ABSTRACTS OF PAPERS

(Abstracts of papers presented at the Central Regional Meeting, Manhattan, Kansas, May 7–9, 1964.)

1. Tables for the Studentized Largest Chi-Square Distribution and Their Applications. J. V. Armitage and P. R. Krishnaiah, Aerospace Research Laboratories.

Let $\chi_1^2, \cdots, \chi_K^2$ be independently distributed chi-squares each with n degrees of freedom and let χ_0^2 be another chi-square with m degrees of freedom distributed independently of the other K chi-squares. Then the distribution of $S = m\chi_{\max}^2/n\chi_0^2$, where $\chi_{\max}^2 = \max(\chi_1^2, \cdots, \chi_K^2)$, is known to be the Studentized largest chi-square distribution. S. S. Gupta (*Ann. Inst. Statist. Math.* **14** 199–216) gave the reciprocals of the upper 25%, 10%, 5% and 1% points for the distribution of S when $K = 1(1)10$ and $m = n = 2(2)50$. In the present paper, upper 10%, 5% and 1% points are given for $K = 2(1)12$, $n = 1(1)20$ and $m = 5(1)50$. Various applications of these tables are also discussed.

2. On Testing the Equality of Parameters to a Specified Value in One or More Rectangular Distributions. D. R. Barr, USAF Academy, University of Iowa and Aerospace Research Laboratories. (By title)

Let $X_1, \cdots, X_k$ be $k \geqq 1$ stochastically independent random variables, where each X_i, $i = 1, \cdots, k$, is distributed uniformly on the interval from 0 to θ_i, and let the hypothesis to be tested be $\{H_0: \theta_1 = \cdots = \theta_k = \theta_0;\ H_1: \theta_i \neq \theta_0$ for at least one $i\}$, where θ_0 is some specified value. The distribution of $-2 \ln \Lambda$ (where Λ is the likelihood ratio test statistic) when the hypothesis is true has previously been found by Hogg (*Ann. Math. Statist.* **27** (1956) 529–532). If the sets A and B are defined by $\{\theta_\alpha: \alpha \varepsilon A\} = \{\theta_\alpha: \theta_\alpha \leqq \theta_0\}$ and $\{\theta_\beta: \beta \varepsilon B\} = \{\theta_\beta: \theta_0 < \theta_\beta\}$, then the power function of the likelihood ratio test is equal to one in case $\prod_{\alpha\varepsilon A} \theta_\alpha^{n_\alpha} \leqq \lambda_0\theta_0^\Sigma$, where $\Sigma = \sum_{\alpha\varepsilon A} n_\alpha$, and is equal to $1 - \prod_{\beta\varepsilon B}(\theta_0/\theta_\beta)^{n_\beta} + \lambda_0\prod_{i=1}^k (\theta_0/\theta_i)^{n_i}\sum_{\gamma=0}^{k-1}[(-1)^\gamma/\gamma!] \ln^\gamma [\lambda_0\prod_{\alpha\varepsilon A}(\theta_0/\theta_\alpha)^{n_\alpha}]$ in case $\lambda_0\theta_0^\Sigma \leqq \prod_{\alpha\varepsilon A}\theta_\alpha^{n_\alpha}$, where $\{\lambda: 0 \leqq \lambda \leqq \lambda_0\}$ is the critical region of the test. In particular, the power function is continuous and the test is unbiased; and if the significance level of the test is α, then $\alpha = \lambda_0\sum_{\gamma=0}^{k-1}[(-1)^\gamma/\gamma!] \ln^\gamma(\lambda_0)$.

3. On Testing the Equality of Parameters of Two Rectangular Distributions. D. R. Barr, USAF Academy, University of Iowa and Aerospace Research Laboratories.

The class of tests of the hypothesis of equality (common value unspecified) of two distributions uniform of the interval from 0 to θ_i, $i = 1, 2$, in which the power function is continuous and the critical region is of the form $\{(w_1, w_2): w_2 \leqq g_*(w_1)$ or $g^*(w_1) \leqq w_2\}$, where w_i, $i = 1, 2$, is the largest item of a random sample of size n_i from the ith distribution, g^* and g^* have inverse functions such that g^* and g^* are continuous and $g^*(t) \leqq t \leqq g^*(t)$ for all $t > 0$, is considered; Murty's test (*J. Amer. Statist. Assoc.* **50** (1955) 1136–1141) and the likelihood ratio test are members of this class. It is proved that the unique unbiased member of this class is the likelihood ratio test. It is shown that if sample sizes are equal, then $-2 \ln \Lambda$, where Λ is the likelihood ratio test statistics, is distributed as the absolute value of a random variable which has a Laplace distribution with certain parameters. The power function of the likelihood ratio test is found; it possesses the monotonicity property.

4. On the Estimation of Contrasts in Linear Models. SUBHA BHUCHONGKUL AND MADAN L. PURI, University of Thailand; New York University.

In linear models with several observations per cell, a class of estimates of all contrasts is defined in terms of rank test statistics such as the Wilcoxon or normal scores statistic. The asymptotic efficiency of these estimates relative to the standard least squares estimates is shown to be the same as the Pitman efficiency of the rank tests on which they are based to the corresponding t-tests. The results obtained are the extensions of the results of Hodges and Lehmann (*Ann. Math. Statist.* 1963) and Lehmann (*Ann. Math. Statist.* 1963).

5. Estimation of a Multivariate Density. THEOPHILOS CACOULLOS, University of Minnesota.

Let $x_1, \cdots, x_n$ be a sample from a population with an absolutely continuous distribution function $F(x)$. Parzen (*Ann. Math. Statist.* **33** 1065–1076) obtained a class of consistent (in quadratic mean), asymptotically unbiased, and asymptotically normal estimates of the density $f(x) = F'(x)$ in the univariate case. This paper gives an extension to the multivariate case. Moreover, it is shown that the estimators, in addition to the above properties, generate an asymptotically Gaussian process on the vector space $\{x\}$. Limits for bias and mean square error are also given.

6. Estimation of the Mean of Symmetrical Distributions (Preliminary report). EDWIN L. CROW, National Bureau of Standards, Boulder, Colorado.

A number of papers have discussed best linear estimates of mean and standard deviation for various distributions. (See Sarhan, A. E. and Greenberg, B. G. (1962), *Contributions to Order Statistics*, Wiley, New York.) Here the point of view is adopted that the distribution is known only to the extent of symmetry and may be subject to contamination. Simple linear combinations of the order statistics, of the form $C \sum i^k(x_{(i)} + x_{(n-i+1)})$ for even sample size n for example, where k is a fixed small positive integer, are proposed for estimation of the mean. Efficiencies of the estimates relative to the best linear estimates are evaluated for small samples from normal, rectangular, triangular, U-shaped, and double-exponential distributions.

7. Approximate Distribution Theory for Some Common Discrete Distributions (Preliminary report). JOHN J. GART, Johns Hopkins University.

