The Annals of Mathematical Statistics
1970, Vol. 41, No. 6, iv–xiii

WILLIAM FELLER, 1906–1970[1]

William Feller died on January 14, 1970, at the age of 63, at Memorial Hospital in New York. He was a member of the National Academy of Sciences, of the American Academy of Arts and Sciences, of the Danish and Yugoslavian Academies of Sciences, fellow of the Royal Statistical Society, past governor of the Mathematical Association of America, former president of the Institute of Mathematical Statistics (1946). A few days before his death he had learned also of his election as honorary member of the London Mathematical Society and of the decision to award him the National Medal of Science, which his wife Clara was to receive in his stead on February 16 at the White House. These outward and visible honors confirm his position in science, to which is added our affection for his gaiety, enthusiasm, gentleness, and responsiveness.

Will Feller was born in Zagreb, Yugoslavia, on July 7, 1906; he attended the University there from 1923 to 1925, leaving with a degree equivalent to our Master of Science. From 1925 to 1928 he worked at the University of Göttingen, where he received the Ph.D. in 1926, at the age of twenty. At Göttingen he had the good fortune to become acquainted with David Hilbert, always his ideal mathematician, as well as with Richard Courant, who recognized his promise and encouraged him to become a mathematician in earnest. In 1928 he went as Privatdozent to the University of Kiel, but left there in 1933 after refusing to sign a Nazi oath. He passed a year in Copenhagen, where he came to know Harold Bohr and his brother Niels, and then five years (1934–1939) at the University of Stockholm, in the vicinity of Marcel Riesz and Harald Cramér. It was during his last year there, on July 27, 1938, that he married Clara Nielsen, who had been his student at Kiel.

In 1939 Will and Clara moved to Providence, Rhode Island, where Will became associate professor at Brown University as well as the first executive editor of *Mathematical Reviews*; he deserves the gratitude of mathematicians for his six years of effort establishing the new journal, now the leading review of mathematics in the world. In 1944, toward the end of the period at Brown, Will became a citizen of the United States (District Court, Providence). The following year he accepted a professorship at Cornell University. Finally, in 1950, he came to Princeton as Eugene Higgins Professor of Mathematics, a position which he held until his death. In 1966 he was appointed also Permanent Visiting Professor at Rockefeller University where, during two years of leave from Princeton in 1965–66 and 1967–68, he found himself serving in part as liaison between geneticists and mathematicians.

Feller's work had great variety and scope, for he contributed to calculus, geometry and functional analysis. But about half of his papers lie in the field of probability. Only these, his best known papers, are discussed here.

[1] Editor's note. This article, prepared by the Editors, is a combination of the memorial resolution by the Faculty of Princeton University and the documents supporting Feller's nomination for the National Medal of Science.

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WILLIAM FELLER

The most notable of Feller's papers on probability before 1950 dealt with the classical limit theorems of probability, which concern the asymptotic behavior of the sums $S_n = X_1 + \cdots + X_n$ of a sequence $X_1, X_2, \cdots$ of independent random variables. One supposed the sequence $S_1, S_2, \cdots$ to be essentially divergent, and in the classical theory one assumes the mean value $m_n = E(S_n)$ and the variance $v_n = s_n^2 = E(S_n^2) - m_n^2$ to exist. In 1930, when Feller was 24 years old, the state of development was this:

Kolmogorov had just given necessary and sufficient conditions for the *law of large numbers* in the classical version, that is to say, conditions on the X_n to ensure that, for every $\epsilon > 0$,

$$(1) \qquad \Pr\{|S_n - m_n| < n\epsilon\} \to 1 \quad \text{as} \quad n \to \infty,$$

or more intuitively that for large n the average $(X_1 + \cdots + X_n)/n$ is pretty sure to be close to its mean value m_n/n.

The *central limit theorem*, whose history goes back to De Moivre and Laplace, seemed to be near its final form. Indeed, the existence of the mean and variance being granted, rather weak additional conditions were known ensuring that the reduced variable $(S_n - m_n)/s_n$ have a distribution close to the unit normal.

