TABLE 1

Exact and Approximate Tail Areas for the t-distribution with n Degrees of Freedom

Exact(*)	Approximation		
Tail Area	$n = 7$	$n = 15$	$n = 40$
.001	.000 816	.001 06	.001 02
.000 05	.000 042 8	.000 051 5	.000 050 3
.000 01	.000 008 66	.000 010 2	.000 010 05
.000 001	.000 000 873	.000 001 02	.000 001 003
.000 000 1	.000 000 087 7	.000 000 102	.000 000 100 1

(*) These tail areas are exact to the extent that Federighi's [1] tabled quantiles are exact.

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A FINITE CRITERION FOR INDECOMPOSABLE CHANNELS[1]

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Let M be the class of all $n \times n$ Markov matrices, $n \geqq 2$, and let $I \subset M$ be the set of all indecomposable matrices. A Markov matrix is indecomposable if, [3] p. 179, it contains only one ergodic class; or equivalently, if, [4] p. 355, it contains only one irreducible set. Let $A(1)$ be a non-empty subset of M, and for $k \geqq 1$ let $A(k)$ be the set of all $m \varepsilon M$ such that m can be expressed as a product of at most k, not necessarily distinct, elements of $A(1)$. Also let $A = U_1^\infty A(k)$. The following theorem clears up a point concerning indecomposable channels [1], [2], [5] p. 74.

THEOREM. *If* $A(2^{n^2}) \subset I$ *then* $A \subset I$.

PROOF. For $m \varepsilon M$ let m' be the $n \times n$ matrix of zeroes and ones obtained by replacing every positive entry of m by a one; and for $B \subset M$ let $B' = \{m' \mid m \varepsilon B\}$. Now if $a_i, b_i \varepsilon M$; $a_i' = b_i'$; $i = 1, 2, \cdots, k$ then $(a_1a_2 \cdots a_k)' = (b_1b_2 \cdots b_k)'$ because the (i, j)th entry $(a_1a_2 \cdots a_k)_{ij}$ of $(a_1a_2 \cdots a_k)$ is positive if and only if there exists a sequence of states $i_1, i_2, \cdots, i_{k-1}$ such that $(a_1)_{ii_1}(a_2)_{i_1i_2} \cdots (a_k)_{i_{k-1}j} > 0$. Also, clearly, $B \subset I$ if and only if $B' \subset I'$; i.e., the locations of the

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zero entries in $m \varepsilon M$ determines whether or not $m \varepsilon I$. Now $A'(1), A'(2), \cdots$ is an increasing sequence of subsets of M', which has less than 2^{n^2} elements, so there must be a smallest r, $1 \leqq r \leqq 2^{n^2}$, such that $A'(r) = A'(r+1)$. To complete the proof we need only show that $A'(r) = A'$ since if $A(r) \subset A(2^{n^2}) \subset I$ then $A' = A'(r) \subset I'$ so $A \subset I$. Thus we need only prove that if $k \geqq 1$ and $A'(k) = A'(k+1)$ then $A'(k+1) = A'(k+2)$. Now if $m \varepsilon A(k+2)$ then $m = bc$, where $b \varepsilon A(k+1)$ and $c \varepsilon A(1)$ so there exists a $d \varepsilon A(k)$ with $b' = d'$ so $m' = (dc)' \varepsilon A'(k+1)$ and the proof is complete.

We conclude with three comments. Clearly $A'(1)$ determines whether or not $A \subset I$ so that if $A(1)$ is an infinite set, which is not the case for indecomposable channels, then $A(1)$ may, for the purpose of determining whether or not $A \subset I$, be replaced by any finite $B \subset M$ with $B' = A'(1)$. If $m \varepsilon A$ has a state which is periodic with period $d > 1$ then $m^d \not\varepsilon I$ and $m^d \varepsilon A$ so $A \not\subset I$. For any $A(1)$, $(A(2^{n^2}))' = A'$.

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NOTE ON QUEUES IN TANDEM[1]

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1. Introduction. Assume that Q_k, $k = 1, 2, \cdots, m$, is a single server queue where customers are served with an exponential service time distribution of mean $1/\mu_k$. We shall assume that the jth customer, C_j, arrives at Q_1 at time t_j, where $\{t_j\}$ are the events of a Poisson process, and λ the number of arrivals per unit time. The queues Q_k are arranged in tandem; that is, after C_j's service at Q_k is completed he proceeds to Q_{k+1} and joins the queue there. We shall extend a result of our previous paper [1] for the foregoing situation.

Let T_{jk} denote C_j's waiting time at Q_k, *including* the duration of C_j's service at Q_k. The purpose of the present note is to show, using the results of [1], that under "equilibrium" conditions the probabilistic description of the random

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