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RANDOM ISHIKAWA ITERATIVE SEQUENCE WITH ERRORS FOR APPROXIMATING RANDOM FIXED POINTS

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Abstract. The purpose of this paper is to construct a random Ishikawa iterative sequence with errors for random strongly pseudo-contractive operator T in separable Banach spaces and to study that under suitable conditions this random iterative sequence with errors converges to a random fixed point of T. As applications, we utilize our results to study the existence problems of solutions for some kinds of nonlinear random operator equations in uniformly smooth separable Banach spaces.

1. Introduction and Preliminaries

Recently many authors have solved the nonlinear operator equations, general variational inequalities and multi-valued variational inclusions by using Ishikawa iterative sequence [12] and Mann iterative sequence [15]. And, concerning the stability and the convergence problems of Ishikawa, Mann, Liu and Xu iteration process for single-valued and set-valued accretive and pseudo-contractive mapping have been studied extensively by many authors for approximating the fixed points of some nonlinear mappings and for approximating solutions of some nonlinear operator equations in Banach spaces (see, for example, [1, 4, 9-11, 23]).

On the other hand, after the publication of the review article by Bharucha-Reid [3], random fixed point theory has been studied extensively by many authors (see, for example, [2, 6, 7, 13, 14, 16-20, 22]).

In 1993, Chang and Huang [5] studied the random iterative sequences for finding random solutions of the random complementarity problems in separable Hilbert

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spaces. In recent years, Choudhury [6, 7] has suggested and analyzed random Mann iterative sequence in separable Hilbert spaces for finding random solutions and random fixed points for some kind of random equations and random operators.

The purpose of this paper is to introduce and construct a random Ishikawa iterative sequence with errors for some kinds of random strictly pseudo-contractive mappings in uniformly smooth separable Banach spaces and to study the approximation problem of random fixed point by using this kind of iterative sequence with errors. As applications, we utilize our results to study the existence problems of solutions for some kinds of nonlinear random operator equations in uniformly smooth separable Banach spaces.

Throughout this paper, we assume that (Ω, Σ) is a measurable space, E is a real uniformly smooth separable Banach space, E^* is the dual space of E, C is nonempty closed convex subset of E, and $J : E \to 2^{E^*}$ is the *normalized duality mapping* defined by

$$J(x) = \{f \in E^* : \langle x, f\rangle = \|x\| \cdot \|f\|, \, \|f\| = \|x\|\}, \quad x \in E.$$

A function $f : \Omega \to C$ is said to be *measurable* if $f^{-1}(B \cap C) \in \Sigma$ for each Borel subset B of E. A function $T : \Omega \times C \to C$ is called a *random operator* if, $T(\cdot, x) : \Omega \to C$ is measurable for every $x \in C$. A measurable function $f : \Omega \to C$ is called a *random fixed point* of the random operator $T : \Omega \times C \to C$ if $T(\omega, f(\omega)) = f(\omega)$ for all $\omega \in \Omega$. A random operator $T : \Omega \times C \to C$ is said to be *continuous* if, for any given $\omega \in \Omega$, $T(\omega, \cdot) : C \to C$ is continuous.

Lemma 1.1. [20] *Let E be a separable metric space and Y be a metric space. Let $T : \Omega \times E \to Y$ be measurable in $\omega \in \Omega$ and continuous in $x \in E$. If $g : \Omega \to E$ is a measurable function, then $f(\cdot, g(\cdot)) : \Omega \to Y$ is measurable.*

Definition 1.1. Let $T : \Omega \times C \to C$ be a continuous random operator, $x_0 : \Omega \to C$ be any given measurable function and $u_n, v_n : \Omega \to C$ be bounded measurable functions for each $n \geq 0$. Then the sequence $\{x_n\}$ with errors define by

$$\begin{aligned} x_{n+1}(\omega) &= (1 - \alpha_n - \gamma_n)x_n(\omega) + \alpha_n T(\omega, y_n(\omega)) \\ &\qquad + \gamma_n u_n(\omega), \quad \omega \in \Omega, \, n \geq 0, \\ y_n(\omega) &= (1 - \beta_n - \delta_n)x_n(\omega) \\ &\qquad + \beta_n T(\omega, x_n(\omega)) + \delta_n v_n(\omega), \quad \omega \in \Omega, \quad n \geq 0, \end{aligned} \tag{1.1}$$

is called the *random Ishikawa iterative sequence with errors*, where $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$ and $\{\delta_n\}$ are real sequences in $[0, 1]$.