The relationship between the partial sum of the Poisson distribution and the χ^2 integral has been used by D. R. Cox (*Biometrika* **40** (1953) 354–360) to approximate the Poisson variate with a χ^2. Analogous approximations are developed here for other common discrete distributions. The relationship between the binomial and the incomplete beta function leads to treating the binomial parameter p as a beta variate with parameters $x + \frac{1}{2}$ and $n - x + \frac{1}{2}$. Thus $[p(n - x + \frac{1}{2})]/[(1 - p)(x + \frac{1}{2})]$ becomes an F. For large numbers the application of Fisher's z transformation to this F leads to the logit with the $\frac{1}{2}$ correction for bias suggested by Haldane and Anscombe. The relationship between the multinomial distribution with k classes and the generalization of the incomplete beta function leads to treating $(p_1, p_2, \cdots p_{k-1})$ as if it is distributed as a multivariate beta (Mosimann, *Biometrika* **49** (1962) 65–82) with parameters $(x_1 + \frac{1}{2}, x_2 + \frac{1}{2}, \cdots x_k + \frac{1}{2})$. In a similar way the negative binomial may be approximated by a beta and a similar relationship for the negative multinomial is under investigation. An alternative argument which leads to these results assumes specific prior distributions for the parameters. All of these priors agree exactly with those found using Perks' indifference rule or Jeffreys' invariance theory (*Theory of Probability* (1961) 179–192).

8. Sequential Analysis of Variance Under Random-Effects and Mixed Models. B. K. GHOSH, Lehigh University.

Consider the general random-effects model $x_i = \mu + y_{1i} + \cdots + y_{si} + z_i$ $(i = 1, \cdots, n, s \geqq 1)$ with $V(y_{ti}) = \sigma_t^2$ $(t = 1, \cdots, s)$ and $V(z_i) = \sigma^2$. In particular, for $s = 1$, 2 and 3 one obtains four well-known statistical designs treated by the classical ANOVA, namely, the one-way layout, the randomized blocks or the second-order nested classification, and the complete two-way layout (with replications). A sequential test is developed to discriminate between the composite hypotheses $H_0: \delta = \sigma_t^2/\sigma^2 = \delta_0$ and $H_1: \delta = \delta_1$. The test is based on the likelihood ratio of a suitably chosen function of the sample observations which are obtained under a specified pattern of sequential sampling. It is shown that in many designs the proposed test rule for some hypotheses of the above type reduces to the sequential version of the classical (fixed-sample) F-test. Some possible merits of the sequential test are judged against its classical counterpart and some alternative sequential tests, with special reference to the four designs mentioned above. Some conjectural ASN are calculated in certain cases. The theory is then extended to test similar hypotheses under a mixed model, that is, when $\{y_{ti}\}$, $t = 1, \cdots, p(<s)$ are constants.

9. Estimation of Parameters of the Logistic Distribution. SHANTI S. GUPTA AND MRUDULLA N. WAKNIS, Purdue University.

The problem of estimating the location and scale parameters of the logistic distribution using sample quantiles is studied for large sample sizes. It has been shown that for estimating the mean μ, the optimum symmetric spacing of the quantiles is given by $\lambda_j = j/(k + 1)$ where k is the number of quantiles on which the best linear estimator has been based. The estimator μ^* of μ given by $\mu^* = \sum_{j=1}^{k} 6j(k + 1 - j)X_{(n_j)}/[k(k + 1)(k + 2)]$ where $n_j = [n\lambda_j] + 1$ and $X_{(\ )}$ denotes the ordered random variable. Again, for fixed small values of k, the optimum spacings for σ, the scale parameter, have been calculated. Nearly best unbiased linear estimators using Blom's method have been tabulated. Some results on the efficiency of these estimators have been obtained. Applications of these estimators in problems of selection and ranking have been outlined.

10. A New Table of Percentage Points of the Beta Distribution. H. LEON HARTER, Aerospace Research Laboratories.

Having encountered applications which required a more extensive and more accurate table of percentage points of the Beta distribution than any previously published, the author set out to compute such a table. The objective was a seven-significant-figure table of the percentage points $X(P; A, B)$ of the Beta distribution with parameters $A = 1(1)40$ and $B = 1(1)40$, accurate to within a unit in the last digit, corresponding to cumulative probabilities $P = .0001, .0005, .001, .005, .01, .025, .05, .1(.1).9, .95, .975, .99, .995, .999, .9995, .9999$. The table of percentage points was obtained by inverse interpolation in a table of the incomplete Beta-function ratio generated from a computational formula which was developed by the use of a relation between the hypergeometric function and the incomplete Beta-function ratio. This paper contains details of the mathematical formulation and method of computation, and a discussion of the use of the table of percentage points, with illustrative examples.

11. **Location and Scale Parameters and Sufficient Statistics.** V. S. Huzurbazar, Iowa State University, University of Poona.

Some properties of distributions depending on location and scale parameters are obtained both for regular (i.e. when the range of the distribution does not depend on the parameters) and non-regular cases. The location-parameter family of distributions admitting a sufficient statistic for the location parameter, and the scale-parameter family of distributions admitting a sufficient statistic for the scale parameter, are derived. It is shown that the two-parameter family of normal distributions is the only family in the regular case which admits a minimal pair of sufficient statistics for both parameters of location and scale. This property is shown to hold good in non-regular cases for the rectangular family, if the range depends on both parameters of location and scale; and for the exponential family if the range depends on only one of the two parameters. These results are proved under the assumption of the existence of the third derivative of the p.d.f. in regular cases, and of the first derivative in non-regular cases.

12. **Small Sample Power of the Bivariate Sign Tests of Blumen and Hodges.** Jerome Klotz, Harvard University.

Exact power under normal location alternatives has been obtained for the bivariate sign tests of Blumen (*J. Amer. Statist. Assoc.* **53** (1958) 448–456) and Hodges (*Ann. Math. Statist.* **26** (1955) 523–527) at sample sizes $n = 8(1)12$. Using a recursive scheme in conjunction with a computer, the power functions have been evaluated to 4 or $5D$ at significance levels $\alpha \leqq 0.1$. The tests have surprisingly good power when compared with the bivariate Hotelling T^2. Efficiency values vary roughly between 94% and 80% in the range covered and appear to be decreasing functions of sample size, significance level, and noncentrality parameter. The results seem comparable (even slightly better) to those in the univariate case as given by Dixon (*Ann. Math. Statist.* **24** (1953) 467–473). The conjectures of Blumen concerning relative performance of the two sign tests seem not to hold. Roughly the Hodges test appears better in the region of higher power with the Blumen test preferable locally, although there is little power difference for $8 \leqq n \leqq 12$. The simplified null distribution (Joffe and Klotz, *Ann. Math. Statist.* **33** (1962) 803–807) and slightly better power in the region of interest suggests preference for the test of Hodges.

13. **An Alternative Approach to Multivariate Ratio Estimation.** J. C. Koop, Institute of Statistics, Raleigh, North Carolina.

