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SOME FAMILIES OF GENERATING FUNCTIONS FOR THE q-KONHAUSER POLYNOMIALS

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Abstract. The q-Konhauser polynomials, which were introduced and investigated in several recent works, are q-biorthogonal with respect to the weight function $x^\alpha\ e_q(-x)$ over the semi-infinite interval $(0, \infty)$. In the present paper, we derive various families of multilinear and multilateral generating functions for these q-Konhauser polynomials. We also briefly consider several special cases and consequences of the results presented in this paper.

1. Introduction, Definitions and Notations

For a real or complex number $q\ (|q| < 1)$, we denote by $(a;q)_n$ the q-Pochhammer symbol (or, alternatively, the q-shifted factorial) given by (see, for example, [14, pp. 346 *et seq.*])

$$(a;q)_n := \begin{cases} 1 & (n=0) \\ \prod_{j=0}^{n-1} \left\{(1-aq^j)\right\} & (n \in \mathbb{N}) \end{cases} \tag{1.1}$$

and

$$(a;q)_\infty = \prod_{j=0}^{\infty} \left\{(1-aq^j)\right\}. \tag{1.2}$$

Furthermore, the q-derivative operator D_q is defined by

$$D_q\{f(x)\} = \frac{f(qx)-f(x)}{(q-1)x}, \tag{1.3}$$

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so that, clearly, we have

$$D_q\left\{x^a\right\} = [a]_q \ x^{a-1} \qquad \left([a]_q := \frac{1-q^a}{1-q};\ |q|<1;\ a \in \mathbb{R}\right) \tag{1.4}$$

and

$$D_q\left\{e_q(ax)\right\} = a\ e_q(ax) \tag{1.5}$$

for the q-exponential function $e_q(x)$ defined by

$$e_q(x) = \sum_{j=0}^{\infty} \frac{\left((1-q)x\right)^j}{(q;q)_j} = \frac{1}{\left((1-q)x;q\right)_\infty}. \tag{1.6}$$

Let $f(x)$ and $g(x)$ be two piecewise continuous functions in (a,b). Then we have

$$\begin{aligned} D_q\{f(x)\ g(x)\} &= f(x)\ D_q\{g(x)\} + g(x)\ D_q\{f(x)\} \\ &\quad +(q-1)\ x\ D_q\{f(x)\}\ D_q\{g(x)\}. \end{aligned} \tag{1.7}$$

For $q \to 1-$, these definitions would reduce to the corresponding relatively more familiar definitions.

The q-integral of a piecewise continuous function $f(x)$ in (a,b) is defined as follows:

$$\int_a^b f(x)\ d_qx = \sum_{n=0}^{\infty}\left(bq^n - bq^{n+1}\right) f(bq^n) - \sum_{n=0}^{\infty}\left(aq^n - aq^{n+1}\right) f(aq^n) \tag{1.8}$$

and

$$\int_0^\infty f(x)\ d_qx = (1-q) \sum_{j=-\infty}^{\infty} q^j\ f(q^j). \tag{1.9}$$

Moreover, for two piecewise continuous functions $f(x)$ and $g(x)$ in the semi-infinite interval $(0,\infty)$, the q-partial integration is defined by

$$\begin{aligned} &\int_0^\infty f(x) D_q\{g(x)\}\ d_qx \\ &\quad = \lim_{n\to\infty}\left\{f\left(q^{-n}\right)\ g\left(q^{-n}\right) - f\left(q^{n+1}\right)\ g\left(q^{n+1}\right)\right\} \\ &\qquad - \int_0^\infty g(x)\ D_q\{f(x)\}\ d_qx \\ &\qquad -(q-1)\int_0^\infty x\ D_q\{f(x)\}\ D_q\{g(x)\}\ d_qx. \end{aligned} \tag{1.10}$$