If $\gamma_n = \delta_n = 0$ for all $n \geq 0$, then we have:

$$\begin{aligned} x_{n+1}(\omega) &= (1-\alpha_n)x_n(\omega) + \alpha_n T(\omega, y_n(\omega)), \quad \omega \in \Omega, \quad n \geq 0, \\ y_n(\omega) &= (1-\beta_n)x_n(\omega) + \beta_n T(\omega, x_n(\omega)), \quad \omega \in \Omega, \quad n \geq 0, \end{aligned} \tag{1.2}$$

is called the *random Ishikawa iterative sequence*, where $\{\alpha_n\}$ and $\{\beta_n\}$ are two real sequences in $[0, 1]$.

Remark 1.1. It should be pointed out that since $T : \Omega \times C \to C$ is a continuous random operator, C is a closed convex subset of E and $x_0 : \Omega \to C$ is a measurable function, and $u_n, v_n : \Omega \to C$ are measurable functions for each $n \geq 0$, by Lemma 1.1, it is easy to see that $\{x_n\}$ and $\{y_n\}$ are sequences of measurable functions from Ω into C.

Definition 1.2. (1) A random operator $T : \Omega \times E \to E$ is said to be *strongly pseudo-contractive* if there exists a function $k : \Omega \to (0, 1)$ such that, for any $x, y \in C$, there exists $j(x - y) \in J(x - y)$ such that

$$\langle T(\omega, x) - T(\omega, y), j(x - y)\rangle \leq k(\omega)\|x - y\|^2, \quad \omega \in \Omega. \tag{1.3}$$

(2) $T : \Omega \times E \to E$ is said to be *strongly accretive* if there exists a function $h : \Omega \to (0, 1)$ such that, for any $x, y \in E$, there exists $j(x - y) \in J(x - y)$ such that

$$\langle T(\omega, x) - T(\omega, y), j(x - y)\rangle \geq h(\omega)\|x - y\|^2, \quad \omega \in \Omega. \tag{1.4}$$

From the definition, it is easy to see that $T : \Omega \times E \to E$ is strongly pseudo-contractive with function $k : \Omega \to (0, 1)$, if and only if $I - T$ is strongly accretive with a function $h : \Omega \to (0, 1)$ defined by $h(\omega) = 1 - k(\omega)$.

The following lemmas will be needed in proving our main results.

Lemma 1.2. *Let E be a real Banach space and $J : E \to 2^{E^*}$ be the normalized duality mapping. Then, for any $x, y \in E$, we have*

$$\|x + y\|^2 \leq \|x\|^2 + 2\langle y, j(x + y)\rangle$$

for all $j(x + y) \in J(x + y)$.

Lemma 1.3. *E is a uniformly smooth Banach space if and only if the normalized duality mapping $J : E \to E^*$ is single-valued and uniformly continuous on any bounded subset of E.*

Lemma 1.4. *Let $\{a_n\}$ and $\{b_n\}$ be two nonnegative real sequences satisfying the following condition*:

$$a_{n+1} \leq (1-\lambda_n)a_n + b_n, \quad n \geq n_0,$$

where n_0 is some nonnegative integer, $\lambda_n \in [0,1]$ with $\sum_{n=0}^{\infty} \lambda_n = \infty$ and $b_n = o(\lambda_n)$. Then $\lim_{n\to\infty} a_n = 0$.

2. The Main Results

Now, we give our main results in this paper.

