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ON QUASI-ARMENDARIZ MODULES

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Abstract. In this paper, we introduce the concept of a (α-) quasi-Armendariz module, principally quasi-Baer module and syudy its some properties. In particular, we show: (1) For an α-quasi-Armendariz module M_R, M_R is a principally quasi-Baer module if and only if $M[x;\alpha]_{R[x;\alpha]}$ is a principally quasi-Baer module. (2) A necessary and sufficient condition for a trivial extensions to be quasi-Armendariz is obtained. Consequently, new families of quasi-Armendariz rings are presented.

1. Introduction

Throughout this work all rings R are associative with identity and modules are unital right R-modules and $\alpha : R \longrightarrow R$ is an endomorphism of the ring R. In [7] Clark called a ring R *quasi-Baer ring* if the right annihilator of each right ideal of R is generated (as a right ideal) by an idempotent. Recently, Birkenmeier et al. [4] called a ring R *right* (resp. *left*) *principally quasi-Baer* [or simply *right* (resp. *left*) $p.q.$*-Baer*] if the right (resp. left) annihilator of a principal right (resp. left) ideal of R is generated by an idempotent. R is called $p.q.$*-Baer* if it is both right and left $p.q.$-Baer. A ring R is called a *right* (resp. *left*) $p.p.$*-ring* if the right (resp. left) annihilator of every element of R is generated by an idempotent. R is called a $p.p.$*-ring* if it is both a right and left $p.p.$-ring. A ring is called *reduced ring* if it has no nonzero nilpotent elements and M_R is called α*-reduced module* by Lee-Zhou [13] if, for any $m \in M$ and $a \in R$, (1) $ma = 0$ implies $mR \cap Ma = 0$, (2) $ma = 0$ iff $m\alpha(a) = 0$, where $\alpha : R \longrightarrow R$ is a ring endomorphism with $\alpha(1) = 1$. The module M_R is called *a reduced module* if M is 1_R-reduced. It is clear that R is a reduced ring iff R_R is a reduced module.

In [13] Lee-Zhou introduced the following notation. For a module M_R, we consider $M[x;\alpha] = \left\{\sum_{i=0}^{s} m_i x^i \ : \ s \geq 0, m_i \in M\right\}$. This set is an abelian group

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under an obvious addition operation. Moreover $M[x;\alpha]$ becomes a module over $R[x;\alpha]$ under the following scalar product operation:
For $m(x)=\sum_{i=0}^{s} m_i x^i \in M[x;\alpha]$ and $f(x)=\sum_{j=0}^{t} a_j x^j \in R[x;\alpha]$, $m(x)f(x)=\sum_{k=0}^{s+t}\left(\sum_{i+j=k} m_i\alpha^i(a_j)\right)x^k$.
The modules $M[x;\alpha]$ is called the *skew polynomial extension* of M. When α is identity, we write $M[x]_{R[x]}$ for $M[x;1_R]_{R[x;1_R]}$.

According to Lee-Zhou [13] a module M_R is called *α-Armendariz* if the following conditions are satisfied:

(1) For $m\in M$ and $a\in R$, $ma=0$ if and only if $m\alpha(a)=0$,

(2) For any $m(x)=\sum_{i=0}^{s} m_i x^i \in M[x;\alpha]$ and $f(x)=\sum_{j=0}^{t} a_j x^j \in R[x;\alpha]$, $m(x)f(x)=0$ implies $m_i\alpha^i(a_j)=0$ for all i and j.

The module M_R is *Armendariz* iff M_R is 1_R-Armendariz. If M_R is α-reduced then M_R is α-Armendariz.

For a subset X of a module M_R, let $r_R(X)=\{r\in R : Xr=0\}$. In [13] Lee-Zhou introduced Baer modules, quasi-Baer modules and *p.p.*-modules as follows.

(1) M_R is called *Baer* if, for any subset X of M, $r_R(X)=eR$ where $e^2=e\in R$.

(2) M_R is called *quasi-Baer* if, for any submodule N of M, $r_R(N)=eR$ where $e^2=e\in R$.

(3) M_R is called *principally projective* (or simply *p.p.*) if, for any $m\in M$, $r_R(m)=eR$ where $e^2=e\in R$.