For $|q| < 1$, let $w(x;q)$ be a positive weight function which is defined on the set $\{aq^n, bq^n;\ n \in \mathbb{N}_0\}$. If the polynomials $\{P_n(x;q)\}_{n\in\mathbb{N}_0}$ satisfy the following property:

$$\int_a^b P_m(x;q)\ P_n(x;q)\ w(x;q)\ d_qx = \begin{cases} 0 & (m \neq n) \\ \neq 0 & (m = n) \end{cases} \quad (m, n \in \mathbb{N}_0), \tag{1.11}$$

then the polynomials $P_n(x;q)$ are called *q-orthogonal polynomials* with respect to the weight function $w(x;q)$ over the interval (a,b). Using this definition, Moak [7] introduced the q-Laguerre polynomials $L_n^\alpha(x;q)$ given explicitly by

$$L_n^\alpha(x;q) = \frac{(q^{\alpha+1};q)_n}{(q;q)_n} \sum_{j=0}^{n} \frac{(q^{-n};q)_j\ q^{\binom{j}{2}}(1-q)^j\ (q^{n+\alpha+1}x)^j}{(q^{\alpha+1};q)_j\ (q;q)_j} \qquad (\alpha > -1). \tag{1.12}$$

These polynomials are *monic polynomials* in the sense that the leading coefficient of the polynomials is 1. The polynomials $L_n^\alpha(x;q)$ are q-orthogonal polynomials with respect to the weight function $x^\alpha\ e_q(-x)$ over the semi-infinite interval $(0,\infty)$, and we have

$$L_n^\alpha(x;q) \to L_n^{(\alpha)}(x) \quad \text{as} \quad q \to 1-,$$

where $L_n^{(\alpha)}(x)$ are the classical Laguerre polynomials given explicitly by

$$L_n^{(\alpha)}(x) := \sum_{j=0}^{n} \binom{n+\alpha}{n-j} \frac{(-x)^j}{j!}. \tag{1.13}$$

Following the definition of q-orthogonal polynomials involving (1.11), we say that the polynomial sets

$$\{P_n(x;q)\}_{n\in\mathbb{N}_0} \quad \text{and} \quad \{Q_n(x;q)\}_{n\in\mathbb{N}_0}$$

are *q-biorthogonal polynomials* with respect to the weight function $w(x;q)$ over the interval (a,b) if they satisfy the biorthogonality condition given below:

$$\int_a^b P_m(x;q)\ Q_n(x;q)\ w(x;q)\ d_qx = \begin{cases} 0 & (m \neq n) \\ \neq 0 & (m = n) \end{cases} \quad (m, n \in \mathbb{N}_0), \tag{1.14}$$

In the year 1967, using his basic results on the general theory of biorthogonal polynomials presented in [5] (see also [8]), Konhauser [6] introduced the following pair of biorthogonal polynomials (that is, the limit case of the q-biorthogonal polynomials when $q \to 1-$):

$$Y_n^\alpha(x;k) \quad \text{and} \quad Z_n^\alpha(x;k) \quad (k \in \mathbb{N};\ \alpha > -1),$$

which are suggested by the classical Laguerre polynomials $L_n^{(\alpha)}(x)$ given explicitly by (1.13). These polynomial sets are biorthogonal with respect to the weight function $x^\alpha e^{-x}$ $(\alpha > -1)$ over the semi-infinite interval $(0, \infty)$ and were subsequently studied rather extensively by several authors (see, for example, [10], [12] and [13]; see also [15] *and* the references cited in each of these earlier works). It should be remarked in passing that the so-called Konhauser *biorthogonal* polynomials $Z_n^\alpha(x;k)$ of the *second* kind were indeed considered earlier in 1956 by Letterio Toscano (1905-1977), but *without* their biorthogonality property which was emphasized upon in Konhauser's investigation ([5] and [6]).