Theorem 2.1. *Let E be a real uniformly smooth separable Banach space, C be a nonempty closed convex subset of E, $T : \Omega \times C \to C$ be a continuous random strongly pseudo-contractive operator with a function $k : \Omega \to (0,1)$ and the range $R(T)$ of T be bounded. Let $u_n, v_n : \Omega \to C$ be bounded measurable functions for each $n \geq 0$. Let $x_0 \in C$ be a given point and $\{x_n(\cdot)\}$ be the random Ishikawa iterative sequence with errors defined by* (1.1). *If the sequences $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}$ and $\{\delta_n\} \subset [0,1]$ appeared in* (1.1) *satisfy the following conditions*:

(*i*) $\alpha_n \to 0$, $\beta_n \to 0$, $\delta_n \to 0$ *as* $n \to \infty$,

(*ii*) $\sum_{n=0}^{\infty} \alpha_n = \infty$,

(*iii*) $\gamma_n = o(\alpha_n)$,

then the sequence $\{x_n\}$ converges strongly to a random fixed point of T.

Proof. First, we point out that, for given $x_0 \in C$, obviously, it is a constant-valued measurable function from Ω to C. By Remark 1.1, we know that $\{x_n\}$ and $\{y_n\}$ are sequences of measurable functions from Ω into C. Since $T : \Omega \times C \to C$ is a continuous random strongly pseudo-contractive operator with a function $k(\omega) : \Omega \to (0,1)$, for any given $\omega \in \Omega$, $T(\omega,\cdot) : C \to C$ is a strongly pseudo-contractive operator with a number $k(\omega) \in (0,1)$. By the well-known result (see, for example, [4]), $T(\omega,\cdot) : C \to C$ has a unique fixed point, that is, there exists a unique point $p(\omega) \in C$ such that

$$p(\omega) = T(\omega, p(\omega)).$$

Next, we have to prove that, for each $\omega \in \Omega$, $\{x_n(\omega)\}$ converges strongly to $p(\omega)$ and $p : \Omega \to C$ is a random fixed point of T. In fact, for any given $\omega \in \Omega$, it follows from Lemma 1.2 and (1.1) that

$$
\begin{aligned}
&\|x_{n+1}(\omega)-p(\omega)\|^2\\
&= \|(1-\alpha_n-\gamma_n)(x_n(\omega)-p(\omega))\\
&\quad + \alpha_n(T(\omega, y_n(\omega))-p(\omega)) + \gamma_n(u_n(\omega)-p(\omega))\|^2\\
&\le (1-\alpha_n-\gamma_n)^2\|x_n(\omega)-p(\omega)\|^2\\
&\quad + 2\langle \alpha_n(T(\omega, y_n(\omega))-p(\omega)) + \gamma_n(u_n(\omega)-p(\omega)), J(x_{n+1}(\omega)-p(\omega))\rangle\\
&= (1-\alpha_n-\gamma_n)^2\|x_n(\omega)-p(\omega)\|^2\\
&\quad + 2\alpha_n\langle T(\omega, y_n(\omega))-T(\omega, p(\omega)), J(x_{n+1}(\omega)-p(\omega))\rangle\\
&\quad + 2\gamma_n\langle u_n(\omega)-p(\omega), J(x_{n+1}(\omega)-p(\omega))\rangle\\
&= (1-\alpha_n-\gamma_n)^2\|x_n(\omega)-p(\omega)\|^2\\
&\quad + 2\alpha_n\langle T(\omega, y_n(\omega))-T(\omega, p(\omega)), J(y_n(\omega)-p(\omega))\rangle\\
&\quad + 2\alpha_n\langle T(\omega, y_n(\omega))-T(\omega, p(\omega)), J(x_{n+1}(\omega)\\
&\quad -p(\omega))-J(y_n(\omega)-p(\omega))\rangle\\
&\quad + 2\gamma_n\langle u_n(\omega)-p(\omega), J(x_{n+1}(\omega)-p(\omega))\rangle\\
&\le (1-\alpha_n-\gamma_n)^2\|x_n(\omega)-p(\omega)\|^2 + 2\alpha_n k(\omega)\|y_n(\omega)-p(\omega)\|^2\\
&\quad + 2\alpha_n\langle T(\omega, y_n(\omega))-T(\omega, p(\omega)), J(x_{n+1}(\omega)\\
&\quad -p(\omega))-J(y_n(\omega)-p(\omega))\rangle\\
&\quad + 2\gamma_n\langle u_n(\omega)-p(\omega), J(x_{n+1}(\omega)-p(\omega))\rangle.
\end{aligned}
\tag{2.1}
$$