2. Quasi-armendariz Modules and Principally Quasi-baer Modules

Our focus in this section is to introduce the concept of a (α-) quasi-Armendariz module, principally quasi-Baer module and study its some properties. It is easy to see that the notation of quasi-Armendariz modules generalize that of Armendariz modules as well as that α-reduced modules. We investigate connections to other related conditions.

Following [16] a ring R is called *Armendariz* if, for any polynomials $f(x)=\sum_{i=0}^{m} a_i x^i$ and $g(x)=\sum_{j=0}^{n} b_j x^j \in R[x]$, $f(x)g(x)=0$ implies $a_ib_j=0$ for all i and j. This notion is generalized by Hirano [8] as the follows; a ring R is called *quasi-Armendariz* if, whenever $f(x)R[x]g(x)=0$, where $f(x)=\sum_{i=0}^{m} a_i x^i$, $g(x)=\sum_{j=0}^{n} b_j x^j \in R[x]$ then $a_iRb_j=0$ for all i and j.

Armendariz rings are quasi-Armendariz. A commutative ring R is Armendariz if and only if it is quasi-Armendariz. The following example shows that there exists a quasi-Armendariz ring R such that R is not Armendariz.

Example 2.1. Let F be a field and consider the ring

$$R = \begin{pmatrix} F & F \\ 0 & F \end{pmatrix}.$$

Then by ([11], Example 1), R is not Armendariz. Since F is a quasi-Armendariz, $R = \begin{pmatrix} F & F \\ 0 & F \end{pmatrix}$ is a quasi-Armendariz by [8, Corollary 3.15].

Following Anderson and Camillo [1], a right R module M is called an *Armendariz module* if, whenever $m(x)f(x) = 0$ where $m(x) = \sum_{i=0}^{n} m_i x^i \in M[x]$ and $f(x) = \sum_{j=0}^{s} a_j x^j \in R[x]$, then $m_i a_j = 0$ for all i and j. Similarly one can define an Armendariz left R-module. Generalizing this definition, we begin the following.

Definition 2.2. A right R-module M is called *quasi-Armendariz* if, whenever $m(x)R[x]f(x) = 0$ where $m(x) = \sum_{i=0}^{n} m_i x^i \in M[x]$ and $f(x) = \sum_{j=0}^{s} a_j x^j \in R[x]$, then $m_i R a_j = 0$ for all i and j.

Clearly, R is a quasi-Armendariz ring if and only if R_R is a quasi-Armendariz right R-module and Armendariz modules are quasi-Armendariz.

Example 2.3. Several easy examples of quasi-Armendariz modules can be given: (1) Every reduced module is a quasi-Armendariz module. (2) For any $n \in \mathbb{Z}$, $\mathbb{Z}_n$ is a quasi-Armendariz $\mathbb{Z}$-module.

Lemma 2.4. *Let M be an R-module.*

(1) *The following are equivalent:*
   (a) For any $m(x) \in M[x]$, $(r_{R[x]}(m(x)R[x]) \cap R)[x] = r_{R[x]}(m(x)R[x])$.
   (b) For any $m(x) = \sum_{i=0}^{n} m_i x^i \in M[x]$ and $f(x) = \sum_{j=0}^{t} a_j x^j \in R[x]$, $m(x)R[x]f(x) = 0$ implies $m_i R a_j = 0$.

(2) *Let M_R be a quasi-Armendariz module and $m(x) \in M[x]$. If $r_{R[x]}(m(x)R[x]) \neq 0$, then $r_{R[x]}(m(x)R[x]) \cap R \neq 0$.*

Proof. (1) $(a) \Rightarrow (b)$ Let $m(x) = \sum_{i=0}^{n} m_i x^i \in M[x]$ and $f(x) = \sum_{j=0}^{t} a_j x^j \in R[x]$ be such that $m(x)R[x]f(x) = 0$. Then $f(x) \in r_{R[x]}(m(x)R[x])$. By (a) $f(x) \in (r_{R[x]}(m(x)R[x]) \cap R)[x]$, and so $a_j \in r_{R[x]}(m(x)R[x]) \cap R$ for all $j = 0, 1, ..., t$. Then $m(x)R[x]a_j = 0$ and so $m_i R a_j = 0$ for all i and j.