If, in the q-biorthogonality property (1.14), we take the weight function

$$w(x;q) = x^\alpha \, e_q(-x)$$

over the semi-infinite interval $(0, \infty)$, we obtain the following q-biorthogonal polynomials known as the q-Konhauser polynomials:

$$Z_n^{(\alpha)}(x,k;q) = \frac{(q^{1+\alpha};q)_{nk}}{(q^k;q^k)_n} \sum_{j=0}^{n} \frac{(q^{-nk};q^k)_j q^{\frac{1}{2}kj(kj-1)+kj(n+\alpha+1)}}{(q^k;q^k)_j \; (q^{1+\alpha};q)_{jk}} \, x^{kj} \tag{1.15}$$

and

$$Y_n^{(\alpha)}(x,k;q) = \frac{1}{(q;q)_n} \sum_{r=0}^{n} \frac{x^r q^{\frac{1}{2}r(r-1)}}{(q;q)_r} \sum_{j=0}^{r} \frac{(q^{-r};q)_j (q^{1+\alpha+j};q^k)_n}{(q;q)_j} \, q^j, \tag{1.16}$$

which were considered by Al-Salam and Verma [2] (see also [3], [4] and [11]), who proved that

$$\begin{aligned} &\int_0^\infty Z_n^{(\alpha)}(x,k;q) \, Y_m^{(\alpha)}(x,k;q) \, x^\alpha \, e_q(-x) \, d_q x \\ &\quad = \begin{cases} 0 & (n \neq m) \\ \neq 0 & (n = m). \end{cases} \qquad (m, n \in \mathbb{N}_0), \end{aligned} \tag{1.17}$$

Equation (1.17) does indeed exhibit the fact that the polynomial sets

$$\left\{ Z_n^{(\alpha)}(x,k;q) \right\}_{n \in \mathbb{N}_0} \quad \text{and} \quad \left\{ Y_n^{(\alpha)}(x,k;q) \right\}_{n \in \mathbb{N}_0}$$

are q-biorthogonal polynomials with respect to the weight function $x^\alpha \, e_q(-x)$ over the semi-infinite interval $(0, \infty)$.

For $k = 1$, the q-Konhauser polynomials in (1.15) and (1.16) reduce to the q-Laguerre polynomials given by (1.12). Moreover, we have the following limit relationships with the aforementioned Konhauser polynomials

$$Z_n^\alpha(x;k) \quad \text{and} \quad Y_n^\alpha(x;k) \quad (k \in \mathbb{N}; \; \alpha > -1),$$

which are biorthogonal with respect to the weight function $x^{\alpha}\ e^{-x}$ over the semi-infinite interval $(0, \infty)$:

$$\lim_{q \to 1-} \left\{ Z_n^{(\alpha)}(x, k; q) \right\} = Z_n^{\alpha}(x; k) \tag{1.18}$$

and

$$\lim_{q \to 1-} \left\{ Y_n^{(\alpha)}(x, k; q) \right\} = Y_n^{\alpha}(x; k). \tag{1.19}$$

Jain and Srivastava [4] derived linear and bilinear generating functions for one of these q-Konhauser polynomials and also suggested the following *alternative* pair of q-Konhauser biorthogonal polynomials:

$$z_n^{(\alpha)}(x, k \mid q) = \frac{(\alpha q; q)_{nk}}{(q^k; q^k)_n} \sum_{j=0}^{n} \frac{(q^{-nk}; q^k)_j}{(\alpha q; q)_{kj}} \frac{(xq)^{kj}}{(q^k; q^k)_j} \tag{1.20}$$

and

$$y_n^{(\alpha)}(x, k \mid q) = \frac{1}{(q; q)_n} \sum_{j=0}^{n} \frac{(xq)^j}{(q; q)_j} \sum_{l=0}^{j} \frac{(q^{-j}; q)_l\ (\alpha q^{l+1}; q^k)_n}{(q\,; q)_l}\ q^{(j-n)l}. \tag{1.21}$$

