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A NOTE ON THE COMPLETE CONVERGENCE OF STABLE DISTRIBUTION FUNCTIONS[1]

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It is shown that if F_n is a stable distribution function for each value of n and if F_n converges completely to F, then F is a stable distribution function.

1. Introduction. A discussion of some of the properties of stable distribution functions, distribution functions of class L, and infinitely divisible distribution functions may be found in [2]. The class of stable distribution functions is properly contained in the class of L distribution functions, and the class of L distribution functions is properly contained in the class of infinitely divisible distribution functions.

It is well known that if F_n is an infinitely divisible distribution function for each value of n and if $F_n \overset{c}{\to} F$, then F is an infinitely divisible distribution function. Kubik has shown ([3] page 248) that if F_n is a distribution function of class L for each value of n and if $F_n \overset{c}{\to} F$, then F is a distribution function of class L. The purpose of this note is to give an elementary proof of the following

THEOREM. *If F_n is a stable distribution function for each value of n and if $F_n \overset{c}{\to} F$, then F is a stable distribution function.*

An erroneous counter-example of this theorem was given by Kubik in ([4] page 402).

2. Proof of Theorem. Assume that F_n is a stable distribution function for each value of n and that $F_n \overset{c}{\to} F$. Then F_n is a distribution function of class L for each value of n, and thus F is a distribution function of class L. It will be assumed that F is non-degenerate as the proof is trivial otherwise. Since every non-degenerate distribution function of class L is absolutely continuous (see [1] page 338), $F_n(x) \to F(x)$ for all values of x.

Let $a > 0$, b, $\alpha > 0$, and β be constants. By the definition of stable distribution function, for each value of n there exists constants $A_n > 0$ and B_n such that

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$$F_n(ax + b)^*F_n(\alpha x + \beta) = F_n(A_n x + B_n) .$$

It follows from this that for all values of x

$$F_n(A_n x + B_n) \to F(ax + b)^*F(\alpha x + \beta) .$$

By a theorem of Khintchine ([2] Theorem 1, page 40) there exist constants $A > 0$ and B such that

$$F(Ax + B) = F(ax + b)^*F(\alpha x + \beta) .$$

Thus, F is a stable distribution function.

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