In the multivariate ratio estimate for the mean $y' = \sum_{i=1}^{p} w_i(\bar{y}/\bar{x}_i)\xi_i \equiv \sum_1^p w_i r_i \xi_i$, given by Olkin (*Biometrika* **45** (1958) 154–165), where the sample mean $\bar{y}$, and those of the p auxiliary characteristics, $\bar{x}_i$ ($i = 1, 2, \cdots, p$) (in regard to which $E(\bar{x}_i) = \xi_i$ and $E(\bar{y}) = \eta$) are based on a simple random sample of size n from a universe of N, the weights w_i, which sum to unity, are determined by minimizing the variance function $V(y')$. The ξ_i's are known. An alternative approach in determining these weights consists in first deriving the upper limit to the absolute value of the bias of this ratio estimate, which is as follows: $|E(y') - \eta| <$ (square root of the sum of the squares of the correlations between the r_i and the $\bar{x}_i$) [the square root of $\sum_1^p w_i^2 V(r_i)\, V(\bar{x}_i)$]. This upper limit is then lowered by finding those w_i's which minimize the function in the square braces. By Cauchy's inequality these minima are given by $w_i = h_i^2/\sum h_i^2$ where $h_i^2 = (V(r_i)\, V(\bar{x}_i))^{-1}$. Estimates of these weights are much simpler to compute than for those given by Olkin's method. However, it is not claimed that this approach is better. Rather the absolute bias here is smaller but the variance is larger, whereas in the former approach it is *vice versa*.

14. On the Estimation of Functional Relationships for a Finite Universe. J. C. Koop, Institute of Statistics, Raleigh, North Carolina. (By title)

The assumptions underlying the classical method of least squares do not apply in the problem of estimating functional relationships for possible causal relationships between the characteristics of the ultimate units (elements) of a finite universe, primarily because all the measurable characteristics observed on each element of a sample, which may be selected in one or more stages and/or phases, with or without replacement, and with (possibly) unequal probabilities, are all random variables. In view of this circumstance, the following method is proposed. Firstly, the ascribed functional (polynomial) relationship is made to pass through the estimated centroid of the sample points, giving rise to a random function with undetermined parameters. Secondly, by the Gauss-Laplace principle of minimum variance, these parameters are determined by minimizing the variance of this random function. The solution of the set of simultaneous equations resulting from this procedure shows that the parameters are functions of the variance and covariance expressions involving the estimated coordinates of the centroid and their powers. The estimates of these parameters are slightly biased. In the case of a linear relation between y and x, based on a simple random sample, the estimate of the slope agrees with the usual least squares estimate. This is a reassuring result. But for stratified sampling the estimate of the slope is a weighted covariance function divided by a similarly weighted variance function.

15. A Fluctuation Theorem and a Distribution Free Test. Charles H. Kraft and Constance van Eeden, University of Minnesota.

Let $(X_1, \cdots, X_n)$ be completely independent random variables with continuous and symmetric (about zero) distributions. It is shown that, if N = number of positive sums in $\{\sum_{j \varepsilon T} X_j \, ; T \subset (1, \cdots, n)\}$, the distribution of N is uniform on $0, 1, \cdots, 2^n - 1$. If the X_i are independent and identically distributed with a continuous distribution $F((x - \theta)/\sigma)$ for some F and for $\theta > 0$, $\sigma > 0$, and if $Y_k = \sum_{i=1}^{k} X_i$, the upper one-sided test based on N is consistent if $\lim_{k \to \infty} P\{(Y_k - k\theta)/\sigma k^{\frac{1}{2}} > -\theta k^{\frac{1}{2}}/\sigma\} = 0$.

16. Simultaneous Tests for Trend and Serial Correlations for Gaussian Markoff Residuals. V. K. Murthy and P. R. Krishnaiah, Douglas Aircraft Company; Wright-Patterson AFB, Ohio.

The model $y_t = m_t + \epsilon_t$ is considered where y_t is the observed value of the random variable at time t, m_t is the expected value of y_t and the residual ϵ_t is a Gaussian Markoff process. The following results were obtained. Assuming $m_t = \alpha + \beta t$ and ϵ_t is a first order stationary Gaussian Markoff process with parameters ρ and σ^2, exact tests were obtained for the parameters α, β, and ρ based on a sample of size N from the process $\{y_t\}$. Also simultaneous confidence limits are obtained for the parameters α, β and ρ. These results are extended to the case of a stationary Gaussian Markoff process of order K. Also attempt is being made to extend the results to the case of a general regression when the residuals follow a stationary Gaussian Markoff process of order K.

17. A Note on Monotonicity Properties of a Class of Ranking and Selection Procedures. M. Haseeb Rizvi, University of Minnesota and Aerospace Research Laboratories.

Consider $k \geqq 2$ single-parameter populations π_i on the real line with c.d.f. $F(x_i, \theta_i)$, $i = 1, 2, \cdots, k$. Let $\theta_{[1]} \leqq \theta_{[2]} \leqq \cdots \leqq \theta_{[k]}$ be the ordered θ-values and suppose $F(x, \theta_{[j]})$

$\leqq F(x, \theta_{[1]})$ for all x and $\theta_{[j]} \geqq \theta_{[1]}$, $(i, j = 1, 2, \cdots, k)$. Consider the following procedures R_1 and R_2 for the problem of selecting a nonempty subset containing the population with the largest parameter and the procedures R_1' and R_2' for the problem of the smallest parameter. Retain π_i in the selected subset if and only if $x_i \geqq x_{[k]} - d_1$, $x_i \geqq d_2 x_{[k]}$, $x_i \leqq x_{[1]} + d_1'$ and $x_i \leqq x_{[1]}/d_2'$ respectively for R_1, R_2, R_1' and R_2' and where $x_{[1]} \leqq x_{[2]} \leqq \cdots \leqq x_{[k]}$ are the ordered x_i's and $d_1 \geqq 0$, $0 < d_2 \leqq 1$, $d_1' \geqq 0$, $0 < d_2' \leqq 1$ are constants (of the procedures) to be determined. Then the following theorems can be stated. *Theorem 1:* $P \cdot \{CS \mid R_m\}$ $(m = 1, 2)$ is a nonincreasing function of $\theta_{[\alpha]}(\alpha = 1, 2, \cdots, k-1)$ and $P\{CS \mid R_m'\}$ $(m = 1, 2)$ is a nondecreasing function of $\theta_{[\beta]}$ $(\beta = 2, 3, \cdots, k)$, where a correct selection (CS) is defined in an obvious manner. *Theorem 2:* Let $q_i(q_i')$ denote the probability of including the population with parameter $\theta_{[i]}$ in the subset selected by $R_m \cdot (R_m')$, $m = 1, 2$. Then $q_j \geqq q_i$ and $q_j' \leqq q_i'$ for $\theta_{[j]} \geqq \theta_{[i]}$.