We recall that the q-Konhauser polynomials $Z_n^{(\alpha)}(x, k; q)$ are generated by [2, p. 5, Equation (4.1)]

$$\sum_{n=0}^{\infty} \frac{Z_n^{(\alpha)}(x, k; q)}{(q^{1+\alpha}; q)_{nk}}\ t^n = \frac{f(tx^k)}{(t; q^k)_\infty} \qquad (|t| < 1;\ |q| < 1), \tag{1.22}$$

where, for convenience,

$$f(\tau) = \sum_{j=0}^{\infty} \frac{q^{\frac{1}{2}kj(kj+j+2\alpha)}}{(q^k; q^k)_j\ (q^{1+\alpha}; q)_{kj}}\ (-\tau)^j. \tag{1.23}$$

The object of the present sequel to the aforecited earlier investigations (see also [1]) is to derive various families of multilinear and multilateral generating functions for the q-Konhauser polynomial sets

$$\left\{ Z_n^{(\alpha)}(x, k; q) \right\}_{n \in \mathbb{N}_0} \quad \text{and} \quad \left\{ Y_n^{(\alpha)}(x, k; q) \right\}_{n \in \mathbb{N}_0}.$$

2. The Main Generating Functions

In our investigation, we need each of the following formulas (*cf.* [2]):

$$Z_n^{(\alpha)}(xy,k;q) = \sum_{j=0}^{n} \frac{(q^{1+\alpha},q)_{kn}}{(q^{1+\alpha},q)_{(n-j)k}} \frac{(1/y^k;q^k)_j}{(q^k;q^k)_j} y^{kj} Z_{n-j}^{(\alpha)}(x,k;q) \tag{2.1}$$

and

$$Y_n^{(\alpha)}(x,k;q) = \sum_{j=0}^{n} \frac{(q^k;q^k)_n \ (q;q)_j \ (q^{\alpha-\beta};q^k)_{n-j}}{(q^k;q^k)_j \ (q;q)_n \ (q^k;q^k)_{n-j}} q^{(\alpha-\beta)j} \ Y_j^{(\beta)}(x,k;q). \tag{2.2}$$

Our first set of generating functions for the q-Konhauser polynomials $Z_n^{(\alpha)}(x,k;q)$ are given by Theorem 1 below.

Theorem 1. *Corresponding to a non-vanishing function* $\Omega_\mu(\xi_1,...,\xi_s)$ *of* s *complex variables* $\xi_1,\cdots,\xi_s$ $(s\in\mathbb{N})$ *and of complex order* μ, *let*

$$\begin{aligned}\Lambda_{\mu,\psi,\alpha}^{n,p}(x,y;\xi_1,...,\xi_s,z,r;q) = &\sum_{k=0}^{[n/p]} a_k \ Z_{n-pk}^{(\alpha)}(xy,r;q) \\ &\cdot \Omega_{\mu+\psi k}(\xi_1,\cdots,\xi_s) \ z^k \\ &(a_k \neq 0;\ n\in\mathbb{N}_0;\ p\in\mathbb{N};\ \psi\in\mathbb{C}),\end{aligned} \tag{2.3}$$

where the notation $[n/p]$ *means the greatest integer less than or equal to*

$$\frac{n}{p} \quad (n\in\mathbb{N}_0;\ p\in\mathbb{N}).$$

Then

$$\begin{aligned}&\sum_{k=0}^{n}\sum_{l=0}^{[k/p]} a_l \frac{(q^{1+\alpha},q)_{r(n-pl)}}{(q^{1+\alpha},q)_{r(n-pl)-r(k-pl)}} \frac{(1/y^r;q^r)_{k-pl}}{(q^r;q^r)_{k-pl}} \\ &\quad\cdot y^{r(k-pl)} Z_{n-k}(x,r;q)\cdot\Omega_{\mu+\psi l}(\xi_1,\cdots,\xi_s)\ z^l \\ &= \Lambda_{\mu,\psi,\alpha}^{n,p}(x,y;\xi_1,\cdots,\xi_s,z,r;q)\end{aligned} \tag{2.4}$$

provided that each member of (2.4) *exists.*

Proof. Let $\mathcal{S}$ denote the left-hand side of the assertion (2.4) of Theorem 1.