Now, we consider the second term on the right side of (2.1). From (1.1) and Lemma 1.2, we have

$$
\begin{aligned}
&\|y_n(\omega) - p(\omega)\|^2\\
&= \|(1-\beta_n-\delta_n)(x_n(\omega)-p(\omega)) + \beta_n(T(\omega, x_n(\omega))\\
&\quad - T(\omega, p(\omega))) + \delta_n(v_n(\omega)-p(\omega))\|^2\\
&\le (1-\beta_n-\delta_n)^2\|x_n(\omega)-p(\omega)\|^2\\
&\quad + 2\langle \beta_n T(\omega, x_n(\omega)) - T(\omega, p(\omega))\\
&\quad + \delta_n(v_n(\omega)-p(\omega)), J(y_n(\omega)-p(\omega))\rangle\\
&\le (1-\beta_n-\delta_n)^2\|x_n(\omega)-p(\omega)\|^2\\
&\quad + 2\beta_n\|T(\omega, x_n(\omega) - T(\omega, p(\omega))\|\|y_n(\omega)-p(\omega)\|\\
&\quad + 2\delta_n\langle v_n(\omega)-p(\omega), J(y_n(\omega)-p(\omega))\rangle\\
&\le (1-\beta_n-\delta_n)^2\|x_n(\omega)-p(\omega)\|^2\\
&\quad + \beta_n\{\|T(\omega, x_n(\omega)) - T(\omega, p(\omega))\|^2 + \|y_n(\omega)-p(\omega)\|^2\}\\
&\quad + 2\delta_n\langle v_n(\omega)-p(\omega), J(y_n(\omega)-p(\omega))\rangle,
\end{aligned}
\tag{2.2}
$$

which implies that

$$\begin{aligned} &(1-\beta_n)\|y_n(\omega)-p(\omega)\|^2 \\ &\le (1-\beta_n-\delta_n)^2\|x_n(\omega)-p(\omega)\|^2+\beta_n\|T(\omega,x_n(\omega))-T(\omega,p(\omega))\|^2 \\ &\quad + 2\delta_n\langle v_n(\omega)-p(\omega), J(y_n(\omega)-p(\omega))\rangle. \end{aligned} \tag{2.3}$$

It is easy to see that $1-\beta_n \ge 1-\beta_n-\delta_n$, and since $\beta_n \to 0$, $\delta_n \to 0$ as $n \to \infty$, there exists a nonnegative integer n_0 such that $\beta_n+\delta_n < \frac{1}{2}$ and so $1-\beta_n-\delta_n > \frac{1}{2}$ for all $n \ge n_0$. Therefore, it follows from (2.3) that

$$\begin{aligned} &\|y_n(\omega)-p(\omega)\|^2 \\ &\le (1-\beta_n-\delta_n)\|x_n(\omega)-p(\omega)\|^2+2\beta_n\|T(\omega,x_n(\omega))-T(\omega,p(\omega))\|^2 \\ &\quad + 4\delta_n\langle v_n(\omega)-p(\omega), J(y_n(\omega)-p(\omega))\rangle, \quad n \ge n_0. \end{aligned} \tag{2.4}$$

Since the range of $R(T)$ of T is bounded, there exists a constant $M_1 > 0$ such that

$$\sup_{\omega\in\Omega, x\in C}\|T(\omega,x)\| \le M_1. \tag{2.5}$$

Let $M = \|x_0\| + \sup\{\|u_n(\omega)\| : \omega \in \Omega, n \ge 0\} + \sup\{\|v_n(\omega)\| : \omega \in \Omega, n \ge 0\} + M_1$. Since $p(\omega) \in C$ and $p(\omega) = T(\omega, p(\omega))$, we can easily prove that

$$\begin{aligned} \sup_{\omega\in\Omega, n\ge 0}&\{\|T(\omega,x_n(\omega))\|, \|T(\omega,y_n(\omega))\|, \|T(\omega,p(\omega))\|, \\ &\|x_n(\omega)\|, \|y_n(\omega)\|\} < M. \end{aligned} \tag{2.6}$$