18. On Mixtures of Perfect Probability Measures (Preliminary report). Robert H. Rodine, Purdue University. (By title)

Let $(X, \mathcal{S}, \mu_y)$ be a probability space for every y belonging to a probability space $(Y, \mathfrak{I}, \nu)$, and suppose that for every S in $\mathcal{S}$, $\mu_y(S)$ is a $\mathfrak{I}$-measurable function of y. Define the mixture measure μ on $\mathcal{S}$ by: $\mu(S) = \int \mu_y(S)\, d\nu$, the integral being taken over Y. It is easily proved that if $(Y, \mathfrak{I}, \nu)$ is discrete, then μ is perfect iff μ_y is perfect for every y in Y, and that the analogous result is false for arbitrary $(Y, \mathfrak{I}, \nu)$. Conditions under which a result of this kind holds are found by exploiting the relationship between mixtures and regular conditional probabilities and the *Theorem*: If $(Z, \mathcal{R}, \lambda)$ is a probability space with $\mathcal{R}$ separable and if $\mathcal{R}'$ is any sub-σ-algebra of $\mathcal{R}$, then λ is perfect iff (i) there exists a regular conditional probability $\lambda(\cdot, \cdot \mid \mathcal{R}, \mathcal{R}')$ and (ii) for every $\mathcal{R}$-measurable, real-valued function f on Z, there is a linear Borel Set $B(f) \subset f(Z)$ and a set $N(f)$ in $\mathcal{R}'$, such that $\lambda(N(f)) = 0$ and $\lambda(B(f), z \mid \mathcal{R}, \mathcal{R}') = 1$ for all z not in $N(f)$.

19. Some New "Goodness of Fit" Criteria and Their Limiting Distributions. M. M. Siddiqui, National Bureau of Standards, Boulder, Colorado.

Anderson and Darling (these *Annals* **23** (1952) 193–212) consider the limiting df's of some generalizations of Cramer-von Mises statistic and Darling (these *Annals* **26** (1955) 1–20) the limiting df of such a statistic when a parameter is estimated. The problem is reduced by finding the limiting cf (characteristic functions). Excepting one case, the inversion of the cf does not seem possible. The purpose of this paper is to observe that in many cases the square of the cf is invertible. Hence if we split the sample into two independent subsamples (or draw two independent samples), construct the statistics for each sample separately and take their sum, the limiting df of this sum may be found. The examples presented by the authors mentioned above are solved to illustrate the method.

20. Comparison of Two Drugs in Multi-Stage Sampling Using Bayesian Decision Theory. Armand V. Smith, Jr., University of Cincinnati. (Introduced by H. A. David)

The general problem considered in this paper is to determine an optimum strategy for deciding how to allocate the observations in each stage of a multi-stage experimental procedure between two binomial populations (e.g., the numbers of successes for two drugs) on the basis of the results of previous stages. After all of the stages of the experiment have been performed, one must make the terminal decision of which one of the two populations has the higher probability of success. The optimum strategy is to be optimum relative to a given loss function; and a prior distribution, or weighting function, for the probabilities of success for the two populations is assumed. Two general classes of loss functions are considered, and it is assumed that the total number of observations in each stage is fixed prior to the experiment. Since the exact procedure for finding the optimum procedure is much too long for practical use for all but small examples, two approximations are developed and compared with the exact procedure.

21. **The Distribution of Products of Independent Random Cauchy Variables** (Preliminary report). MELVIN D. SPRINGER AND WILLIAM E. THOMPSON, General Motors Corporation; University of New Mexico.

The p.d.f.'s of products of independent random Cauchy variables are derived in pseudo-closed form for general n and in closed form for $n \leqq 10$. These results are obtained through the application of fundamental methods for deriving product distributions. The methods, previously discussed by the same authors (*Ann. Math. Statist.* **34** (1963) Abstract 1129) are based on a modification of the Mellin transform. Evaluation of the modified Mellin inversion integral is discussed in detail. Although no moments of order equal to or greater than one exist, the limiting distribution as $n \rightarrow \infty$ appears to approach a Dirac delta function.

(Abstracts of papers to be presented at the Annual Meeting, Amherst, Massachusetts, August 26–29, 1964. Additional abstracts appeared in the December, 1963 and March, 1964 issues, and others will appear in the September, 1964 issue.)

3. **Alternative Analyses of Contingency Tables** (Preliminary report). JOHN J. GART, Johns Hopkins University.

The approximate distribution theory for the multinomial developed in the author's previous abstract is applied to contingency tables. For the 2×2 table the odds (or cross product) ratio, $\Psi = (p_1q_2)/(p_2q_1)$, can be treated as having the distribution of a constant multiple of the ratio of two independent F variates. When one set of marginal totals is large and the corresponding proportions small, Ψ may be treated as a constant multiple of an F. This leads to confidence limits for Ψ similar to those derived by the author (*J. Roy. Statist. Soc. Ser. B* **24** (1962) 454–463) from a different argument. When all the numbers are large, Fisher's z transform applied to the F's yields a logit analysis. For the $r \times c$ table the likelihood ratio test criterion may be written, using our approximate distribution theory, as a function of χ^2's similar in form to Bartlett's test statistic for the homogeneity of variances. We adjust the test statistic so that its mean is $(r - 1)(c - 1)$ except for terms of the second order in the observed cell entries. The resulting χ^2 test is compared to the usual contingency table χ^2 test for 2×2 and 2×3 tables where exact test results are readily available. Also derived are generalizations to three dimensional tables which are similar to those of Lindley (invited address, IMS 1963 annual meeting).

4. **Large-Sample Estimation of an Unknown Discrete Waveform Which Is Randomly Repeating in Gaussian Noise.** MELVIN HINICH, Carnegie Institute of Technology. (Introduced by Herman Chernoff)

Suppose we have an input $X(t)$ made up of an unknown waveform $\theta(t)$ of known fixed length, which is repeated randomly, and is imbedded in Gaussian noise with a known covariance function. The rate of recurrence of the waveform is a known small constant. In addition, the signal-to-noise ratio of the input $X(t)$ is quite low. We wish to estimate the waveform $\theta(t)$ and its autocorrelation $\psi(\tau) = \int \theta(t + \tau)\, \theta(t)\, dt$, restricting ourselves to discrete-time observations on $X(t)$. We derive an asymptotically efficient estimator of the discrete version of $\psi(\tau)$. This estimator is a weighted average of the sample autocorrelation and the square of a linear estimator of the time average (the zero-frequency or DC value) of the waveform. The estimation of θ is more difficult. The problem stated above was motivated by a problem of electronic surveillance of an enemy communication system based upon pulse position modulation, PPM.

5. On the Probability of Large Deviations of the Mean for Random Variables in $D[0, 1]$. J. SETHURAMAN, Michigan State University.

