BOOK REVIEWS

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C. R. RAO, *Linear Statistical Inference and Its Applications*, John Wiley and Sons, Inc., New York, 1965. $14.95. xviii +522 pp.

Review by FRANKLIN A. GRAYBILL

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In the preface the author has stated that his purpose in writing this book is "... to present up-to-date theory and techniques of statistical inference in a logically integrated and practical form." In addition he has stated "Essentially, it incorporates the important developments in the subject that have taken place in the last three decades. It is written for readers with a background knowledge of mathematics and statistics at the undergraduate level."

The book is divided into eight chapters and consists of 522 pages. It has an author index in addition to a subject index. At the end of each chapter there is a set of references and a large number of problems. Some illustrative examples are included. Below is the title and a brief outline of each chapter.

CHAPTER 1. *Algebra of Vectors and Matrices.* In this chapter vector spaces and subspaces are defined; matrices and determinants and solutions of sets of linear equations are discussed, as well as techniques for reducing matrices to triangular form, echelon form, diagonal form, etc. Transformations, projections and generalized inverses are discussed. Some of the pertinent theorems on quadratic forms which include characteristic roots and canonical matrices are discussed. Also included are: some theorems and discussion of convex sets in vector spaces; inequalities based on vectors, matrices, and determinants; extrema of quadratic forms. This chapter has a very good set of problems. Chapter 1 contains a great deal of both necessary and useful material for anyone who intends to study statistics at a graduate level. However, the material is supplemental in nature and one should certainly have had a least a one-semester course in matrix algebra in order for this chapter to be a significant help. The chapter is very valuable in that it contains many of the theorems that the reader will find useful in the remainder of this book. It, however, does more; the chapter contains many theorems, not necessary for the remainder of the book, that the reader will find useful as a reference. A student who plans to do graduate study in statistics could well refer to the theorems and the problems in this chapter at the time he takes his first course in undergraduate matrix or linear algebra. The proofs are too skimpy for someone who has not had such a formal course.

CHAPTER 2. *Probability Theory, Tools and Techniques.* This chapter includes

the topics: calculus of probabilities, mathematical expectations and moments of random variables, limit theorems, family of probability measures and problems of statistics. This chapter contains the following material in the appendix: Stieltjes and Lebesgue integrals and some important theorems in measure theory. The mathematical level of this chapter is slightly above the level of the book *Mathematical Methods of Statistics* by H. Cramér, but of course the results are more modern and certainly the material is more condensed. The contents of this chapter are essentially the present-day calculus of probability a la Kolmogorov. Much of the material in Chapter 2 seems not to be necessary for the remainder of the book. For example the author includes the Radon-Nikodym theorem, the Borel-Cantelli lemma, and some other material that appears not to be integrated into the remainder of the book. However, it seems that for much of the material in this chapter, especially in the appendices, it was not intended to discuss what its importance is in the mainstream of statistical theory and application. Some of the material appears to be out of context relative to the remainder of the book.

CHAPTER 3. *Continuous Probability Models.* This chapter contains the conventional material on distributions of continuous random variables including the normal, gamma, beta, Cauchy, t, and F. The following material is also included: distributions of linear and quadratic functions of normal random variables; symmetric normal random variables, and the bivariate normal distribution. In Chapter 3 the author discusses the theoretical derivation of a number of the basic distributions, which includes the normal, the gamma, the Cauchy, the circular (spherical) normal and others. A good set of problems is included in this chapter.

CHAPTER 4. *The Theory of Least Squares and Analysis of Variance.* In this chapter the author has derived much of the theory of the linear model. In the first section he has discussed point estimation via least squares. He derives linear, unbiased, minimum variance estimators of the parameters that are linear functions of the expected value of the observations. The situation where these parameters are subject to known linear restrictions is also discussed. Confidence intervals and tests of hypotheses on the linear model are also discussed. Then the author considers special cases such as: one-way analysis of variance models; two-way models with single, multiple (but equal), and multiple (not equal) number of observations in the cells; non-additivity tests; variance components; regression and other models. Chapter 4 is one of the strongest in the book and, of course, the author has contributed generously to the statistical literature in the area of the linear model. Matrices are used extensively and a good set of problems is included, as well as illustrative examples.

CHAPTER 5. *Criteria and Methods of Estimation.* This chapter contains a number of results on minimum variance unbiased estimation which includes theorems on sufficient statistics, completeness, Rao-Cramér inequality, Fisher's information, and some additional topics. The author briefly discusses Bayes estimators, empirical Bayes procedures, fiducial probability, and the minimax principle.

He then turns to a discussion of criteria for estimation in large samples and methods of estimation which include the method of moments, minimum chi-square, and maximum likelihood. A section on the estimation of the multinomial distribution, the estimation of parameters in the general case, and the method of scoring is included. This chapter contains a good set of problems.

CHAPTER 6. *Large Sample Theory and Methods.* In this chapter the author discusses chi-square goodness of fit, contingency tables, order statistics, transformations, standard errors of moments, and related topics. A number of illustrative examples and a small set of problems are contained in this chapter.

CHAPTER 7. *Theory of Statistical Inference.* In this chapter the author has included testing hypotheses (both simple and composite), confidence intervals and the relation between the two. He also has discussed briefly such topics as the Fisher-Behrens problem; robustness, sequential analysis, distribution free procedures, Stein's two-step procedure, and the principle of randomization. A set of problems has been included.

CHAPTER 8. *Multivariate Analysis.* Some of the topics included in this chapter are the following: some characterizations of the multivariate normal distribution, the Wishart distribution, analysis of dispersion, test for dimensionality and discriminatory analysis. In addition the author discusses many of the conventional problems concerned with tests and estimation. The author develops multivariate normal theory from the point of view of linear combination of univariate random variables. This chapter contains a good set of problems.

This book has a very large number of excellent qualities. There are a number of topics that are to be found in no other single textbook. I believe that every graduate student in statistics would do well to read portions of this book, particularly Chapters 4, 5, 6 and 8. I am not certain why the author included the word "linear" in the title since the book covers much more general inference than what is generally considered to be "linear."

My main criticism of the book is the inconsistency in level of presentation. This is particularly true if the book is intended for graduate students in colleges and universities in the United States. For example on page 2 the author defines the addition of vectors and on page 113 he states the Radon-Nikodym theorem. He undoubtedly presented material such as this for completeness and for reference later on, but on the other hand some theorems such as above have been stated without proof and never referred to again.

I believe that many students will encounter difficulty understanding the proofs of some of the theorems (this may be valuable), and I submit that many instructors will spend considerable time in supplying the details of a number of proofs (this may be valuable also).

This book appears to be similar (at least some sections) in style and spirit to the author's book *Advanced Statistical Methods in Biometric Research*, although I could find no reference to this book in the list of References.

Some readers will be disappointed because there is no development from a decision-theoretic point of view, some will be disappointed because there is no

general development of Bayesian procedures, and some will be disappointed because proofs are not tighter.

I found a number of errors in the book, some of which appeared to be typographical and some not. This, of course, is common to most books at this level and with the amount of notation present. Most of these can be corrected in the second printing.

This book contains a large amount of very pertinent material for statisticians and for those training to become statisticians, and I predict that it will be found on many of their bookshelves.