

CORRECTION NOTE AND ACKNOWLEDGMENT OF PRIORITY

CORRECTION TO "MULTIVARIATE CORRELATION MODELS WITH MIXED DISCRETE AND CONTINUOUS VARIABLES"

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The authors are indebted to C. G. Khatri for pointing out two errors in the above paper (*Ann. Math. Statist.* **32** (1961) 448–465).

On page 456, the definition of τ^2 in Theorem 5.1 should be changed from

$$\tau^2 = \sum_0^k n_m (\mu_m - \sum_0^k p_j \mu_j)^2 / \sigma^2$$

to

$$\tau^2 = \sum_0^k n_m (\mu_m - \sum_0^k (n_j/n) \mu_j)^2 / \sigma^2.$$

On page 461, the complications of Section 6.1 arose from the fact that the $b_{i,j}$ were given as independently and identically distributed, each as $N(0, 1)$. However, the elements of B are not independent, but are correlated so that the necessary distributions are easily obtained. Section 6.1 should be replaced by the following.

6.1 *Remarks on sample canonical correlations.* Define

$$b_{im} = (\bar{y}_i^{(m)} - \bar{y}_i), \quad B = (b_{im}) : p \times (k + 1),$$

$$D = \text{diag}(n_0/n, \dots, n_k/n), \quad D_p = \text{diag}(p_0, \dots, p_k),$$

$$s_{ij} = \sum_{m=0}^k \sum_{\lambda=1}^{n_m} (y_{i\lambda}^{(m)} - \bar{y}_i^{(m)})(y_{j\lambda}^{(m)} - \bar{y}_j^{(m)}),$$

$$S = (s_{ij}) : p \times p, \quad h_{ij} = \sum_{m=0}^k n_m b_{im} b_{jm} / n, \quad H = (h_{ij}) : p \times p.$$

Then $H = BDB'$ is an estimate of UD_pU' , and S/n is an estimate of Σ , so that we are interested in the joint distribution of the roots of $|nH - \theta S| = 0$. Because of the invariance of the roots under the transformation $H \rightarrow \Sigma^{-\frac{1}{2}} H \Sigma^{-\frac{1}{2}}$, $S \rightarrow \Sigma^{-\frac{1}{2}} S \Sigma^{-\frac{1}{2}}$, we can assume that $\Sigma = I$.

When the canonical correlations are zero, i.e., $\mu^{(0)} = \dots = \mu^{(k)}$, with n_0, \dots, n_k fixed, the vectors $n^{\frac{1}{2}}(b_{j0}, \dots, b_{jk})D^{\frac{1}{2}}$ for $j = 1, \dots, p$ are independently and identically distributed, each as $N(0, I - a'a)$, where $a = n^{-\frac{1}{2}}(n_0^{\frac{1}{2}}, \dots, n_k^{\frac{1}{2}})$. Since $I - a'a$ is an idempotent matrix of rank k , by the usual reduction we have that $nBDB'$ can be written as WW' , where $W : p \times k$, and the elements of W are independently and identically distributed as $N(0, 1)$. With $\Sigma = I$, the distribution of S is $W(I, p, n - k - 1)$. Consequently, the problem now is to find the joint distribution of the roots of $|WW' - \theta S| = 0$. The two cases $p \leq k$ and $p > k$ have to be distinguished, but in either case the distribution

of the roots is known. When the $\mu^{(i)}$ are not equal, $EW = \Omega$, and for fixed n_0, \dots, n_k , we require the distribution of the roots of $|WW' - \theta S| = 0$. This is the non-central case; for a resumé of recent results see A. T. James (Distributions of matrix variates and latent roots derived from normal samples, *Ann. Math. Statist.* **35** (1964) 475–501).

The material of this section may also be obtained by an application of C. G. Khatri's "A note on the interval estimation related to the regression matrix," *Ann. Inst. Stat. Math.*, **13** (1961) 145–146.

In our paper we introduced a vector correlation coefficient. We take this opportunity to point out that several other possible extensions of vector correlation coefficients and correlation ratios have recently been considered by R. F. Tate in a paper, "Conditional normal regression models", which has been submitted for publication.

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Regularity Condition D' of the author's paper "A New Proof of the Pearson-Fisher Theorem", *Ann. Math. Statist.* **35** (1964) 817–824, is not new as stated but has been shown to be a sufficient condition by C. R. Rao, "A Study of Large Sample Test Criteria Through Properties of Efficient Estimates", *Sankhyā* **23** (1961) 25–40. The author regrets having overlooked this prior contribution of Professor Rao.