Let the space, $D[0, 1]$, of all real-valued functions on $[0, 1]$ with discontinuities of the first kind only be endowed with the J_1-topology of Skorokhod. Let $X_1(t, \omega)$, $X_2(t, \omega)$, $\cdots$ be a sequence of independently and identically distributed random variables in $D[0, 1]$ measurable w.r.t. the Borel σ-field in the J_1-topology. Let $E[\exp(s\|X_1\|)] < \infty$ for all s. Then, for each $\epsilon > 0$, there is a $\rho(\epsilon)$ with $0 \leqq \rho(\epsilon) < 1$ such that

$$\lim_n n^{-1} \log P\{\omega:\sup_{0 \leqq t \leqq 1} | [X_1(t, \omega) + \cdots, + X_n(t, \omega)]/n - E(X_1(t)) | \geqq \epsilon\} = \log \rho(\epsilon).$$

This generalizes the corresponding result for Banach spaces shown in Sethuraman "On the Probability of Large Deviations for Families of Sample Means" (to appear in the *Annals*) and the result of Ranga Rao "The Law of Large Numbers for $D[0, 1]$ valued random variables," *Theor. Veroyotnost i. Primen.* **8** (1963) under the added restriction of the existence of a moment generating function.

6. Classification Into Multivariate Normal Populations When the Population Means Are Linearly Restricted. M. S. SRIVASTAVA, Stanford University and University of Toronto.

Let x_i be a p-dimensional random vector which is distributed in the population π_i as multivariate normal with mean vector μ_i and covariance matrix (non-singular) Δ; i.e. $x_i \sim N(\mu_i, \Delta)$. The μ_i's are unknown but it is known that (i) $\mu_0 = \mu_i$ for only one i; $i \varepsilon (1, 2, \cdots k)$; (ii) $EX_{(p\times k)} = \mu_{(p\times k)} = A_{(p\times m)} \xi_{(m\times k)}$. The problem is to find the value of i for which $\mu_0 = \mu_i$, where $x = (x_1, \cdots, x_k)$, ξ is a matrix of order $p \times m$ of unknown parameters and A is a known matrix of rank $r \leqq m \leqq k$. Let S/n' be the usual estimate of Δ; $n' = (k + 1)(n - 1)$. Let A_1 be a basis of the matrix A; $A_{1\,(r\times k)} = (c_1, c_2, \cdots, c_k)$ (say). Define b_i and $Z^{(i)}$ by $c_i'(A_1A_1')^{-1}c_i$ and $A_1'(A_1A_1')^{-1}c_i$ respectively. The procedure to take the decision i if i is the smallest integer for which the minimum of the statistics $(1/(1 + b_i)) \cdot (x_0 - xZ^{(i)})' (A^{-1} + W)^{-1} (x_0 - xZ^{(i)})$ is attained is admissible in the whole class of procedures (A, λ known, $\lambda > 0$). For invariant procedures A drops off; $W = \lambda S + XX' + X_0X_0'$.

7. Renewal Theory in the Plane (Preliminary report). JOSEPH A. YAHAV AND PETER J. BICKEL, University of California, Berkeley. (By title)

Let $U[A]$ denote the expected number of visits of a transient 2-dimensional non-arithmetic random walk to a Borel set A in R^2. Let $S(a, y)$ denote the sphere of radius a about the point y for a given norm $\|\cdot\|$ of the Euclidean topology. Then, the elementary renewal theorem for the plane, we have obtained, states that $\lim_{a\to\infty} U[S(a, 0)]/a = 1/\|E(\mathbf{X}_1)\|$ where $\mathbf{X}_1 = (X_{11}, X_{21})$ is the first step of the walk, if $E(\mathbf{X}_1)$ exists. This generalizes Feller's renewal theorem. The main result generalizes the Blackwell renewal theorem in the case of the L_1 and L_∞ norms for random walks which have both $E(X_{11})$ and $E(X_{21})$ finite and one of them different from 0. The theorem states that $\lim_{a\to\infty} \{U[S(a + \Delta, \mathbf{0})] - U[S(a, \mathbf{0})]\} = \Delta/\|E(\mathbf{X}_1)\|$ for every $\Delta \geqq 0$ and $\|\cdot\|$ specified above. Some renewal theorems for (a) bounded regions lying outside a cone about the line defined by $E(\mathbf{X}_1)$ and (b) regions of polygonal type are established. The Blackwell theorem for totally symmetric transient walks with finite step expectations, both of whose marginal walks are recurrent is also proved. Extensions are being considered.

(Abstracts to be presented at the European Regional Meeting, Bern, Switzerland, September 14–16, 1964. Additional abstracts will appear in future issues.)

1. Some Statistical Characteristics of a Peak to Average Ratio. Milton Morrison and Filbert Tobias, ITT Data and Information Systems Division.

The peak to average ratio is a measure used in engineering design. Peak refers to the greatest observation of a population or sample and average refers to one of the familiar measures of central tendency. In this paper the median is chosen as the average. The density function of the peak to median ratio is derived in general for non-negative variates using applicable order statistics techniques. It is then derived specifically for a uniform distribution with lower limit, a, where a is non-negative; and upper limit, $a + b$. It is shown that when the lower limit is zero the distribution of the peak to median ratio is independent of b, but when $a \neq 0$, the density function of the peak to median ratio contains b and a as parameters. The effect of sample size on the density of the peak to median ratio (sample drawn from the uniform distribution) is noted, and densities are shown for several sample sizes and values of b, with a taken as zero and unity. The behavior of the peak to median ratio when the underlying distributions are exponential, Pareto or triangular, and the sample size is large is briefly noted.

2. Rank Test for Scale Parameter for Asymmetrical One-Sided Distributions. Mohamed A. H. Taha, Swiss Federal Institute of Technology and Swiss Forest Research Institute.

The class of one-sided asymetrical distributions is easy to identify in the praxis and is of considerable importance. A typical example of this class is the Γ-distribution with small shape parameter. For this class a two-sample rank test is suggested to test the null-hypothesis: $F(x) = G(y)$ against the alternative $G(y) < F(x)$ generally, or specially against the alternative $G(y) = F(\beta x)$, $\beta < 1$. The test is a linear combination of Wilcoxon's test and that of Mood. The critical values of the test for two samples, any of which is not greater than 8, the first four moments as well as a recurrence formula for both the moment-generating function and the frequency function, are given. The test is also compared with others and is shown to be of higher efficiency.

(Abstracts of papers invited by the I.M.S. Committee for the Symposium on System and Control Optimization, Monterey, California, January 28 and 29, 1964.)

1. Distribution-Free Methods of Signal Detection. C. B. Bell, San Diego State College.

For a variety of practical and theoretical reasons it is sometimes not feasible to assume that the underlying noise distribution is of a specified functional form. The approach to signal detection presented here is an extension of the thesis of J. Capon (1959); and consists of setting up the detection problem as 1-sample, 2-sample, and k-sample distribution-free hypothesis testing models. The detectors, then, correspond to statistical tests and are classified as follows for each of the models.

(A) *Sample Cpf detectors:* (1) Kolmogorov-Smironov type, (2) Cramer-von-Mises type, (3) sign-quantile type.

(B) *Run-block detectors:* (1) spacing type, (2) empty cell type, (3) runs type, (4) slippage type.

(C) *Rank-sum detectors:* (1) percentile-van der Waerden type, (2) moment-Fisher-Yates type, (3) randomized-Doksum type.