In fact, for $n = 0$, since $x_0 \in C$ and $y_0(\omega) = (1-\beta_0-\delta_0)x_0 + \beta_0 T(\omega, x_0) + \delta_0 v_0(\omega) \in C$, we have

$$\|x_0\| \le M, \quad \|T(\omega,x_0)\| \le M, \quad \|T(\omega,y_0(\omega))\| \le M$$

for each $\omega \in \Omega$. For $n = 1$, we have

$$x_1(\omega) = (1-\alpha_0-\gamma_0)x_0 + \alpha_0 T(\omega, y_0(\omega)) + \gamma_0 u_0(\omega) \in C, \quad \omega \in \Omega,$$

and so, for each $\omega \in \Omega$,

$$\begin{aligned} \|x_1(\omega)\| &\le (1-\alpha_0-\gamma_0)\|x_0\| + \alpha_0\|T(\omega,y_0(\omega)\| + \gamma_0\|u_0(\omega)\| \le M, \\ \|y_1(\omega)\| &\le (1-\beta_1-\delta_1)\|x_1(\omega)\| + \beta_1\|T(\omega,x_1(\omega))\| + \delta_1\|v_1(\omega)\| \le M. \end{aligned}$$

Thus, by induction, we can prove that (2.6) is true. It follows from (2.6) and (2.4) that

$$\|y_n(\omega)-p(\omega)\|^2 \le \|x_n(\omega)-p(\omega)\|^2 + 8M^2\beta_n + 16M^2\delta_n, \quad n \ge n_0. \tag{2.7}$$

Now, we consider the third term on the right side of (2.1). By using (2.6), we have

$$
\begin{aligned}
&2\alpha_n\langle T(\omega, y_n(\omega)) - T(\omega, p(\omega)), J(x_{n+1}(\omega) - p(\omega)) - J(y_n(\omega) - p(\omega))\rangle \\
&\le 2\alpha_n\|T(\omega, y_n(\omega)) - T(\omega, p(\omega))\|\|J(x_{n+1}(\omega) - p(\omega)) \\
&\quad - J(y_n(\omega) - p(\omega))\| \\
&\le 4M\alpha_n\|J(x_{n+1}(\omega) - p(\omega)) - J(y_n(\omega) - p(\omega))\|.
\end{aligned} \tag{2.8}
$$

Since $\alpha_n \to 0$ and $\beta_n \to 0$ as $n \to \infty$, we have

$$
\begin{aligned}
&\|x_{n+1}(\omega) - p(\omega) - (y_n(\omega) - p(\omega))\| \\
&= \|x_{n+1}(\omega) - y_n(\omega)\| \\
&= \|(1 - \alpha_n - \gamma_n)x_n(\omega) + \alpha_n T(\omega, y_n(\omega)) + \gamma_n u_n(\omega) \\
&\quad - \{(1 - \beta_n - \delta_n)x_n(\omega) + \beta_n T(\omega, x_n(\omega)) + \delta_n v_n(\omega)\}\| \\
&\le |\alpha_n + \gamma_n - \beta_n - \delta_n|\|x_n(\omega)\| + M(\alpha_n + \gamma_n \\
&\quad + \beta_n + \delta_n) \to 0 \quad (n \to \infty).
\end{aligned} \tag{2.9}
$$

Since E is uniformly smooth, by Lemma 1.3, we know that the normalized duality mapping $J : E \to 2^{E^*}$ is single-valued and uniformly continuous on any bounded subset of E. Therefore, it follows from (2.9) that

$$
e_n(\omega) := \|J(x_{n+1}(\omega) - p(\omega)) - J(y_n(\omega) - p(\omega))\| \to 0 \quad (n \to \infty) \tag{2.10}
$$

for each $\omega \in \Omega$. From (2.1), (2.6), (2.7), (2.8) and (2.10), we have