Then, by some simple calculations, we find that

$$\begin{aligned}\mathcal{S} :&= \sum_{l=0}^{[n/p]} \sum_{k=0}^{n-pl} a_l \frac{(q^{1+\alpha}, q)_{r(n-pl)}}{(q^{1+\alpha}, q)_{r(n-pl)-rk}} \frac{(1/y^r; q^r)_k}{(q^r; q^r)_k} y^{rk} Z_{n-k-pl}^{(\alpha)}(x, r; q) \\ &\quad \cdot \Omega_{\mu+\psi l}(\xi_1, \cdots, \xi_s)\, z^l \\ &= \sum_{l=0}^{[n/p]} a_l\ \Omega_{\mu+\psi l}(\xi_1, \cdots, \xi_s)\, z^l \\ &\quad \cdot \sum_{k=0}^{n-pl} \frac{(q^{1+\alpha}, q)_{r(n-pl)}}{(q^{1+\alpha}, q)_{r(n-pl)-rk}}\ \frac{(1/y^r; q^r)_k}{(q^r; q^r)_k}\ y^{rk}\ Z_{n-k-pl}^{(\alpha)}(x, r; q),\end{aligned}$$

which, in light of the formulas (2.1) and (2.3), yields

$$\begin{aligned}\mathcal{S} &= \sum_{l=0}^{[n/p]} a_l\ \Omega_{\mu+\psi l}(\xi_1, \cdots, \xi_s)\ Z_{n-pl}^{(\alpha)}(xy, r; q)\ z^l \\ &= \Lambda_{\mu,\psi,\alpha}^{n,p}(x, y; \xi_1, \cdots, \xi_s, z, r; q),\end{aligned}$$

which is precisely the right-hand side of the assertion (2.4) of Theorem 1.

In a similar manner, by using the formula (2.2), we can prove the following result.

Theorem 2. *Corresponding to a non-vanishing function* $\Omega_{\mu}(\xi_1, \cdots, \xi_s)$ *of* s *complex variables* $\xi_1, \cdots, \xi_s$ $(s \in \mathbb{N})$ *and of complex order* μ, *let*

$$\Lambda_{\mu,\psi}^{(1)}(\xi_1, \cdots, \xi_s; \tau) = \sum_{k=0}^{\infty} a_k\ \Omega_{\mu+\psi k}(\xi_1, \cdots, \xi_s)\ \tau^k \quad (a_k \neq 0;\ \psi \in \mathbb{C}) \tag{2.5}$$

and

$$\begin{aligned}&\Theta_{n,p}^{\alpha,\mu,\psi}(x; \xi_1, \cdots, \xi_s; \zeta; r; q) \\ &= \sum_{k=0}^{[n/p]} a_k\ \frac{1}{(q^{1+\alpha}; q)_{r(n-pk)}}\ Z_{n-pk}^{(\alpha)}(x, r; q) \\ &\quad \cdot \Omega_{\mu+\psi k}(\xi_1, \cdots, \xi_s)\ \zeta^k \quad (n \in \mathbb{N}_0;\ p \in \mathbb{N}).\end{aligned} \tag{2.6}$$

Then

$$\begin{aligned}&\sum_{n=0}^{\infty} \Theta_{n,p}^{\alpha,\mu,\psi}\left(x; \xi_1, \cdots, \xi_s; \frac{\eta}{t^p}; r; q\right)\ t^n \\ &= \frac{f(tx^r)}{(t; q^r)_\infty}\ \Lambda_{\mu,\psi}^{(1)}(\xi_1, \cdots, \xi_s; \eta),\end{aligned} \tag{2.7}$$

provided that each member of (2.7) *exists*, *the function* f *being defined by* (1.23).