$(b) \Rightarrow (a)$ Let $g(x) = \sum_{j=0}^{s} b_j x^j \in (r_{R[x]}(m(x)R[x]) \cap R)[x]$. Then $b_j \in r_{R[x]}(m(x)R[x])$ and so $m(x)R[x]b_j = 0$ for all j. Then $m(x)R[x]g(x) = 0$. Hence $g(x) \in r_{R[x]}(m(x)R[x])$. Therefore $(r_{R[x]}(m(x)R[x]) \cap R)[x] \subseteq r_{R[x]}(m(x)$

$R[x])$. Let $h(x) = \sum_{j=0}^{k} c_j x^j \in r_{R[x]}(m(x)R[x])$. Then $m(x)R[x]h(x) = 0$. By (b) $m_i R c_j = 0$. Therefore $m(x)R[x]c_j = 0$ for all j. Hence $c_j \in r_{R[x]}(m(x)R[x]) \cap R$ for all j, and so $h(x) \in (r_{R[x]}(m(x)R[x]) \cap R)[x]$. Thus $r_{R[x]}(m(x)R[x]) \subseteq (r_{R[x]}(m(x)R[x]) \cap R)[x]$. Hence $(r_{R[x]}(m(x)R[x]) \cap R)[x] = r_{R[x]}(m(x)R[x])$.

(2) Clear from (1) $(b) \Rightarrow (a)$. ∎

A generalization of a zero commutative ring is a semicommutative ring. A ring R is *semicommutative* if $ab = 0$ implies $aRb = 0$ for $a, b \in R$. Historically, some of the earliest results known to us about semicommutative rings (although not so called at the time) was due to Shin [17].

McCoy [15] proved that if R is a commutative ring, then whenever $g(x)$ is a zero-divisor in $R[x]$, there exists a non-zero element $c \in R$ such that $cg(x) = 0$ and Hirano [8] proved that if R is a semi-commutative ring, then whenever $f(x)$ is a zero-divisor in $R[x]$ there exists a non-zero element $c \in R$ such that $f(x)c = 0$. We shall extend these results to module case.

Proposition 2.5. *Let M be a reduced module. If $m'(x)$ is a torsion element in $M[x]$ (i.e. $m'(x)h(x) = 0$ for some $0 \neq h(x) \in R[x]$), then there exists a non-zero element c of R such that $m'(x)c = 0$.*

Proof. Let $m'(x) = \sum_{i=0}^{n} m_i x^i$ and $h(x) = \sum_{j=0}^{s} h_j x^j$ and $m'(x)h(x) = 0$. Then

(1) $m_0 h_0 = 0$;

(2) $m_0 h_1 + m_1 h_0 = 0$;

(3) $m_0 h_2 + m_1 h_1 + m_2 h_0 = 0$;

$\vdots \qquad \vdots$

$(n+s)$ $m_n h_s = 0$.

Note that for a reduced module M for any $m \in M$ and any $a \in R$, $ma = 0$ implies $mRa = 0$ and $ma^2 = 0$ implies $ma = 0$ by Lemma 1.2 in [13]. By (1) $m_0 R h_0 = 0$ since M is reduced. Multiplying (2) by h_0 from the right and using hypothesis we obtain $m_1 R h_0 = 0$ and so $m_0 R h_1 = 0$. Multiplying (3) by h_0 from the right and using hypothesis, from (1) and (2), we have $m_2 h_0 = 0, m_1 h_1 = 0, m_0 h_2 = 0$, and so $m_2 R h_0 = 0, m_1 R h_1 = 0, m_0 R h_2 = 0$. By induction, $m_i R h_j = 0$ for all i and j. Assume that $h(x) \neq 0$. Then at least one of coefficients of $h(x)$ is nonzero, say $h_{j_0} \neq 0$. Then $m'(x) h_{j_0} = 0$. This completes the proof. ∎

Now, we give the following new definition which is connected with Lee-Zhou definitions.

Definition 2.6. The module M is called *principally quasi-Baer module (p.q.-Baer for short)* if, for any $m \in M$, $r_R(mR) = eR$ where $e^2 = e \in R$.

It is clear that R is a right $p.q.$-Baer ring iff R_R is a $p.q.$-Baer module. If R is a $p.q.$-Baer ring, then for any right ideal I of R, I_R is a $p.q.$-Baer module. Every submodule of a $p.q.$-Baer module is $p.q.$-Baer module. Moreover, every quasi-Baer module is $p.q.$-Baer, and every Baer module is quasi-Baer. If R is commutative then M_R is $p.p.$-module iff M_R is $p.q.$-Baer module.

