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J. G. KEMENY, L. J. SNELL, AND A. W. KNAPP. *Denumerable Markov Chains*. Van Nostrand, New York, 1967. xi + 439 pp. $12.50.

Review by STEVEN OREY

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This book introduces the reader to Markov chains with stationary transition probabilities, denumerable state space, and discrete time. The emphasis is on the potential theory of these chains, to which the last five of the eleven chapters in the book are devoted.

No knowledge of the subject is presupposed. The first three chapters provide the necessary background in measure theory and probability. Chapters four through six provide the basic facts about Markov chains, including a discussion of duality, cycles, and convergence theorems.

Motivation and historical background for the study of the potential theory of Markov chains is given in Chapter seven. This includes a brief discussion of some of the inter-relations between classical potential theory and Brownian motion. The discussion of the potential theory of Markov chains is divided into two cases, transient and recurrent.

Transient potential theory is discussed in Chapter eight. This is a specialization of the theory which is now well developed for general Markov processes. The Martin boundary is treated in Chapter 10. It is welcome to have a leisurely discussion of these results in the text-book literature.

Recurrent potential theory is given in Chapter nine. In this case, unlike the transient one, there is no obvious potential kernel and one must first be constructed and shown to have desirable properties. For special cases such constructions are classical or have recently been systematically studied. The authors succeed in developing a theory for a reasonably wide class of recurrent chains, the so called *normal* chains. These include ergodic chains and recurrent random walk. Chapter 11 introduces recurrent boundary theory. The results of this chapter are mostly taken from the work of the two senior authors.