(D) *Pitman detectors:* (1) location type, (2) scale type.

Detectors considered are compared as to unbiasedness, consistency, maximum and minimum power; and asymptotic relative efficiency corresponding to the behavior of the detector in the presence of increasing weak signals of a specified form. A stochastic processes model based on the Chebyshev inequalities of Birnbaum and Marshall (1961) is also considered.

2. On the Estimation of State Variables and Parameters. Henry Cox, U. S. Navy, Washington, D. C.

A fundamental problem in modern control theory is that of estimating state variables and parameters of dynamic systems which are subject to random disturbances when available measurements are corrupted by noise. If estimates are to be used for control purposes, there is the severe requirement that the estimation problem be solved in real time. The problem of real time estimation of state variables is discussed for a class of nonlinear discrete-time systems. Through dynamic programming a sequential procedure is developed which yields the known results obtained by Kalman for linear systems and leads to an approximation technique for nonlinear systems which is suitable for real time implementation on a digital computer. The approximation technique is applied to a simple example.

3. Sequential Control Processes. Cyrus Derman, Columbia University.

Of interest are dynamic systems which are observed periodically and classified into one of a finite number of states. After each observation one of a finite number of possible decisions is made; the choice of the decision influences the behavior of the system. More specifically, it is assumed that if a state i is observed and a decision k is made at any time t, there is a known conditional probability $q_{ij}(k)$, independent of t, that the next observed state will be j. The problems that arise are concerned with finding optimal rules for making the decisions. A variety of problems can be treated by dynamic and linear programming methods. Fundamental questions are concerned with existence of optimal rules and, when they do exist, whether there is an optimal rule which is stationary. Some theorems will be given which are useful in settling these questions for certain classes of problems. Examples can be given where optimal rules do not have a stationary character.

4. An Approach to Empirical Time Series Analysis. Emanuel Parzen, Stanford University.

Empirical time series analysis has aims such as the following: (i) to develop efficient computer programs for the statistical treatment of empirical time series, paying especial attention to flexibility of input and output, (ii) to develop a philosophy, based on statistical theory, for judging and interpreting the statistical data reduction provided by the computer output, (iii) to provide experience in the small sample applicability of statistical procedures derived from asymptotic theory. This paper attempts to develop a philosophy for empirical time series analysis, involving the routine use of four data handling procedures (covariance estimation, spectral estimation, autoregressive model fitting and spectral estimation, and trend elimination and estimation), embodied in a computer program. To indicate the speed of the program, we note that the cross-spectral analysis of a pair of time series, each consisting of 4000 observations, requires approximately 10 minutes on a 7090, including computation of covariances. Several examples of empirical time series analysis are given. The paper is to be published in the *Journal of Radio Science.*

5. The Minimum-Delay Concept in the Design of Discrete-Time Filters. ENDERS A. ROBINSON, University Institute of Statistics, Uppsala.

The minimum-delay concept, and the associated concepts of mixed-delay and maximum-delay, are developed to serve as reference points in the classification of filters. Application is made to the optimum design of discrete-time filters, such as sampled-data filters in which the signals appear as trains of pulses, and digital computer filters in which the signals appear as sequences of numbers. Design specifications are given for spiking filters, shaping filters, and Wiener filters, under various practical restrictions.

6. Bispectral Estimation. M. ROSENBLATT AND J. W. VAN NESS, Brown University.

Let X_t, $EX_t \equiv 0$, be a real sixth-order stationary random process ($EX_t^6 < \infty$). Assume that the second and third order moment functions $r(t) = EX_\tau X_{\tau+t}$, $r_3(t_1, t_2) = EX_\tau X_{\tau+t_1} \cdot X_{\tau+t_2}$ can be written as integrals

$$r(t) = \int_{-\infty}^{\infty} e^{it\lambda} f(\lambda)\, d\lambda,\ r_3(t_1, t_2) = \int_{-\infty}^{\infty}\int \exp\,(it_1\lambda_1 + it_2\lambda_2) g(\lambda_1, \lambda_2)\, d\lambda_1\, d\lambda_2$$

in terms of continuous spectral density $f(\lambda)$ and bispectral density $g(\lambda_1, \lambda_2)$. Given observations x_t, $0 \leqq t \leqq N$, the following class of estimates of the bispectral density are studied. Let $\rho_N(v_1, v_2) = N^{-1} \int x_t x_{t+v_1} x_{t+v_2}\, dt$, $0 \leqq |t|, |t + v_1|, |t + v_2| \leqq N$. The estimate is of the form

$$g_N^*(\lambda_1, \lambda_2) = \int_{-N}^{N}\int \exp\,(-i\lambda_1 v_1 - i\lambda_2 v_2) a(B_N v_1, B_N v_2) \rho_N(v_1, v_2)\, dv_1\, dv_2$$

with $B_N \to 0$, $NB_N^2 \to \infty$ as $N \to \infty$ and $a(v_1, v_2)$ a normalized ($a(0, 0) = 1$) continuous bounded and uniformly integrable weight function satisfying certain intrinsic symmetry conditions. Such a sequence of estimates is shown to be asymptotically unbiased and consistent with a covariance function which to the first order is $(NB_N^2)^{-1}$ times a cubic expression in the spectral density $f(\cdot)$. The estimates are shown to be asymptotically normal under certain additional conditions.

7. On Optimal Stopping. JOSEPH A. YAHAV, University of California, Berkeley.

The paper considers the following problem. One takes independent and identically distributed observations from a population obeying a probability law $F_\theta(x)$. However, one does not know $F_\theta(x)$. What is known is a family of distribution functions. $\Theta = \{F_\theta(x)\}$ and one assumes that there exists a prior probability measure $\mu(d\theta)$ on Θ. One is interested in a procedure which is optimal. We define "procedure" and "optimal procedure." We show that under some conditions an "optimal procedure" exists. We show that these conditions are implied by the following two conditions: (i) $E_\theta|x^2| < \infty$ for all $\theta \in \Theta$; (ii) $\int E_\theta|x|\mu\,(d\theta) < \infty$. The optimal procedure turns out to be: Stop at stage j if $y_j = \alpha(j)$ where $\alpha(j)$ is a function of X_1, X_2, $\cdots$, X_j, $\mu(d\theta)$.

The function $\alpha(j)$, although easy to describe, is quite difficult to calculate, hence we give two other functions which again are functions of $X_1, \cdots, X_j, \mu(d\theta)$ and are somewhat easier to calculate. These functions, denoted by $\beta_1(j)$, $\beta_2(j)$ have the following properties: $\beta_1(j) \leqq \alpha(j) \leqq \beta_2(j)$ and $\beta_2(j) - \beta_1(j) \to 0$ as $j \to \infty$.

Furthermore, we give an example for which

$$y_j < \alpha(j) \Leftrightarrow y_j < \beta_1(j) \Leftrightarrow y_j < \beta_2(j).$$

(*Abstracts not connected with any meeting of the Institute.*)

1. Estimates of Reliability for Some Distributions Useful in Life Testing. A. P. Basu, Bell Telephone Laboratories, Inc., Murray Hill, New Jersey.