We can give the following definition by considering definition of α-Armendariz module.

M_R is called *α-quasi-Armendariz* if the following conditions are satisfied:

(1) For any $m \in M$ and any $a \in R$, $ma = 0$ if and only if $m\alpha(a) = 0$,

(2) For any $m(x) = \sum_{i=0}^{s} m_i x^i \in M[x;\alpha]$ and $f(x) = \sum_{j=0}^{t} a_j x^j \in R[x;\alpha]$, $m(x)R[x;\alpha]f(x) = 0$ implies $m_i R\alpha^i(a_j) = 0$ for all i and j.

Note that the module M_R is quasi-Armendariz if and only if M_R is 1_R-quasi-Armendariz.

Theorem 2.7. *Let M be an α-quasi-Armendariz module. Then M_R is a p.q.-Baer module if and only if $M[x;\alpha]_{R[x;\alpha]}$ is a p.q.-Baer module.*

Proof. Assume that $M[x;\alpha]_{R[x;\alpha]}$ is a $p.q.$-Baer module. Let $m \in M$. Then there exists an idempotent $f(x) \in R[x;\alpha]$ such that $r_{R[x;\alpha]}(mR[x;\alpha]) = f(x)R[x;\alpha]$. Note that $f(x)R[x;\alpha] \subseteq r_{R[x;\alpha]}(mR) = r_R(mR)[x;\alpha]$ always holds. Let $g(x) = b_0 + ... + b_t x^t \in r_R(mR)[x;\alpha]$. Then $mRb_j = 0$ for all $0 \leq j \leq t$. By hypothesis $mR\alpha^i(b_j) = 0$ for all i and $0 \leq j \leq t$. Let $h(x) = \sum_{k=0}^{s} c_k x^k \in R[x;\alpha]$. Then $mh(x)b_j = \sum_{k=0}^{s} mc_k\alpha^k(b_j)x^k = 0$ for all j, and so $mh(x)g(x) = 0$ for all $h(x) = \sum_{k=0}^{s} c_k x^k \in R[x;\alpha]$. Hence $g(x) \in r_{R[x;\alpha]}(mR[x;\alpha])$. Thus $r_{R[x;\alpha]}(mR[x;\alpha]) = f(x)R[x;\alpha] = r_R(mR)[x;\alpha]$. Let $f(x) = a_0 + a_1x + ... + a_nx^n$ where all $a_i \in r_R(mR)$. Note that, for any $a \in r_R(mR)$, $f(x)a = a$. Hence $f(x)a = (a_0 + a_1x + ... + a_nx^n)a = a_0a + a_1xa + ... + a_nx^na = a$ implies that $a = a_0a$. Since $a_0^2 = a_0$ and $r_R(mR) = a_0R$, M_R is a $p.q.$-Baer module.

For the converse, assume that M_R is $p.q.$-Baer. Let $m(x) = m_0 + m_1x + ... + m_nx^n \in M[x;\alpha]$. Then $r_{R[x]}(m(x)R[x]) = (r_{R[x]}(m(x)R[x]) \cap R)[x] = r_R(m(x)R[x])[x]$ by Lemma 2.4. Let C_{mR} the set of all coefficients of $m(x)R[x]$, i.e., $C_{mR} = \{m_iR : i = 0, ..., n\}$. $r_{R[x]}(m(x)R[x]) \cap R = r_R(m(x)R[x]) = r_R(C_{mR})$. Since M_R is $p.q.$-Baer, $r_R(C_{mR}) = \cap_{i=0}^{n} r_R(m_iR) = \cap_{i=0}^{n} e_iR$, where $e_i^2 = e_i \in R$ and $r_R(m_iR) = e_iR$. We claim that $\cap_{i=0}^{n} e_iR = eR$, where $e^2 = e \in R$. Since $m_1Re_1 = 0$, $m_1Re_0e_1 = 0$ and so $e_0e_1 \in r_R(m_1R) = e_1R$.