$$
\begin{aligned}
&\|x_{n+1}(\omega) - p(\omega)\|^2 \\
&\le (1 - \alpha_n - \gamma_n)^2\|x_n(\omega) - p(\omega)\|^2 \\
&\quad + 2\alpha_n k(\omega)\{\|x_n(\omega) - p(\omega)\|^2 + 8\beta_n M^2 + 16\delta_n M^2\} \\
&\quad + 8M^2\gamma_n + 4M\alpha_n e_n(\omega) \\
&= \{1 - 2\alpha_n(1 - k(\omega))\}\|x_n(\omega) - p(\omega)\|^2 \\
&\quad + \{\alpha_n^2 + \gamma^2 + 2\gamma_n + 2\alpha_n\gamma_n\}\|x_n(\omega) - p(\omega)\|^2 \\
&\quad + 16M^2\alpha_n\beta_n k(\omega) + 32M^2\alpha_n\delta_n k(\omega) + 8M^2\gamma_n + 4M\alpha_n e_n(\omega) \\
&\le \{1 - 2\alpha_n(1 - k(\omega))\}\|x_n(\omega) - p(\omega)\|^2 \\
&\quad + 4M^2\{\alpha_n^2 + \gamma^2 + 2\gamma_n + 2\alpha_n\gamma_n\} \\
&\quad + 16M^2\alpha_n\beta_n k(\omega) + 32M^2\alpha_n\delta_n k(\omega) \\
&\quad + 8M^2\gamma_n + 4M\alpha_n e_n(\omega), \quad n \ge n_0.
\end{aligned} \tag{2.11}
$$

In (2.11), putting

$$
a_n(\omega) = \|x_n(\omega) - p(\omega)\|^2, \quad \lambda_n(\omega) = 2\alpha_n(1 - k(\omega))
$$

$$b_n(\omega) = 4M^2\{\alpha_n^2 + \gamma^2 + 2\gamma_n + 2\alpha_n\gamma_n\}$$
$$+ 16M^2\alpha_n\beta_n k(\omega) + 32M^2\alpha_n\delta_n k(\omega) + 8M^2\gamma_n + 4M\alpha_n e_n(\omega),$$

then we have

$$a_{n+1}(\omega) \leq (1 - \lambda_n(\omega))a_n(\omega) + b_n(\omega), \quad \omega \in \Omega,\ n \geq n_0. \tag{2.12}$$

Since $\alpha_n \to 0$ as $n \to \infty$, there exists a positive integer $n_1 \geq n_0$ such that $\lambda_n(\omega) \in [0,1]$ for all $n \geq n_1$ and $\sum_{n=0}^{\infty} \lambda_n(\omega) = \infty$ for each $\omega \in \Omega$. Further, it is easy to see that $b_n(\omega) = o(\lambda_n(\omega))$ for each $\omega \in \Omega$. Therefore, by Lemma 1.4 and (2.12), for each $\omega \in \Omega$, $a_n(\omega) \to 0$ as $n \to \infty$, that is, for each $\omega \in \Omega$,

$$\|x_n(\omega) - p(\omega)\| \to 0 \quad (n \to \infty).$$

Since C is closed, it follows that p is a mapping from Ω into C. Since $p : \Omega \to C$ is the pointwise limit of the sequence $\{x_n\}$ of measurable functions from $\Omega \to C$, p is also measurable ([8]) and so, by Lemma 1.1, $T(\cdot, p(\cdot)) : \Omega \to C$ is measurable. Since $p(\omega) = T(\omega, p(\omega))$, $p : \Omega \to C$ is a random fixed point of T. This completes the proof. ■

Theorem 2.2. *Let E be a real uniformly smooth separable Banach space, C be a nonempty closed convex subset of E, $T : \Omega \times C \to C$ be a continuous random strongly pseudo-contractive operator with a function $k : \Omega \to (0,1)$ and the range $R(T)$ of T be bounded. Let $x_0 \in C$ be a given point and $\{x_n(\cdot)\}$ be the random Ishikawa iterative sequence with errors defined by* (1.2). *If the sequences $\{\alpha_n\}$, $\{\beta_n\}, \{\gamma_n\}$ and $\{\delta_n\} \subset [0,1]$ appeared in* (1.2) *satisfy the following conditions*:

(*i*) $\alpha_n \to 0$, $\beta_n \to 0$ *as* $n \to \infty$,

(*ii*) $\sum_{n=0}^{\infty} \alpha_n = \infty$,

then the sequence $\{x_n\}$ converges strongly to a random fixed point of T.