In this paper the Rao-Blackwell and Lehmann-Scheffé theorems are used to derive the minimum variance unbiased estimates of reliability for a number of distributions useful in life testing. Estimates are also obtained for the case of censored sample in the (one and two parameter) exponential case. A result of Pugh (*Operations Res.* **11** (1963) 57–61) comes out as a special case of the gamma and censored one-parameter exponential model and Laurent's result (*Ann. Math. Statist.* **34** (1963) 652–657) as a special case of the censored two-parameter exponential model.

2. On Some Tests of Hypotheses Relating to the Exponential Distribution When Some Outliers Are Present. A. P. Basu, University of Minnesota.

In this paper the robustness of some of the existing tests for the exponential distribution when some outliers are present has been reviewed. It was noted that the tests proposed by Epstein and Sobel (*J. Amer. Statist. Assoc.* **48** (1953) 486–502) can be used without any major modification in the presence of outliers whereas a more suitable flexible test criterion has been proposed for the exponential lower limit which is superior to Carlson's statistic (*Skand. Aktuarietidskr.* **41** (1958) 47–54) in all situations. Distribution of this statistic has been derived and tables have been constructed to facilitate the use of this statistic. Also some tests for outliers have been proposed in the exponential case.

3. Multisample Nonparametric Tests for Equality of Scales. Jayant V. Deshpande, University of Poona. (Introduced by V. P. Bhapkar)

The notation of abstract no. 2 (*Ann. Math. Statist.* **34** 1624) is used throughout the following. Two nonparametric tests for the problem of equality of scale parameters of c populations are offered in this paper. The first test is based on the statistic $V = N(2c-1)[\sum_{i=1}^{c} \cdot p_i(u_{i1} - c^{-1})^2 - \{\sum_{i=1}^{c} p_i(u_{i1} - c^{-1})\}^2]$, and the second test is based on the statistic $D = [N(2c-1)(c-1)\binom{2c-1}{c-1}/2\{c^2 + \binom{2c-1}{c-1}(c^2-4c+2)\}]\ [\sum_{i=1}^{c} p_i(d_i - 2c^{-1})^2 - \{\sum_{i=1}^{c} p_i(d_i - 2c^{-1})\}^2]$, where $d_i = u_{i1} + u_{ic}$. It is shown that both V and D have asymptotically χ^2 distributions with $c - 1$ d.f. under the $H_0: F_1 = F_2 = \cdots = F_c$. Both of these tests are shown to be consistent for a large class of pertinent alternatives. It is shown that under the alternative of different scales, both these statistics have noncentral χ^2 distribution with $c - 1$ d.f. The D test is shown to be more efficient than the V test for all symmetric distributions, the limiting efficiency as the number of samples tends to ∞ being 2. The D test is shown to be more efficient than a parametric test proposed by Lehmann (*Testing of Statistical Hypotheses* (1959) 273–275) for normal distributions. The statistic D is shown to retain its original asymptotic distribution even if all the observations are centred at the respective sample medians.

4. Evaluation of Hodges-Lehmann Estimates. Arnljot Høyland, University of Oslo.

The Hodges-Lehmann estimate of shift and the Lehmann estimate of contrasts (*Ann. Math. Statist.* **34** 602 and 958) are based on medians of sets of differences. A shortcut method for computing such medians is the following. Establish the rectangle

$$\begin{array}{cccc} & y_{(1)} & \cdots & y_{(n)} \\ x_{(1)} & a & & \\ \cdots & & b & \\ x_{(m)} & & & \end{array}$$

where $x_{(1)} \leqq \cdots \leqq x_{(m)}$, and $y_{(1)} \leqq \cdots \leqq y_{(n)}$, denote the two separately ordered sets of observations. Imagine the differences $y_{(j)} - x_{(i)}$ computed and entered in the corresponding places in the rectangle. The "diagonal" elements a, b, etc. now have ranks $\geqq m$, $\geqq 2(m-1)$ etc. respectively. Utilizing the partial ordering of the differences $y_{(j)} - x_{(i)}$, the wanted median usually is found after only few computations. Hodges and Lehmann also suggest an estimate of location for symmetric distributions (*Ann. Math. Statist.* **34** 601), namely the median of the $\frac{1}{2}n(n+1)$ averages $\frac{1}{2}(Z_r + Z_s)$, $r \leqq s \leqq n$ of "pairs" of the n observations Z_i. A shortcut method is here easily obtained by establishing the upper right half of a similar rectangle with $z_{(1)} \leqq \cdots \leqq z_{(n)}$ along the sides, imagining the sums $z_{(r)} + z_{(s)}$ computed and entered in the corresponding places in the rectangle and utilizing the partial ordering of the sums $z_{(r)} + z_{(s)}$.

5. On Some Related Combinatorial Results (Preliminary report). J. P. Imhof, University of Geneva.

A theorem of Takács (*Operations Res.* **9** (1961) 402–407) also obtained by Dwass is generalized, using an elementary geometric argument suggested by a combinatorial result of Baxter (*Ann. Math. Statist.* **32** (1961) 901–905). If for positive integers $s, r, n, X_1, X_2, \cdots, X_{s+rn}$ are non-negative integer-valued exchangeable random variables, then $P[S_{s+jr} > j$ for $j = 0, 1, \cdots, n-1 \mid S_{s+rn} = n] = s/(s+rn)$, where $S_i = X_1 + \cdots + X_i$. This has applications to queuing theory which generalise some results of Takács (*J. Amer. Statist. Assoc.* **57** (1962) 327–337). When $s = 1$ a more precise result following from a more precise statement of Baxter's lemma is $P[S_i > (i-1)/r$ for k of the indices $i = 1, \cdots, rn \mid S_{1+rn} = n] = 1/(1+rn)$, for $k = 0, 1, \cdots, rn$. This implies Theorem 1 of Gnedenko and Mihalevic (IMS-AMS Selected Translations **1** 55–57).