Finally, we present a set of generating functions for the q-Konhauser polynomials $Y_n^{(\alpha)}(x, k; q)$. The proof of Theorem 1, which we have already detailed above fairly adequately, can be applied *mutatis mutandis* to derive of the result contained in Theorem 3 below. Indeed, in place of the formula (2.1) used in proving Theorem 1, the proof of Theorem 3 would make use of the formula (2.2).

Theorem 3. *Corresponding to a non-vanishing function* $\Omega_{\mu}(\xi_1, \cdots, \xi_s)$ *of* s *complex variables* $\xi_1, \cdots, \xi_s$ $(s \in \mathbb{N})$ *and of complex order* μ, *let*

$$\Delta_{\mu,\psi,\alpha}^{n,p}(x; \xi_1, \cdots, \xi_s; r; q) = \sum_{k=0}^{[n/p]} a_k \, Y_{n-pk}^{(\alpha)}(x, r; q) \cdot \Omega_{\mu+\psi k}(\xi_1, \cdots, \xi_s) \, z^k \tag{2.8}$$

$$\left(a_k \neq 0;\; n \in \mathbb{N}_0;\; p \in \mathbb{N};\; \psi \in \mathbb{C}\right).$$

Then

$$\begin{aligned} &\sum_{k=0}^{n} \sum_{l=0}^{[k/p]} a_l \, \frac{(q^r; q^r)_{n-pl} \, (q; q)_{k-pl} \, (q^{\alpha-\beta}; q^r)_{(n-pl)-(k-pl)}}{(q^r; q^r)_{k-pl} \, (q; q)_{n-pl} \, (q^r; q^r)_{(n-pl)-(k-pl)}} \\ &\cdot q^{(k-pl)(\alpha-\beta)} \, Y_{k-pl}^{(\beta)}(x, r; q) \, \Omega_{\mu+\psi l}(\xi_1, \cdots, \xi_s) \, z^l \\ &= \Delta_{\mu,\psi,\alpha}^{n,p}(x; \xi_1, \cdots, \xi_s; r; q), \end{aligned} \tag{2.9}$$

provided that each member of (2.9) *exists.*

3. Concluding Remarks and Observations

For every suitable choice of the coefficients $a_k \quad (k \in \mathbb{N}_0)$, if the multivariable function

$$\Omega_{\mu}(\xi_1, \cdots, \xi_s) \quad (s \in \mathbb{N} \setminus \{1\})$$

is expressed as an appropriate product of several simpler functions, the assertion of each of the above theorems can be applied in order to derive various families of multilinear and multilateral generating functions for the q-Konhauser polynomials

$$Z_n^{(\alpha)}(x, k; q) \quad \text{and} \quad Y_n^{(\alpha)}(x, k; q),$$

as the case may be.

Since

$$Z_n^{(\alpha)}(x, 1; q) = L_n^{\alpha}(x; q) = Y_n^{(\alpha)}(x, 1; q),$$

as we pointed out in Section 1, the special case $r = 1$ of each of our results (Theorems 1 to 3 above) would yield the corresponding families of multilinear and multilateral generating functions for the q-Laguerre polynomials given explicitly by (1.12).

In view of the limit relationships (1.18) and (1.19), each of our results would also yield the corresponding families of multilinear and multilateral generating functions for the Konhauser polynomials $Z_n^\alpha(x;k)$ and $Y_n^\alpha(x;k)$, which were investigated recently by Rassias and Srivastava [10]. Furthermore, since

$$\lim_{q\to 1-}\{L_n^\alpha(x;q)\} = L_n^{(\alpha)}(x),$$

as we pointed out in Section 1, the limit case of each of our results can easily be shown to yield the corresponding families of multilinear and multilateral generating functions for the classical Laguerre polynomials $L_n^{(\alpha)}(x)$, which were considered earlier by Rassias and Srivastava [9]

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