The foregoing investigations treated only the distribution measures of the sums. Much more delicate is the study of the asymptotic behavior of the sequence $S_1, S_2, \cdots$ itself, first properly formulated in 1909 by Emil Borel. After notable advances by Hausdorff, Hardy and Littlewood, and Khinchin, Kolmogorov arrived in 1929 at what seemed the last work, the famous *law of the iterated logarithm*, which says that almost certainly

$$(2) \qquad \limsup_{n\to\infty} \frac{S_n - m_n}{\sqrt{2v_n \log \log v_n}} = 1,$$

provided the X_n satisfy rather strong conditions of boundedness.

Other problems concerning the sums S_n had not received such elaboration. In particular, problems of renewal theory, so important in biology, population research or engineering, had been solved in many particular instances but had not yet been brought to the attention of mathematicians capable of establishing the general theory. (In this context, X_n is the lifetime of the nth replacement; all X_n have the same distribution; the quantity of interest is the age at time t of the replacement then in use, that is to say, $Y(t) = t - S_n$, where n is determined by the inequalities $S_n < t \leq S_{n+1}$, and one seeks conditions for the distribution of $Y(t)$ to converge as t increases.)

Let us follow the development of these topics after 1930.

In 1935 both Feller [12] and Paul Lévy, quite separately, showed the mean m_n and variance s_n^2 to be altogether foreign to the proper statement of the central limit theorem, then proceeded to give simple conditions necessary and sufficient for the distribution of S_n to be nearly normal. Feller was not content with this 'best possible' result; he returned to the problem time and again to simplify the proof, to calculate explicit estimates [32], [94], to study particular examples [38].

In 1937 Feller [16] similarly pointed out the lack of naturality of the mean

m_n and the scaling factor n in the formulation (1) of the law of large numbers, then established the definitive version.

The law of the iterated logarithm had a startling continuation. First Lévy gave new insight into the situation by noting that any sequence (a_n) of constants must be either a 'lower sequence' for the S_n or else an 'upper sequence'; it is a lower (or upper) sequence if almost certainly the inequality $S_n < a_n$ (or $S_n > a_n$) holds for only finitely many values of n. The discovery initiated the problem of finding a criterion for upper and lower sequences, which was accomplished by Kolmogorov for the simplest choices of the X_n. (His proof was never published, but one can be found hidden in a paper by Petrovsky, or explicit in a paper of 1942 by Erdös.) Finally, in 1943, Feller [33] established the generalization of Kolmogorov's criterion to arbitrary random variables, under mild and reasonable conditions of regularity. The paper remains certainly one of the most intense, profound arguments in mathematics. Although difficult to read, because of the complexity of the reasoning, it has still served as the model in searching for similarly complete answers to such questions as the growth of $M_n = \max(S_1, \cdots, S_n)$ or the number of changes of sign of the S_n. In recent lectures [103] Feller has recast the argument and one must admit, in tribute to his exposition, that the new version not only displays the author as the one best understanding his own work but also provides the key to using his technique. He has also simplified the proof of Kolmogorov's law (2) in order to make it accessible to the beginning student.

Characteristically, Feller has investigated what can be said when the restrictions imposed in [33] are not assumed; for example, [40] and the recent paper [96] both contain unexpected results.

The development of renewal theory illustrates both Feller's power as a mathematician and his interest in the applications of probability theory. His paper [27] presented what was known in 1940, furnishing proofs, unifying the theory, bringing the problems to the attention of mathematicians. Exact results were known in several examples, asymptotic expansions had been found under stringent assumptions, but the ultimate simplicity and beauty of the subject were still concealed. The main problem turned out to be, in the notation introduced above, the proof that $E(Y(t))$ approached $E(X_1)$, a problem that can be phrased in integral equations or power series without mention of probability; it was the decided opinion of most experts that some supplementary hypothesis (beyond a certain obvious one) was needed to ensure convergence. However, in the paper [47] of 1949, Erdös, Feller and Pollard succeeded in showing convergence to hold universally, a result of extreme importance in theoretical applications, as shown by the paper [49] on the theory of recurrent events. Feller had never tired of the subject; for example, among his papers of 1961, [85] gives a simple proof based on an idea of Hunt and Choquet and Deny, while [84] introduces new methods which yield a generalization of the standard theorem.