Thus $e_1e_0e_1 = e_0e_1$. Let $f_1 = e_0e_1$ Then $f_1^2 = (e_0e_1)(e_0e_1) = e_0e_1 = f_1$ and $e_0R \cap e_1R = f_1R$. Since $m_2Re_2 = 0$, $m_2Rf_1e_2 = 0$ and so $f_1e_2 \in r_R(m_2R) = e_2R$. Hence $e_2f_1e_2 = f_1e_2$. Let $f_2 = f_1e_2$. Then $f_2^2 = f_2$ and $f_1R \cap e_2R = f_2R$. Continuing this process, we obtain $f_n^2 = f_n \in R$ such that $\cap_{i=0}^{n} e_iR = f_nR$. Thus $r_{R[x;\alpha]}(m(x)R[x;\alpha]) = r_R(C_mR)[x;\alpha] = f_nR[x;\alpha]$. ∎

Theorem 2.8. *Let M_R be a reduced module. Then the following statements are equivalent;*

(1) *M_R is a p.p.-module.*

(2) *M_R is a p.q.-Baer module.*

(3) *$M[x]_{R[x]}$ is a p.p.-module.*

(4) *$M[x]_{R[x]}$ is a p.q.-Baer module.*

Proof. (1) ⇔ (3) By [13, Corollary 2.12].

(2) ⇔ (4) Clear by Theorem 2.7 since every reduced module is quasi-Armendariz.

(1) ⇔ (2) Let $m \in M$. If $a \in r_R(m)$ then $ma = 0$ and by [13, Lemma 1.2], $mRa = 0$ and so $a \in r_R(mR)$. Then $r_R(m) \subseteq r_R(mR)$. But $r_R(mR) \subseteq r_R(m)$ obviously holds. Consequently, $r_R(mR) = r_R(m) = eR$. Hence the claim follows. ∎

3. When Is a Trivial Extension Quasi-armendariz?

Given a ring R and a bimodule ${}_RM_R$, *the trivial extension* of R by M is the ring $T(R, M) = R \oplus M$ with the usual addition and multiplication

$$(r_1, m_1)(r_2, m_2) = (r_1r_2, r_1m_2 + m_1r_2).$$

This is isomorphic to the ring of all matrices $\begin{pmatrix} a & m \\ 0 & a \end{pmatrix}$ where $a \in R, m \in M$.

Lemma 3.1. ([14, Lemma 2.1]) *Let M be an (R, R)-bimodule. Then $M[x]$ is an $(R[x], R[x])$-bimodule and $T(R[x], M[x]) = T(R, M)[x]$.*

Proposition 3.2. *Let M be an (R, R)-bimodule. If the trivial extension $T(R, M)$ is a quasi-Armendariz ring, then M is a quasi-Armendariz left and right R-module.*

Proof. Let $m(x) = m_0 + m_1x + ... + m_sx^s \in M[x]$, $f(x) = a_0 + a_1x + ... + a_nx^n \in R[x]$ and suppose that $f(x)R[x]m(x) = 0$. For an arbitrary $c \in R$, $n \in M$

we have the following equation:

$$\left[\sum_{i=0}^{n}\begin{pmatrix} a_i & 0 \\ 0 & a_i \end{pmatrix}x^i\right]\begin{pmatrix} c & n \\ 0 & c \end{pmatrix}\left[\sum_{j=0}^{s}\begin{pmatrix} 0 & m_j \\ 0 & 0 \end{pmatrix}x^j\right]$$
$$=\begin{pmatrix} f(x) & 0 \\ 0 & f(x) \end{pmatrix}\begin{pmatrix} c & n \\ 0 & c \end{pmatrix}\begin{pmatrix} 0 & m(x) \\ 0 & 0 \end{pmatrix}$$
$$=\begin{pmatrix} f(x)c & f(x)n \\ 0 & f(x)c \end{pmatrix}\begin{pmatrix} 0 & m(x) \\ 0 & 0 \end{pmatrix}=\begin{pmatrix} 0 & f(x)cm(x) \\ 0 & 0 \end{pmatrix}=0.$$

Since $T(R, M)$ is quasi-Armendariz,

$$\begin{pmatrix} a_i & 0 \\ 0 & a_i \end{pmatrix}\begin{pmatrix} c & n \\ 0 & c \end{pmatrix}\begin{pmatrix} 0 & m_j \\ 0 & 0 \end{pmatrix}=0$$

for all i and j. Therefore $a_i c m_j = 0$ for all i and j. Consequently, M is a quasi-Armendariz left R-module. Similarly, M is a quasi-Armendariz right R-module. ■

Letting ${}_RM_R = {}_RR_R$ yields the following:

Corollary 3.3. *If the trivial extension $T(R, R)$ is a quasi-Armendariz ring, then also R is quasi-Armendariz.*

Theorem 3.4. *Let M be an (R, R)-bimodule such that*

(1) *R is a quasi-Armendariz ring.*

(2) *M is an Armendariz left and quasi-Armendariz right R-module.*

(3) *If $f(x)Rg(x) = 0$ in $R[x]$, then $f(x)M[x] \cap M[x]g(x) = 0$.*

Then the trivial extension $T(R, M)$ is a quasi-Armendariz ring.

