John Riordan, *Stochastic Service Systems* (SIAM Series in Applied Mathematics). John Wiley and Sons, New York, 1962. $6.75, £2.8.3. x + 139 pp.

Review by David G. Kendall

University of Cambridge

One might say that this is "yet another book on queueing theory", but the author would very properly protest. He holds the rather curious view that the use of the adjective "queueing" implies "first come, first served", and with his prior interests in the field of telephony he is anxious to emphasize other orders of service, which he says are elsewhere rarely considered. This is in fact not quite true; in steel technology, for example, the use of the hot customer (= ingot) at the end of the line in preference to others who have waited longer (and so are now unsuitably cold) is not uncommon, although presumably in this example "last come, first served" would be combined with "defections among those waiting" (the cold customers will be taken away and re-heated).

A more serious reason for speaking of *stochastic service theory* rather than queueing theory is that this then includes the theory of loss-systems where there may be no queueing or waiting at all. I think in fact that the author has coined a happy phrase; 'stochastic service' is far more appropriate and intelligible than 'mass service' which is often used instead.

There are several very personal views expressed in the author's introductory chapter (Chapter 1) with which I found myself in disagreement, but I will just mention the only one which seems to me really unfortunate; the author's opening remarks are: "The problems responsible for the theory of stochastic service systems seem to have first arisen in the telephone business. As reported in R. I. Wilkinson, 1956b, a central problem for telephone engineers in the United States shortly after the turn of the present century was how to determine the number of telephone lines between central offices so as to give callers reasonable service. . . . Although there seem to be no similar reports from other parts of the world, it may be taken for granted that history as usual repeated itself." This will appear to some an inadequate appreciation of the role played by A. K. Erlang [3] in the development of this branch of applied probability theory. (It will be recalled that Erlang's first published paper on these matters is dated 1909.) I am convinced however that this is entirely the result of an unfortunate and unintentional use of words, for the author refers to Erlang constantly throughout the book with evident appreciation.

I found Chapter 2 ("Traffic Input and Service Distributions") the most interesting, possibly because of the contact with current work [1] on point-processes of all sorts. Here again some statements must be challenged. Thus the author says that he will not discuss inputs with non-independent inter-arrival times "because such processes have not yet arisen in practical studies of service systems"; oil-men and air-traffic engineers will wish that this were true (indeed

Riordan in a very interesting comment at the foot of page 3 shows that he is aware of delayed scheduled arrivals in the air-traffic application). The definition of the Poisson input on page 9 is incautious and would include, for example, regular arrivals with random phase. The treatment of the Bernoullian input on pages 18–19 seems invalid to me; the author could have established what he wants here very easily by folding the interval $(0, T)$ round the perimeter of a circle of radius $T/2\pi$, and appealing to symmetry. The treatment of the unexpended service time on page 19 is a little cavalier. However, a really critical reading of this chapter agreeably forces one back to first principles, and it can be recommended as a source for the main examples of point-processes in traffic-like applications.

Chapter 3 deals with the infinite-server system; here there can be no "lost calls" and no delays, so that interest is focussed mainly on the occupancy statistics and on the character of the output. The author says something about the artifice of "lost calls held", but his explanation only confused me. The system $M/M/\infty$ (Riordan does not use this shorthand) is described as a birth-and-death process (it is however a death-and-immigration process), and extensive formulae are obtained for it. The elegant result (12) of page 27 for $M/G/\infty$ (due to Beneš and the author), can be neatly explained by the device of decomposing the input (cf. [5]). We may write (in the author's notation)

$$\rho u(t) = a \int_0^\infty \min (s, t) \, dB(s),$$

and then the result (12) becomes obvious if we decompose the incoming Poisson stream into independent component Poisson streams of which a typical member is of intensity $a\, dB(s)$ and consists of those callers having service times within ds of s. (This argument was used in [5] to obtain the known result that the output of $M/G/\infty$ is Poissonian.) The author goes on to obtain similar formulae for other infinite-server queues, now with more general input (and exponential service). The whole of this chapter is exceedingly interesting, and one has the distinct impression that the infinite-server case has been inadequately studied in the past.

Single-server systems are the subject of Chapter 4, which contains a very representative selection of results and methods, both for delay-systems, loss-systems, and combinations of the two. One misses a reference to the valuable work of F. Spitzer here ([10]; see also [8], [9]). The important results of J. F. C. Kingman [6], [7] and others on heavy traffic asymptotics were probably too recent for inclusion in this work.

In Chapter 5 the author proceeds to many-server systems. The general treatment of $GI/G/s$ is dismissed with references to F. Pollaczek, and to J. Kiefer and J. Wolfowitz, but a vast amount of valuable information is given about specific systems not only in the 'delay' but also in the 'loss' case.

Finally in Chapter 6 the author acquaints the reader with the uncomfortable fact that the identification in practice of the stochastic service system one is confronted with may be more difficult and troublesome than one might expect.

Some work by Beneš, by Clarke, and by Kosten is reviewed in detail. Further references which the author does not give are [2], [4]. (The theme of these papers is estimation from simulations, but analogous problems arise in real-life studies.)

One special feature of this book is that, over and over again, the author does not content himself with the remark that "expansion of the generating function will give the moments etc.". A master of combinatorics, expansion procedures, and Pickwick polynomials, he actually carries out the expansion wherever it is at all possible, and so the book is much richer in explicit formulae than is usual. This gives it a characteristic flavour, and ensures it a permanent place in the literature.

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