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ON A CERTAIN RECURRENCE RELATION

BY J. C. AHUJA

Portland State University

1. Introduction. Let $x_1, x_2, \cdots, x_n$ be a random sample of n observations drawn from the truncated Poisson distribution

$$f(x;\theta) = \alpha\theta^x/x!, \qquad x \in T \tag{1}$$

where $\alpha = 1/[\exp(\theta)-1]$, $0 < \theta < \infty$, and T is the set of positive integers. The minimum variance unbiased estimator $g(t, n)$ for the parameter θ in (1), based on a sample size n and sample total t, has been obtained by Roy and Mitra [2] in terms of the numbers $\Delta^n O^t$ where Δ is the forward difference operator, while Tate and Goen [3] have obtained $g(t, n)$ in the form

$$\begin{aligned} g(t,n) &= tC(t,n)/n && \text{for} \quad t > n \\ &= 0 && \text{for} \quad t = n \end{aligned} \tag{2}$$

where $C(t, n) = 1-[S(t-1, n-1)/S(t, n)]$, and $S(t, n)$ is the Stirling number of the second kind with arguments t and n as defined by Riordan ([1] page 33). It may be pointed out that the numbers $S(t, n)$ are related to the numbers $\Delta^n O^t$ by means of $n!\,S(t, n) = \Delta^n O^t$.

The minimum variance unbiased estimator $g(t, n)$ defined by (2) has been tabulated to three decimal places by Roy and Mitra [2] for $n = 2(1)10$ and $t = 2(1)96$, whereas Tate and Goen [3] have furnished a table, correct to five decimal places, of the function $C(t, n)$ for $n = 2(1)49$ and $t = 3(1)50$, using Miksa's unpublished table of Stirling numbers of the second kind. Tate and Goen point out (see [3] page 760) that an extended table of $C(t, n)$ would be quite useful but they have been unable to devise a method for computing $C(t, n)$ which does not depend on entries in the table of Stirling numbers of the second kind.

The purpose of this note is to provide a recurrence relation for $C(t, n)$ which does not involve Stirling numbers of the second kind and which would be quite useful for either extending the existing table of $C(t, n)$ or tabulating the function $g(t, n)$ itself.

2. Recurrence relation for $C(t, n)$. The definition of $C(t, n)$ in (2) gives us

$$C(t+1, n) = 1-[S(t, n-1)/S(t+1, n)]. \tag{3}$$

Using the recurrence formula for Stirling numbers of the second kind available in Riordan ([1] page 33) as

$$S(t+1, n) = S(t, n-1)+nS(t, n) \tag{4}$$

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we find, after some algebraic manipulations, that (3) may be written in the form

$$C(t+1,n) = nC(t,n-1)/nC(t,n-1)-(n-1)C(t,n)+(n-1) \tag{5}$$

where $C(t, n) = 0$ for $t = n \geqq 1$, and $C(t, 1) = 1$ for $t \geqq 2$.

Thus (5) provides the desired recurrence relation for $C(t, n)$. It may be remarked that (2) and (5) enable us to write down a recurrence relation for $g(t, n)$ also.

REFERENCES

[1] RIORDAN, JOHN (1958). *An Introduction to Combinatorial Analysis.* Wiley, New York.

[2] ROY, J. and MITRA, S. K. (1957). Unbiased minimum variance estimation in a class of discrete distributions. *Sankhyā* **18** 371–378.

[3] TATE, R. F. and GOEN, R. L. (1958). Minimum variance unbiased estimation for the truncated Poisson distribution. *Ann. Math. Statist.* **29** 755–765.