Proof. Letting $\gamma_n = \delta_n = 0$ for all $n \geq 0$ in the proof of Theorem 2.1. This completes the proof. ■

3. Applications

As applications, in this section we shall utilize Theorem 2.1 to study the existence of random solutions for some kinds of random nonlinear operator equations in uniformly smooth separable Banach spaces.

Theorem 3.1. *Let X be a real uniformly smooth separable Banach space and $T : \Omega \times X \to X$ be a continuous random strongly accretive operator with a* *function $h : \Omega \to (0,1)$. Let $f : \Omega \to X$ be any given measurable function and a mapping $S : \Omega \times X \to X$ be denoted by $S = I - T + f$. Let $u_n, v_n : \Omega \to C$ be bounded measurable functions for each $n \geq 0$. Let $x_0 \in X$ be a given point and $\{x_n\}$ be the random Ishikawa iterative sequence defined by*

$$\begin{aligned} x_{n+1}(\omega) &= (1-\alpha_n-\gamma_n)x_n(\omega) + \alpha_n S(\omega, y_n(\omega)) + \gamma_n u_n(\omega), \quad n \geq 0, \\ y_n(\omega) &= (1-\beta_n-\delta_n)x_n(\omega) + \beta_n S(\omega, x_n(\omega)) + \delta_n v_n(\omega), \quad n \geq 0, \end{aligned} \tag{3.1}$$

for each $\omega \in \Omega$. If the range of S is bounded and the sequences $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$ and $\{\delta_n\}$ satisfying the conditions (i), (ii) *and* (iii)*in Theorem* 2.1*, then, for any given measurable function $f : \Omega \to X$, the random equation*

$$T(\cdot, x(\cdot)) = f(\cdot) \tag{3.2}$$

has a unique random solution $p : \Omega \to X$ and the random Ishikawa iterative sequence $\{x_n(\omega)\}$ defined by (3.1) *converges strongly to $p(\omega)$ for each $\omega \in \Omega$.*

Proof. By the assumption, $S : \Omega \times X \to X$ is a continuous random strongly pseudo-contractive mapping with a function $k : \Omega \to X$ defined by $k(\omega) = 1 - h(\omega)$. By Theorem 2.1, the random Ishikawa iterative sequence $\{x_n\}$ defined by (3.1) converges strongly to a random fixed point $p : \Omega \to X$ of S for each $\omega \in \Omega$. Therefore, we have

$$S(\omega, p(\omega)) = p(\omega), \quad \omega \in \Omega,$$

that is,

$$T(\omega, p(\omega)) = f(\omega), \quad \omega \in \Omega,$$

which implies that $p(\omega)$ is a random solution of the equation (3.2). This completes the proof. ■

Theorem 3.2. *Let X be a real uniformly smooth separable Banach space and $T : \Omega \times X \to X$ be a continuous random strongly accretive operator with a function $h : \Omega \to (0,1)$. Let $f : \Omega \to X$ be any given measurable function. Denote by $S = I - T + f : \Omega \times X \to X$. Let $x_0 \in X$ be a given point and $\{x_n\}$ be the random Ishikawa iterative sequence defined by*

$$\begin{aligned} x_{n+1}(\omega) &= (1-\alpha_n)x_n(\omega) + \alpha_n S(\omega, y_n(\omega)), \quad n \geq 0, \\ y_n(\omega) &= (1-\beta_n)x_n(\omega) + \beta_n S(\omega, x_n(\omega)), \quad n \geq 0, \end{aligned} \tag{3.3}$$

for each $\omega \in \Omega$. If the range of S is bounded and the sequences $\{\alpha_n\}$ and $\{\beta_n\}$ satisfying the conditions (i) *and* (ii) *in Theorem* 2.2*, then, for any given measurable function $f : \Omega \to X$, the random equation*

$$T(\cdot, x(\cdot)) = f(\cdot)$$

has a unique random solution $p : \Omega \to X$ *and the random Ishikawa iterative sequence* $\{x_n(\omega)\}$ *defined by* (3.3) *converges strongly to* $p(\omega)$ *for each* $\omega \in \Omega$.

Proof. Letting $\gamma_n = \delta_n = 0$ for all $n \geq 0$ in the proof of Theorem 3.1. This completes the proof. ■

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