6. Some Basic Properties of the Incomplete Gamma Function Ratio. S. H. Khamis, Food and Agriculture Organization, Rome.

The difference-differential properties of the incomplete gamma function ratio [Khamis, *Bull. Internat. Statist. Inst.* **37** part 3 (1960)] defined by $P(N, X) = \int_0^X t^{N-1}e^{-t/2}\,dt / \int_0^\infty t^{N-1} \cdot e^{-t/2}\,dt, N, X > 0$, are used to prove the recurrence relation (1) $2NP(N+1, X) = (2N+X) \cdot P(N, X) - XP(N-1, X)$ which holds for all $N > 1$ and all positive X. Recurrence (1) is used to derive, *inter alia*, the following properties of $P(N, X)$: (2) $P(N+S, X) = P(N, X) + [P(N+1, X) - P(N, X)] \sum_{r=1}^{S} (\Gamma(N+1)/\Gamma(N+r))(X/2)^{r-1}$, (3) $P(N, X+2h) = \sum_{r=0}^{n} (h^r/r!) S_{n-r} P(N-r, X) + R_{n+1}$, (4) $P(N, X+2h) = e^{-h} \sum_{r=0}^{n} (h^r/r!) P(N-r, X) + K_{n+1}$ where $S_t = \sum_{j=0}^{t} (h^j/j!)$ and R_{n+1} and K_{n+1} are remainder terms for which suitable expressions are obtained. Rough but useful upper bounds for the remainder terms are also given. The recurrence (1) is used to extend the definition of $P(N, X)$ for all $N \leqq 0$. Numerical illustrations of the practical usefulness of properties (1) to (4) are given. In particular, (1) or (2) enables N-wise interpolation for small values of N and (3) or (4) enable X-wise interpolation for small X, cases where ordinary polynomial interpolation methods are known to be unsatisfactory. Relation (2) is useful in the summation of an important type of finite and infinite series and when $S \to \infty$ it leads to the known infinite series expansion for $P(N, X)$.

7. Tables of the Incomplete Gamma Function, Chi-Squared and Poisson Distributions. S. H. KHAMIS, Food and Agriculture Organization, Rome.

The finite difference and differential properties of the incomplete gamma function ratio [Khamis, *Bull. Internat. Statist. Inst.* **37** part 3 (1960)], $P(N, X) = \int_0^X t^{N-1}e^{-t/2}dt / \int_0^\infty t^{N-1} \cdot e^{-t/2}\,dt$, were used to tabulate $P(N, X)$ correct to ten decimals for the following tabular values of N and X: $N = 0.05(0.05)10.00(0.1)20(0.25)70$; $X = 0.0001(0.0001)0.001(0.001)0.01(0.01)1(0.05)6(0.1)16(0.5)66(1)166(2)250$. The general uses of the tables as well as known and new properties of the tabulated function are discussed and their application to interpolation in the tables is indicated in an introduction to the tables. A subsidiary table of the gamma function $\Gamma(N)$ for $1 \leqq N \leqq 2$ is also included in the introduction to facilitate the use of the tables. The method of computation was developed by the author in 1947 and adapted for use on an IBM 7090 electronic computer by Mr. Wilhelm Rudert of the mathematical staff of the Institute for Practical Mathematics (IPM), Darmstadt, Germany, in consultation with the author and under the general guidance of Professor Alwin Walther of the same Institute.

8. Uniqueness and Monotone Operators in Markov Processes. J. MACQUEEN AND R. M. REDHEFFER, University of California, Los Angeles.

On a finite or infinite interval $a < x < b$ let $Tu = g(x, u') + u - \alpha \int u\, dF_x$ where g and α, $0 \leqq \alpha \leqq 1$ are given functions and F_x is a distribution function for each x. (Equations of the type $Tu = 0$ characterize certain expectations associated with Markov processes.) Let $\alpha(x)F_x(x) < 1$. Then T is "monotone," in that $Tu \leqq Tv$ in (a, b) and $\sup(u - v) \leqq 0$ at $a+$ and $b-$ imply $u \leqq v$. Hence $Tu = Tv$ and $\sup |u - v| = 0$ at $a+$ and $b-$ implies $u = v$ (uniqueness). If $\alpha(x) \leqq 1 - \theta < 1$, then $|Tu - Tv| \leqq \epsilon$ in (a, b) and $\sup|u - v| \leqq \delta$ at $a+$ and $b-$ imply $|u - v| \leqq \max[\epsilon/\theta, \delta]$ (stability). These results extend to integral equations on a topological measure space and to vector-valued functions. In dynamic programming problems, the method gives upper and lower bounds for the expected return from the optimal stationary policy. The proofs, which are very easy, follow Redheffer (*Arch. Rational Mech. Anal.* (1963) 196–212) except that instead of the first point where $u = v$ we consider the last point where $u - v$ attains its maximum.

9. On Some Aspects of a Two Stage Group-Screening Method. M. S. PATEL, M. S. University of Baroda.

In this paper, conditions are given under which a two stage group-screening method gives at least as many correct decisions on the average as that given by a single stage experiment without losing its main objective of reducing the number of runs. Caution is also given when this method should not be used.

10. Properties of a Test in MANOVA. RICHARD E. SCHWARTZ, Cornell University.

In the canonical form of the multivariate linear hypothesis Y_i $(i = 1, \cdots, r)$, U_j $(j = 1, \cdots, s)$ and Z_k $(k = 1, \cdots, n)$ are independent p-variate normal column vectors, distributed as $N(\xi_i, \Sigma)$, $N(\eta_j, \Sigma)$ and $N(0, \Sigma)$ respectively. Let $Y = (Y_1, \cdots, Y_r)$, $\xi = (\xi_1, \cdots, \xi_r)$ and $Z = (Z_1, \cdots, Z_n)$. The test of $H_0:\xi = 0$ with rejection region $\{\text{tr } YY' \cdot (ZZ' + YY')^{-1} > \text{const}\}$ has the following properties: (1) For testing H_0 vs. $H_1:\xi \neq 0$ it is admissible by an argument which follows Stein's proof of the admissibility of Hotelling's T^2 test (*Ann. Math. Statist.* **27** 616–623). (2) For testing H_0 vs. $H_1:\text{tr } \xi\xi' \Sigma^{-1} = \delta$ it is locally minimax as $\delta \to 0$ in the sense of Giri and Kiefer (*Ann. Math. Statist.* **33** 1490–1491). (3) Among invariant level α tests of H_0 which depend only on the latent roots of $YY'(ZZ')^{-1}$, and which therefore have power function of the form $\alpha + c \text{ tr } \xi\xi' \Sigma^{-1} + o(\xi\xi' \Sigma^{-1})$ the above test maximizes the value of c (cf. Constantine, A. G. *Ann. Math. Statist.* **34** 1270–1285 esp. equation 41).

11. A Rank-Order Procedure for Testing Interviewer Agreement. HANS K. URY, California Department of Public Health, Berkeley.

A rank-order procedure is presented for determining whether a number of interviewers, out of a total of n, tend to obtain either positive or negative responses with undue frequency in a survey whose questions admit only these two answers. The method therefore deals with interviewer variability over an entire survey, in contrast to standard techniques which do so on a question-by-question basis (but for more general questions). The procedure is based on the use of Friedman's χ^2 (*J.A.S.A.* **32** (1937) 675) or Kendall and Smith's W (*Ann. Math. Statist.* **10** (1939) 275). When each interviewer questions the same number of subjects, the quantities ranked are the proportions of positive responses obtained on each question; in the general case, the conditional "attainment probabilities" are ranked. Methods for spotting all outliers among the interviewers are discussed. Extensions of the procedure are given to surveys containing multiple-choice questions with answers which can be ranked. Survey designs which will tend to minimize or equalize the dependence between questions and ensure similarity of the sub-populations interviewed are discussed, as are applications of the procedure to quality control situations and to diagnostic comparisons in which the conditions of independence and similarity are automatically met.