K_1 \phi\mu + K_2}{K_1 \phi + K_2 \mu}.$$

The parameter μ may be estimated by $F_1 + F_2 - 1$ and, under H_0, $\phi = 1$ so that a numerical estimate of ψ, ψ_0, may be calculated. If the confidence interval on ψ does not include ψ_0 tle null hypothesis would be rejected.

Applications. Hypotheses about ϕ may be of interest in genetic experiments and in the choice of sampling plans [2]. The REM model may also apply in some comparisons of the sensitivities of experiments—for example in situations where *different* sets of treatments are used in the two experiments but both sets are regarded as random samples from the same population of treatments.

The distinction between FEM and REM tests is important here because they can lead to different conclusions. As a numerical illustration of this point consider the data on the rating of canned tomatoes by two judges used by Schumann and Bradley in [3]. For this data $n_{ei} = 24$, $n_{ti} = 8$, and $w = 3.53$. Using Table I [1] the critical value, w_0, is 4.12 for the REM test so that w is not significant at the 5% level. For the FEM test recommended in [3] the critical value is less than 2.87 so that w *is* significant. In general the critical values for the FEM test will be smaller but there is an "ultraconservative" [3] FEM test which is the same as the REM test.

Whether a FEM or a REM test is appropriate is a tricky question involving the *context* of the study. Thus since the *same* set of treatments was used in the comparison of judges a FEM test would seem to be implied. However, as noted in [3], there is a possible judge $\times$ treatment interaction to consider. In other words the judges might make *self-consistent* subjective ratings but there might be little correlation between the two sets of ratings. If this happened the ratings would behave more or less *as if* two *different* sets of treatments were used (implying a REM test). Actually the judge $\times$ treatment interaction does not appear to be serious so the FEM test seems more appropriate. However, this is an issue which must be carefully considered in each specific application.

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ON A THEOREM OF LÉVY-RAIKOV

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Theorem of Lévy-Raikov. *If ϕ_1, ϕ_2 are characteristic functions and $\phi = \phi_1 \phi_2$ is analytic, then so are ϕ_1 and ϕ_2 and the strips in the complex plane where ϕ_1 and ϕ_2 may be extended analytically are at least as large as the strip where ϕ may be extended analytically.*

This theorem was originally proved by P. Lévy [2, 3] for the case where ϕ may be extended analytically over the entire plane, and by Raikov [5]. Another very simple proof may be found in [1].

The purpose of this note is to give a sharpened version of this result.

Theorem. *If ϕ_1, ϕ_2 are characteristic functions and $\phi = \phi_1 \phi_2$ is differentiable $2n$ times, then so are ϕ_1 and ϕ_2. For any real a let $\psi_a(x) = e^{iax}\phi(x)$; then there exist numbers a_j, m_j such that*

$$|\phi_j^{(2k)}(0)| \leqq m_j\, |\psi_{a_j}^{(2k)}(0)|, \qquad j = 1, 2,\ 0 \leqq k \leqq n. \tag{1}$$

If ϕ is infinitely differentiable and the Hamburger moment sequence

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