Proof. Suppose that $\alpha(x)T(R, M)\beta(x) = 0$ where

$$\alpha(x)=\begin{pmatrix} a_0 & m_0 \\ 0 & a_0 \end{pmatrix}+\begin{pmatrix} a_1 & m_1 \\ 0 & a_1 \end{pmatrix}x+\ldots+\begin{pmatrix} a_n & m_n \\ 0 & a_n \end{pmatrix}x^n \in T(R, M)[x],$$

$$\beta(x)=\begin{pmatrix} b_0 & l_0 \\ 0 & b_0 \end{pmatrix}+\begin{pmatrix} b_1 & l_1 \\ 0 & b_1 \end{pmatrix}x+\ldots+\begin{pmatrix} b_s & l_s \\ 0 & b_s \end{pmatrix}x^s \in T(R, M)[x],$$

Let

$$f(x) = a_0 + a_1x + \ldots + a_nx^n,\ g(x) = b_0 + b_1x + \ldots + b_sx^s,$$
$$m(x) = m_0 + m_1x + \ldots + m_nx^n,\ l(x) = l_0 + l_1x + \ldots + l_sx^s.$$

Then $f(x), g(x) \in R[x]$ and $m(x), l(x) \in M[x]$. For an arbitrary $\begin{pmatrix} a & m \\ 0 & a \end{pmatrix} \in T(R, M)$, it follows that

$$0 = \begin{pmatrix} f(x) & m(x) \\ 0 & f(x) \end{pmatrix} \begin{pmatrix} a & m \\ 0 & a \end{pmatrix} \begin{pmatrix} g(x) & l(x) \\ 0 & g(x) \end{pmatrix}$$
$$= \begin{pmatrix} f(x)ag(x) & f(x)al(x) + f(x)mg(x) + m(x)ag(x) \\ 0 & f(x)ag(x) \end{pmatrix}.$$

Thus $f(x)ag(x) = 0$ and $f(x)al(x) + f(x)mg(x) + m(x)ag(x) = 0$. Since $a \in R$ arbitrary, $f(x)Rg(x) = 0$. Since R is a quasi-Armendariz by (1), $a_iRb_j = 0$ for all i and j. Since $f(x)[al(x) + mg(x)] + [m(x)a]g(x) = 0$, $f(x)[al(x) + mg(x)] = -[m(x)a]g(x) \in f(x)M[x] \cap M[x]g(x) = 0$, so $f(x)[al(x) + mg(x)] = [m(x)a]g(x) = 0$. since $a \in R$ arbitrary $m(x)Rg(x) = 0$. Then by (2), $m_iRb_j = 0$ for all i and j. And $f(x)[al(x)] = -[f(x)m]g(x) \in f(x)M[x] \cap M[x]g(x) = 0$ by (3).So $f(x)al(x) = 0$ and hence $f(x)Rl(x) = 0$. Then by (2), M is an Armendariz left R-module and hence M is a quasi-Armendariz left R-module. Therefore $a_iRl_j = 0$ for all i and j. For arbitrary $m \in M$, we have $f(x)mg(x) = 0$. But $f(x)m \in M[x]$ and since M is an Armendariz left R-module by (2), we obtain $a_imb_j = 0$ for all i and j. Therefore

$$\begin{pmatrix} a_i & m_i \\ 0 & a_i \end{pmatrix} \begin{pmatrix} c & n \\ 0 & c \end{pmatrix} \begin{pmatrix} b_j & l_j \\ 0 & b_j \end{pmatrix} = \begin{pmatrix} a_icb_j & a_icl_j + a_inb_j + m_icb_j \\ 0 & a_icb_j \end{pmatrix} = 0$$

for all i, j and $\begin{pmatrix} c & n \\ 0 & c \end{pmatrix} \in T(R, M)$. Consequently the trivial extension $T(R, M)$ is a quasi-Armendariz ring. ■

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