Although Feller continued his contributions to classical problems, the years 1950 to 1962 saw him engaging all his effort in work of another spirit—the creation of a theory of diffusion which combines functional analysis, differential equations and probability. There was plenty of work for all: During this period twelve students wrote their theses under Feller, most of them on topics related to diffusion. Mathematicians outside Princeton also took part in the development, so that today diffusion in one dimension is perhaps the most thoroughly investigated of stochastic phenomena.

A few historical remarks are needed to place this accomplishment in the right setting.

Kolmogorov, in a remarkable paper of 1931, showed that a Markov process satisfies certain integro-differential equations, the best known instance being Brownian motion and the heat equation (an early result of Einstein). Consequently there arose, for equations of a certain type, the questions of the existence and the uniqueness of a corresponding Markov process, treated in detail by Feller in [14] and [26]. The exact relations remained unclear, as Feller himself often pointed out in attempts to engage other mathematicians in fruitful research. In addition, the study of other fields, especially genetics, had acquainted him with a number of situations concerning differential operators where it was important to classify the totality of corresponding processes.

Thus Feller was led to study, from a new point of view, the parabolic equation $\partial u/\partial t = Lu$, with L a second-order differential operator on a linear interval. It turned out that an associated Markov process corresponded to a 'boundary condition', but one which might differ markedly from those familiar in the theory of differential equations. The theory of semigroups of operators entered as an essential tool in formulating and solving the analytic problems; the idea was to view a boundary condition as a restriction on the domain of L which made the restricted operator the infinitesimal generator of an appropriate semigroup. The papers [53], [57], [58], [59] settled the main problems, [62], [63] and much later [63] dealt with related questions about abstract semigroups of operators, and [64] presented the interpretation of the various boundary conditions in terms of the behavior of the paths of the Markov processes. Feller's next step was to abstract and generalize the notion of differential operator appearing in diffusion theory, and especially to avoid reference to an irrelevant differentiable structure. He found that every such operator (provided certain degeneracies are excluded) can be written, essentially uniquely, in the form $(d/d\mu)(d/dx)$, where x is a coordinate function (the scale parameter) and μ is an increasing function (the speed measure), both intrinsically associated to the operator; these papers [65], [66], [67], [72], [75], [77], [79] made the study of the most general linear diffusion hardly more difficult than the study of Brownian motion.

Feller himself was the first to apply the ideas to other situations. In [81] he showed how simply one can treat the birth-and-death process by introducing the right parameters. In [71], [73], returning to the Kolmogorov equations for discontinuous processes, he initiated the general theory of boundaries for Markov processes, a subject which has had a tremendous growth in the last decade. In two recent papers [89], [91], the outcome of discussions with Th. Dobzhansky at the Rockefeller Institute, he dealt with the 'Haldane paradox': evolution by natural selection involves as 'cost' a number of 'genetic deaths', often so great that evolutionary change can occur only very slowly. Feller succeeded in showing that the Haldane paradox is spurious; rather amusingly, as early as 1952, in the address [57], he had already analyzed the assumptions implicit in the current mathematical theory of evolution and had warned against the one of constant population size, which was later to lead to the Haldane paradox.

Feller's activity increased with the years, and he never cast aside a problem once begun, even when the solution appeared complete. Many of the twelve papers written or published in his last two years treat interests going back to the thirties. Among them is the paper of which he was proudest, *General analogues of the law of the iterated logarithm* [103]; surpassing the great paper [33] of 1943 in simplicity and generality, it is the outcome of thirty years of struggle to clarify his understanding.

His own research will of course remain Feller's principal contribution to mathematics, but he has helped the progress of science and humanity in many other ways. He always seemed to have time for discussion with students or colleagues—indeed, with almost anyone and on any subject. His students, especially, recall the patience with which he guided them and the endless hours he spent listening to their conjectures or voicing his own. His expository lectures and articles have done a great deal to spread the knowledge and recognition of probability throughout the world. His books catch the flavor of his mathematics, though only his presence could convey the full enthusiasm of his lectures. The jacket cover of the third edition of Volume I bears this appreciation from Gian-Carlo Rota of MIT: '. . . one of the great events in mathematics of this century. Together with Weber's *Algebra* and Artin's *Geometric Algebra* this is the finest text book in mathematics in this century. It is a delight to read and it will be immensely useful to scientists in all fields.' The book achieves a remarkable popular appeal with no sacrifice of mathematical rigor, and is good evidence of Feller's contention that accurate theory has great practical value and interest.

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